Recent developments in lattice and effective field theory for hard probes

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Physical Motivation on the lattice

The quark-antiquark potential at $T > 0$

- Real time spectral function for quark-antiquark systems extracted from quarks moving in imaginary time on the lattice
  - Zero T subtraction: Rasmus Larsen, Peter Petreczky, Johannes Heinrich Weber
  - Pade fit: Gaurang Parkar, Alexander Rothkopf

The Heavy Quark Momentum Diffusion Coefficient κ

- In effective theory to leading order in $1/m$, current current correlator, related to the heavy quark momentum diffusion coefficient κ, can be expressed in a color-Electric correlators, now done with dynamical quarks
  - Luis Altenkort, Hai-Tao Shu, Olaf Kaczmarek, Peter Petreczky

Heavy Quakonium at $T > 0$

- NRQCD used to extract behavior above critical temperature, signal improved by using sources with finite size
  - Extended sources: Stefan Meinel, Rasmus Larsen, Swagato Mukherjee, Peter Petreczky
  - Bayesian BR: Seyong Kim, Alexander Rothkopf

Lattice connection to real time physics

- All results in this talk uses that real time spectral functions $\rho(\omega)$ can be expressed in an imaginary time formulation on the lattice

$$C(\tau) \quad = \quad \int_{-\infty}^{\infty} \rho(\omega) K(\omega,\tau) d\omega \tag{1}$$

- $C(\tau)$ is the correlation on the lattice

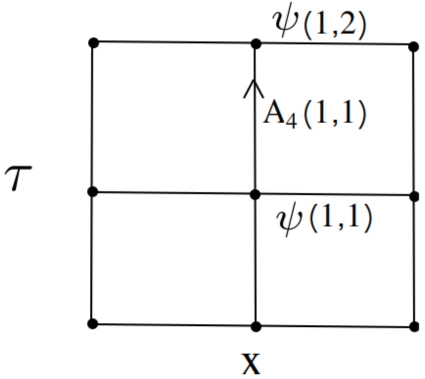
- $\rho(\omega)$ is the real time spectral function

- $K(\omega,\tau)$ is a kernel that transforms spectral information into correlation information
- For real time $K = \exp(i\omega t)$
- For imaginary time non-periodic effective-theories measurements $K = \exp(-\omega\tau)$
- For imaginary time periodic measurements $K = \frac{\cosh(\omega\tau - \omega/(2T))}{\sinh(\omega/(2T))}$

Lattice QCD

- The gauge fields A_μ are on the lattice exponentiated to a link $U_\mu = \exp(iaA_\mu)$

- Fermions ψ lives on the lattice points, and $U_\mu(x)$ lives between the points connecting the fermions at $x+\mu$ to x



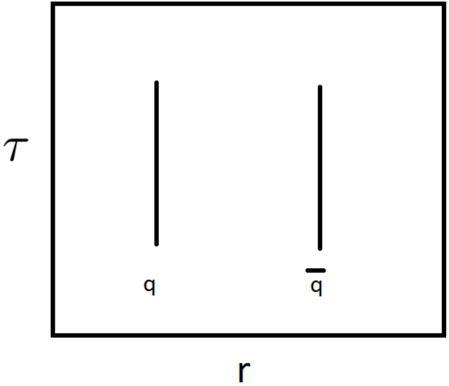
$$Z = \int DU_\mu D\psi D\bar{\psi} \exp(-S_E(U_\mu, \psi, \bar{\psi}))$$

- Electric and magnetic interactions $F_{\mu\nu}$ comes from links going around in a circle in the μ, ν plane

The quark-antiquark potential at $T > 0$

- The quark-antiquark potential is found from static quarks moving through time
- Transport a quark(antiquark) in time is done with $U_4(U_4^\dagger)$
- transportation of several lattice sites is called a wilson line W

$$W(t,x) \quad = \quad \prod_{i}^{t} U_4(i,x) \tag{2}$$



- The real time spectral function $\rho(\omega, r)$ is related to the correlation of 2 wilson lines

$$C(\tau, r) =< Tr(W(\tau,0)W(\tau,r)^\dagger) >= \int \rho(\omega, r)\exp(-\omega\tau)d\omega \tag{3}$$

