

Jet medium modifications

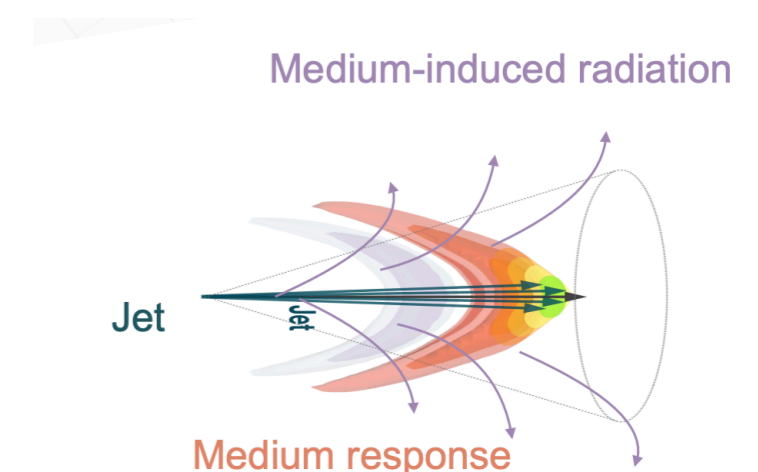
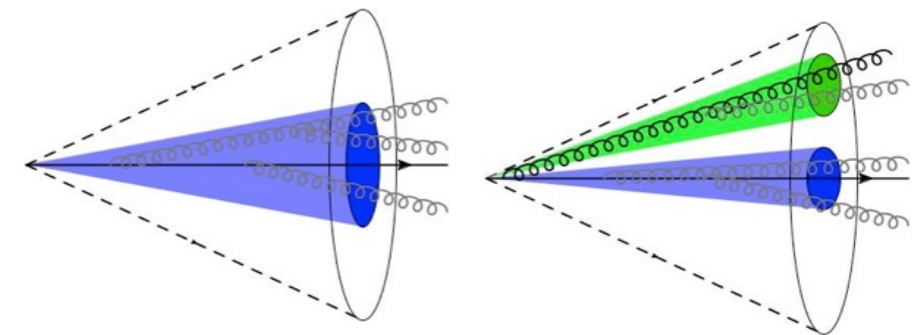
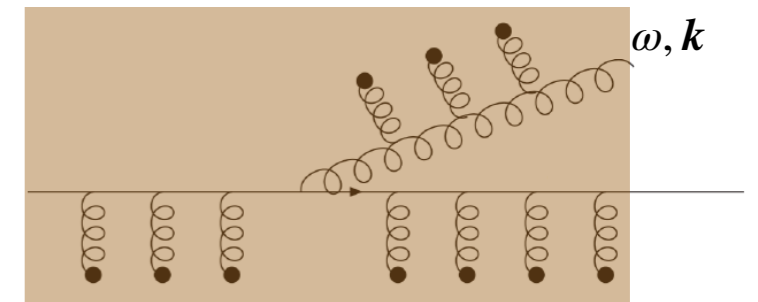
Carlota Andres

CPHT, École polytechnique
HP2023, Aschaffenburg, March 26 -31



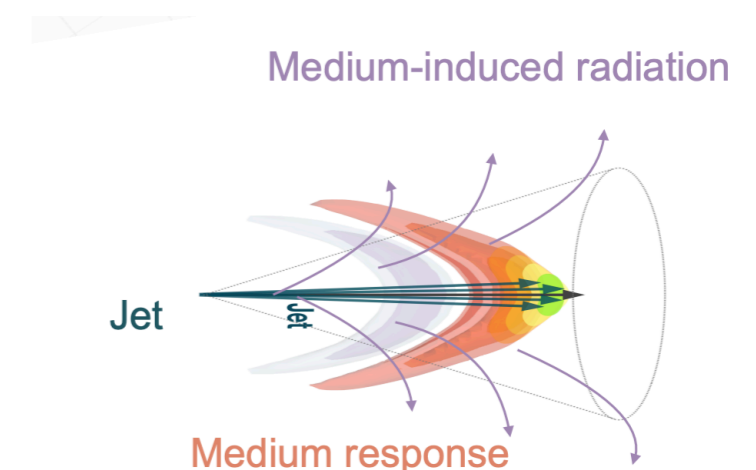
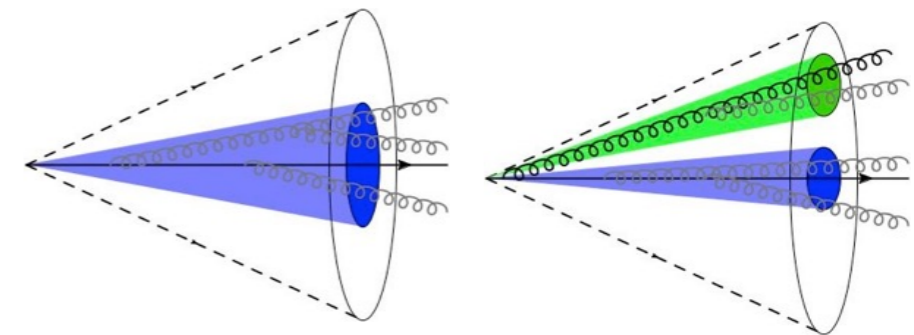
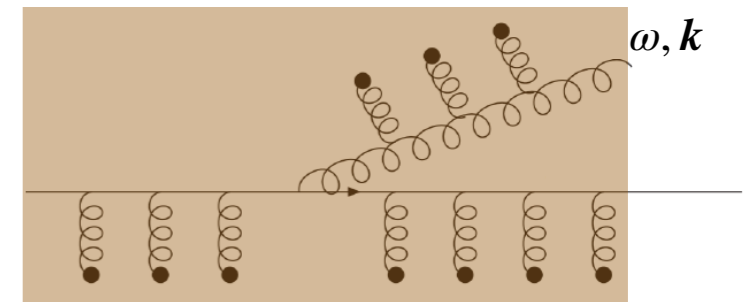
Jet modification in HI

