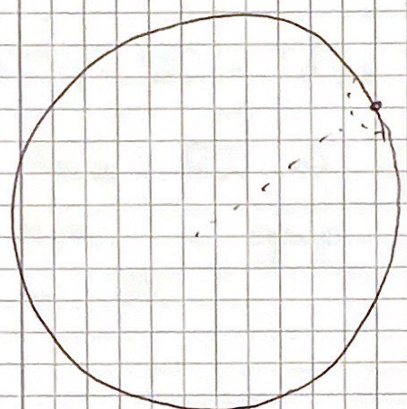
Genevieve Walsh   Talk 2

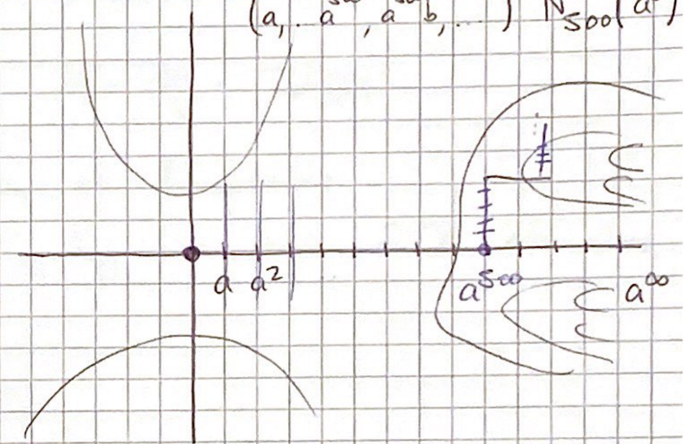
Topology on the boundary

X proper geodesic hyp metric space, $x \in \partial X$.

$N_r(x) = \{ y \mid (x,y)_w > r \}$

$(x,y)_w = \sup_{\substack{x_i \to x \\ y_i \to y}} \{ \liminf (x_i, y_i) \}$ for $y \in \partial X$



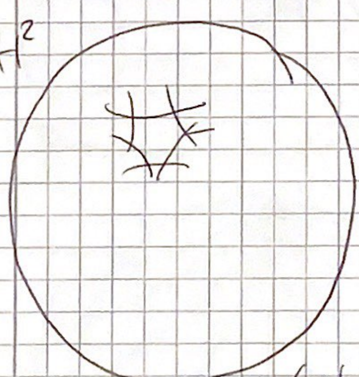


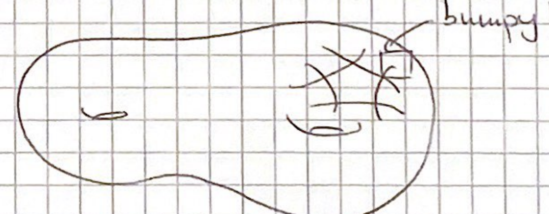
$(x,y)_w = \sup_{x_i \to x} \{ \liminf (x_i, y) \}$ for $y \in X$.

For the example of F_2, topology on the boundary is the cantor set (exercise!)

The boundary is a QI invariant

$\mathbb{H}^2$





$f: X \to X'$ is a (a,b) quasi-isometry between hyp. metric spaces

∂X = quasi-geo / bounded Hsdff distance

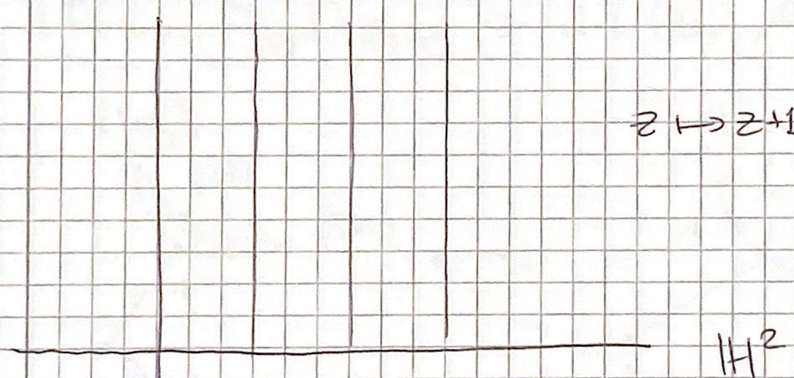
$\partial f : \partial X \to \partial X'$ defined by $[\gamma] \to [f(\gamma)] \in \partial X'$.
Does not depend on the choice of γ.
$\gamma_1 \sim \gamma_2 \Rightarrow d_H(\gamma_1, \gamma_2) < c$

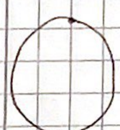
$d_H(f(\gamma_1), f(\gamma_2)) \leq ac + b$ $\quad g : X' \to X$ inverse

$d_H(g \circ f(\gamma), \gamma)$ is bounded so $[g \circ f(\gamma)] = [\gamma]$.