Extracting $\rho(\omega, r)$

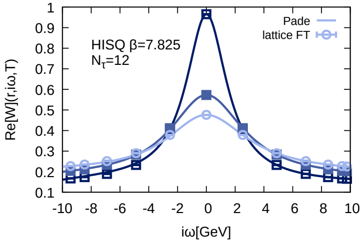
- Extracting $\rho(\omega, r)$ from $C(\tau, r)$ is an inversion problem
- We will focus on the results from 2 techniques at finite temperature T

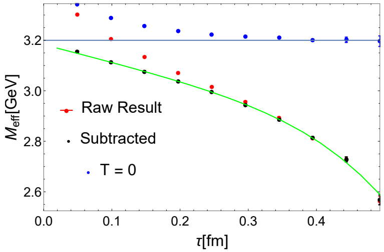
Pade fit

- Fit rational function to fourier transform of correlator $C(\tau, r) \to C(\omega_n, r)$
- Rotate $i\omega_n \to \omega$ to obtain spectral function

Zero temperature subtraction

- Find continuum at zero temperature and subtract it from finite T correlator
- Fit simple spectral function to subtracted correlator to extract position and width

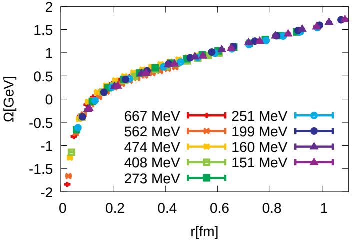




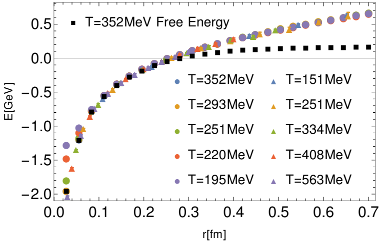
Energy from Wilson Line Correlator

- Real part of potential E or Ω found from peak position of spectral function $\rho(\omega, r)$
- Almost no change in position of peak E [arxiv:2110.11659]
- New results on larger lattices 96^3 included for zero temperature subtraction

Pade fit



Zero temperature subtraction

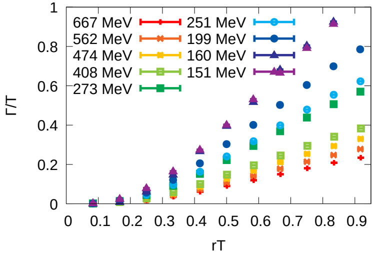


- No Screening in the potential observed
- Distortions observed on large lattice at small r due to the use of smearing

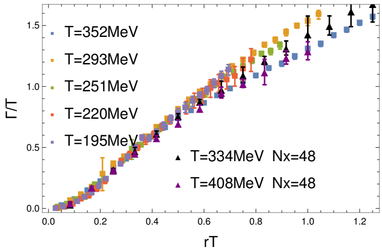
Width from Wilson Line Correlator

- Width Γ of spectral function $\rho(\omega, r)$ dependent on shape of model
- Consistency seems to be in the cumulants of the correlator where the second cumulant $\langle\omega^2\rangle - \langle\omega\rangle^2$ controls the width

Pade fit



Zero temperature subtraction



The Heavy Quark Momentum Diffusion Coefficient

- The heavy quark momentum diffusion coefficient κ controls the size of random kicks to quarks coming from interactions with the medium
- For an effective theory where $M >> T$ and Λ_{QCD} integrating out the heavy quarks, gives κ from a color-electric correlator, instead of a current current correlator

$$G_E(\tau) \quad = \quad -\sum_{i=1}^{3} \frac{\langle ReTr[U(1/T,\tau)E_i(\tau,0)U(\tau,0)E_i(0,0)]\rangle}{3ReTr[U(1/T,0)]} = \int_0^\infty \rho(\omega) \frac{\cosh(\omega\tau - \omega/(2T))}{\sinh(\omega/(2T))} \frac{d\omega}{\pi} \tag{4}$$

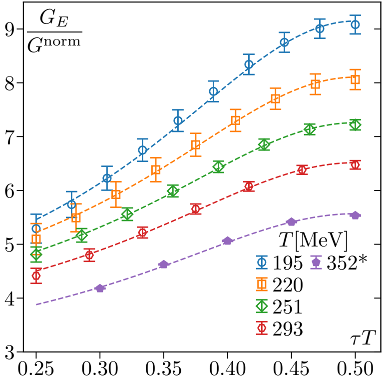
- $U(\tau_1, \tau_2)$ is the temporal wilson line between τ_1 and τ_2 that connects the color-electric E_i measurements.
- κ related to color-electric correlators spectral function at $\lim_{\omega\to 0} \rho(\omega)/\omega$
- $\lim_{\omega\to 0} \rho(\omega)/\omega$ hard to obtain due to inversion problem

Extracting the signal from noisy lattices

- κ extracted from $2+1$ simulations on fine lattices $96^3 N_\tau$ by Altenkort et. al. [Arxiv:2302.08501]
- light quarks slightly heavy $m_s/m_l = 5$, physical corresponds to ratio of 27.
- See talk by Olaf Kaczmarek "The heavy quark diffusion coefficient from 2+1 flavor lattice QCD"

- Noisy correlator $G_E(\tau)$ extrapolated to the continuum and zero smearing
- 3 different ansatz used to interpolate low ω and high ω regions

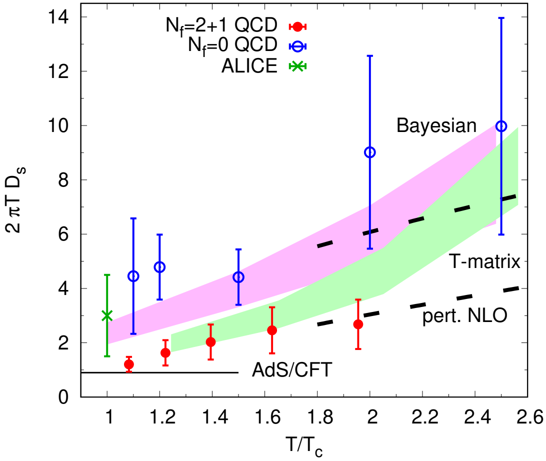
$$\begin{aligned} \rho_{low} &= \kappa\omega/(2T) \\ \rho_{high} &= K\rho_{pert,LO} \end{aligned}$$



Diffusion coefficient

- Results for $D_s = 2T^2/\kappa$ shows lower than quenched behavior

- $6D_s$ is the mean distance squared traveled by unit time
- T-Matrix results updated compared to figure in paper, R. Rapp et al. [arxiv:1612.09318][arxiv:1711.03282]



Heavy Quarkonium at $T > 0$

- We want to learn about heavy quarkoniums behavior in high temperature mediums
- We do this by creating a quarkonium state at $\tau = 0$ and then propagate it though imaginary time

$$\begin{aligned} \int d^3x < O(\tau, x) O^\dagger(0,0) > &= C(\tau) \\ \int_{-\infty}^{\infty} \rho(\omega) \exp(-\omega\tau) d\omega &= \end{aligned} \tag{5}$$

- We are interested in the zero momentum state, so we integrate over space
- Bottomonia too heavy for first principle lattice calculations
- Non Relativistic QCD used, since it only cares about energy differences

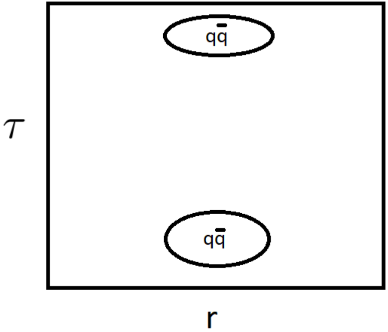
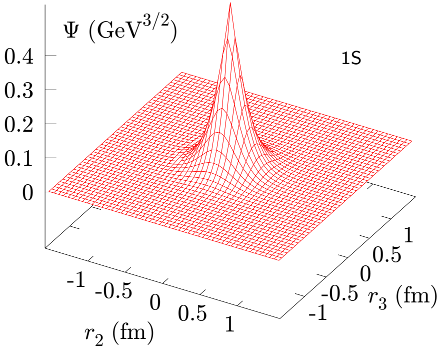


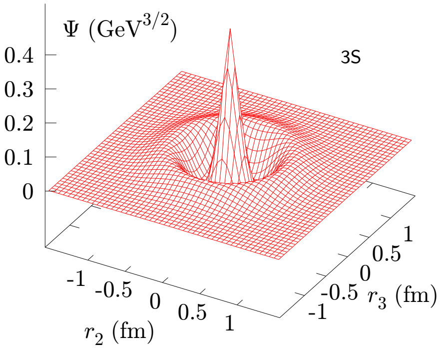
Figure: Illustration of a Upsilon correlation measurement.