$\Rightarrow \partial g \circ \partial f = Id_{\partial X} \qquad \partial f \circ \partial g = Id_{\partial X'}$

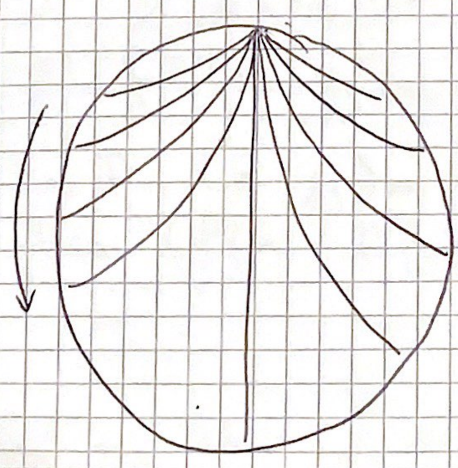
Hence QI $\Rightarrow$ bij map on boundaries
Note: a group acting by isometries on X also acts on ∂X.





$\partial \mathbb{H}^2 \setminus \{0\} / G = \bigcirc$

Poincaré model



Continuity $f : X \to X'$ a λ-c quasi-isometry

$x_i \to \bar{x}$ $y_i \to \bar{y}$

Fact X Gromov hyp $\exists \delta$ $(x, y)_p \leq d(p, [x,y])$
$\leq (x,y)_p + \delta$

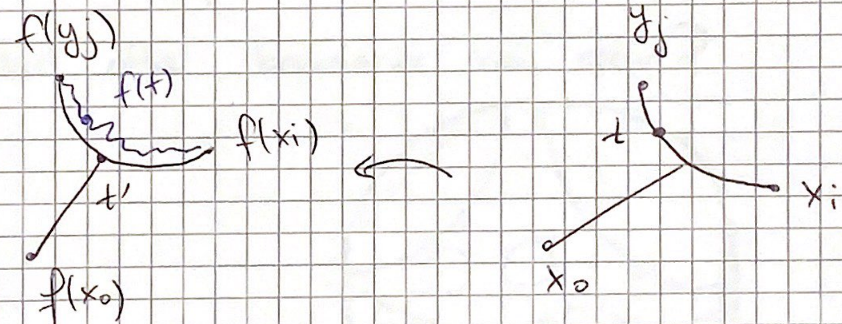
in X': $(f(x_i), f(y_j))_{f(x_0)} \geq d(f(x_0), [f(x_i), f(y_j)]) - \delta$

$= d(f(x_0), t') - \delta$ for some $t' \in [f(x_i), f(y_j)]$.

$[x_i, y_j]$ maps to a quasi geodesic

$\exists t \in [x_i, y_j]$: $f(t)$ is close to t'.

$(f(x_i), f(y_j))_{f(x_0)} \geq d(f(x_0), f(t)) - K - \delta$



λ-c q.i. $\geq \frac{1}{\lambda} d(x_0, t) - c - K - \delta$

$\geq \frac{1}{\lambda} d(x_0, [x_i, y_j]) - c - K - \delta$

$\geq \frac{1}{\lambda} (x_i, y_j)_{x_0} - (c + K + \delta)$

So, if $(x_i, y_j)_{x_0} > \lambda(n + c + K + \delta)$
then $(f(x_i), f(y_j))_{f(x_0)} > \frac{1}{\lambda}(\lambda(n + c + K + \delta)) - (c + K + \delta)$

$= n$

Cont map btwn cpt Hsdff spaces $\Rightarrow$ homeomorphism.

Why is this great?

G acts geometrically on X, G acts geometrically on Y, G hyperbolic, then $\partial X \cong \partial Y$

We can define $\partial G = \partial X \cong \partial Y$

This opens the field

Look at ∂X, what possible G have this boundary?

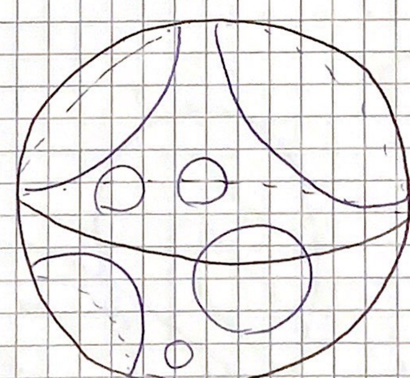
If $\partial G = \bullet \quad \bullet$ then G is virtually $\mathbb{Z}$

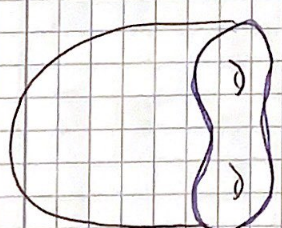
$\partial G =$ cantor set then G is virtually free ($rk \geq 2$)

$\partial(\pi_1(S_g)) = S^1$

<u>Theorem</u> (Tukia, Gabai, ...) if $\partial\Gamma \cong S^1$ then Γ is virtually Fuchsian.

What other boundaries can occur?

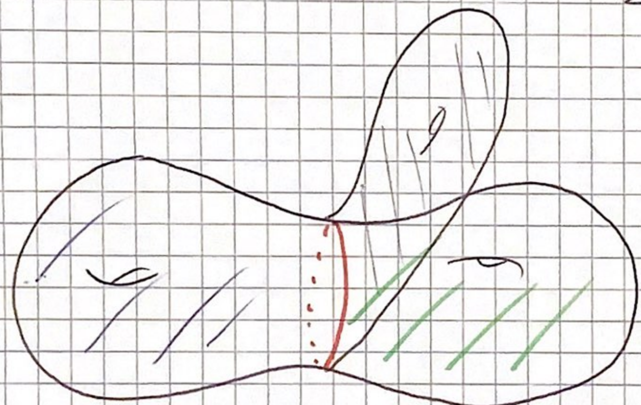




$\partial(\pi_1(M^3$ w/ t.g boundary$))$ is a Sierpinski carpet

__Fact__ connected hyperbolic boundaries do not have cut points.

(Bowditch) local cut points $\Longleftrightarrow$ splittings over $\mathbb{Z}$



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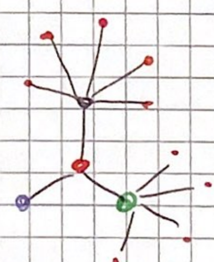

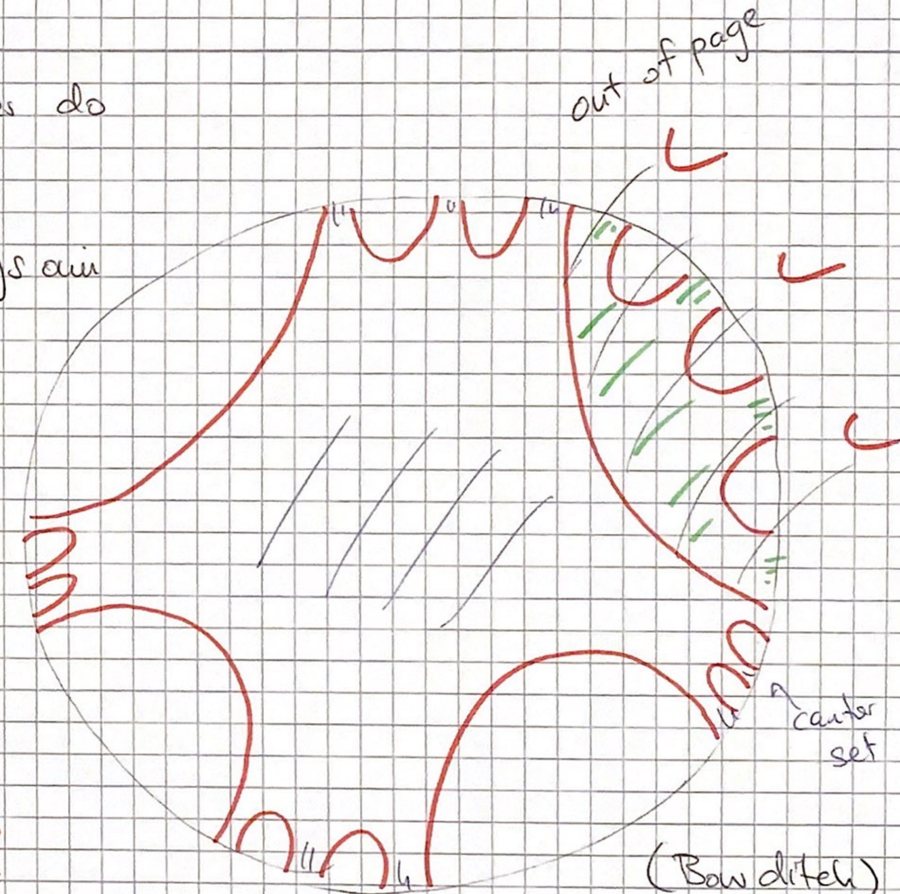

(Bowditch)

This splitting yields vertex stabilisers

- elementary ($\mathbb{Z}$)
- hanging Fuchsian
- rigid (don't admit any more splittings)