Extended Sources

- Sources that approximate the shape of the state, improves signal
- Source are calculated from discretized schroedinger equation with confining potential that reproduces zero temperature spectrum

$$O_i(\mathbf{x},t) = \sum_{\mathbf{r}} \Psi_i(\mathbf{r})\bar{q}(\mathbf{x}+\mathbf{r},t)\Gamma q(\mathbf{x},t) \tag{6}$$



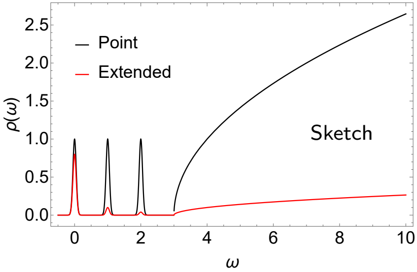


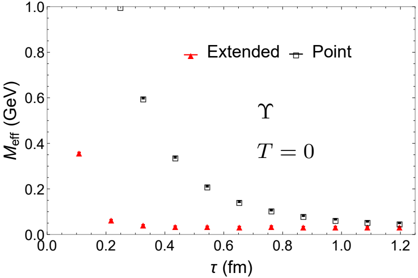
Effect of extended sources

- Extended sources projects into specific state better

- Plateaus of the effective mass $M_{eff} -> $ Mass state exists in $\rho(\omega)$

$$M_{eff} = \frac{1}{a}\log[C(\tau)/C(\tau+a)] = -\frac{\partial}{\partial_\tau}\log(C(\tau)) \qquad (7)$$





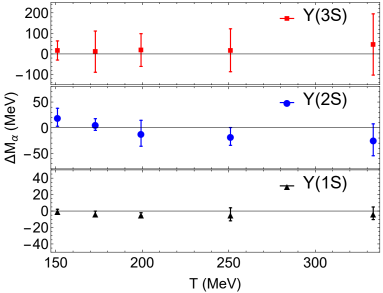
Mass of Bottomonia states at finite temperature

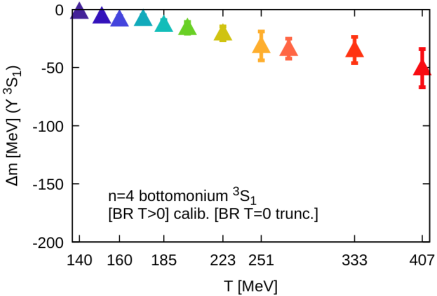
Extended Sources Result

- Results from extended sources consistent with no shift to peak position
- Larsen et al[arXiv:1910.07374]

Bayesian Method

- Results from Bayesian BR method shows a decrease up to $40MeV$ for ground state
- Seyong Kim et al [arxiv:1808.08781]

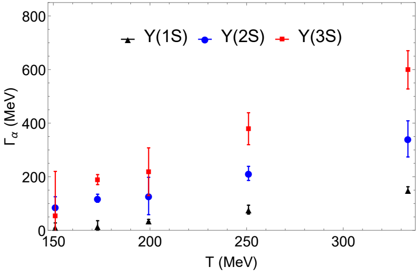


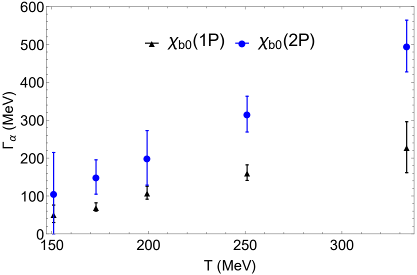


Spectral Width

Extended Sources Result

- Results from extended sources shows that the larger the state, the larger the width
- Larsen et al[arXiv:1910.07374]
- Extended sources allows for projection onto excited states of Υ and χ_b





Summary

- Lattice calculations are able to constrain parameters for effective models from first principle calculations
  - The heavy quark momentum diffusion coefficient has been calculated with dynamical quarks
  - Recent studies of lattice QCD with dynamical fermions indicate no screening in static quark-antiquark potential for $T > T_c$
  - Large spectral width, indicating strong nonlinear interactions with the medium

Potential at $T > 0$

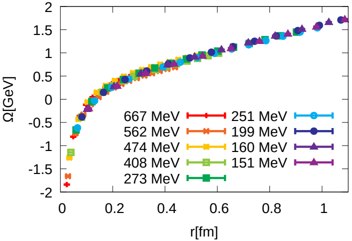
Lattice QCD simulations with dynamical fermions show no screening for the static quark-antiquark potential

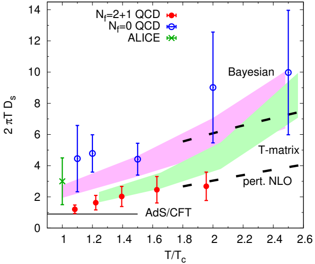
The Heavy Quark Momentum Diffusion Coefficient κ

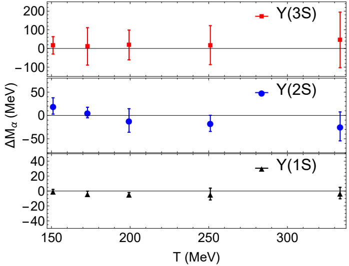
κ calculated with dynamical quarks $D_s = 2T^2/\kappa$

Quakonium at $T > 0$

Extended sources improves signal for Bottomonia

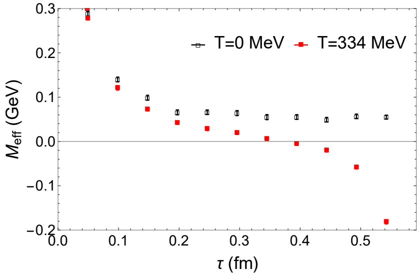






Continuum Subtracted S-wave

- Extended sources greatly reduces continuum contribution

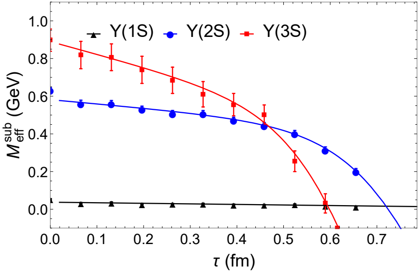


- Small τ behavior similar at $T = 0$ and $T \neq 0$
- Extract continuum $C_{high}(\tau)$ from $T = 0$ results
- 0 Corresponds to mass of η_b at $T = 0MeV$.

$$\begin{aligned} C(\tau) &= Ae^{-M\tau} + C_{high}(\tau) \\ C_{sub}(\tau, T) &= C(\tau, T) - C_{high}(\tau) \end{aligned} \quad (8)$$

Finite Temperature Subtracted Effective Mass

- Drop in effective mass as $\tau \to 1/T$
- Linear behavior at small to mid range τ



$T = 251 MeV$

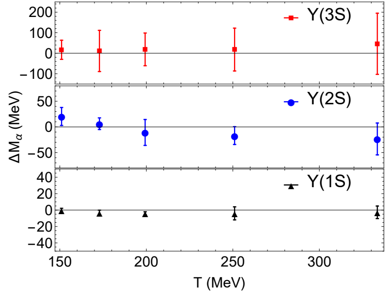
- Information in correlation function is thus

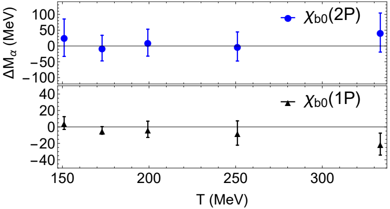
$$C_{sub}(\tau, T) \sim \exp(-M_\alpha \tau + \frac{1}{2}\Gamma_\alpha^2 \tau^2 + O(\tau^3)) \quad (9)$$

$$\rho_\alpha(\omega, T) = A_\alpha(T) \exp\left(-\frac{[\omega - M_\alpha(T)]^2}{2\Gamma_\alpha^2(T)}\right) + A_\alpha^{\text{cut}}(T)\,\delta\left(\omega - \omega_\alpha^{\text{cut}}(T)\right)$$

Mass

- The mass is found to be consistent with zero temperature results [R. Larsen et al., arXiv:1910.07374], $\Delta M_\alpha = M_\alpha(T) - M_\alpha(0)$





- Seyong Kim et al, arxiv:1808.08781