- **Medium-induced energy loss**
 - Out-of-cone energy loss
 - Jet and hadron suppression
- **Color coherence effects** [Soto-Ontoso's talk](#)
Mon 17:05
 - Expected to modify the jet inner structure
 - Not yet unequivocally seen in observables
- **Medium response** [Go's talk](#) Mon 17:30
 - Medium recoils become part of the jet
 - Not yet unequivocally seen in observables



Jet modification in HI

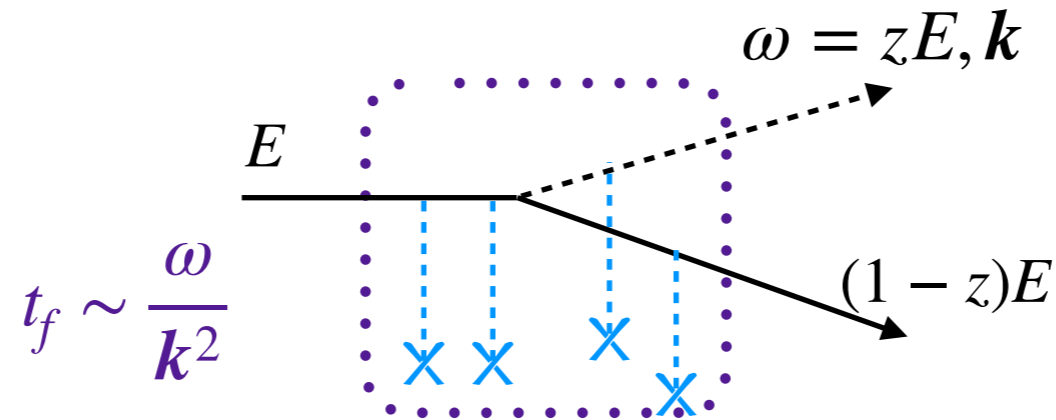
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Medium-induced radiation

- During the formation time of the gluon **multiple scatterings** act **coherently**

LPM effect



Landau, Pomeranchuk, Migdal

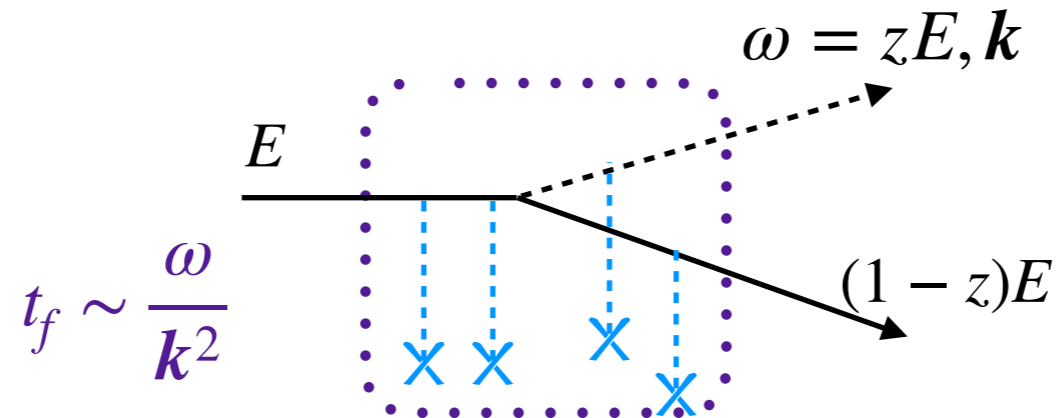
Suppression of the spectrum for large formation times

- Resummation of multiple scatterings: **BDMPs-Z formalism**

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 - For **energy loss** calculations, we only need the **soft limit** ($z \ll 1$):

Formally : $E \rightarrow \infty, z \rightarrow 0$ ($\omega = zE$ finite)

Emission kernel

Broadening

$$\omega \frac{dI}{d\omega d^2\mathbf{k}} = \frac{2\alpha_s C_R}{(2\pi)^2 \omega^2} \text{Re} \int_0^\infty dt' \int_0^{t'} dt \int_{pq} \mathbf{p} \cdot \mathbf{q} \mathcal{K}(t', \mathbf{q}; t, \mathbf{p}) \mathcal{P}(\infty, \mathbf{k}; t', \mathbf{q})$$

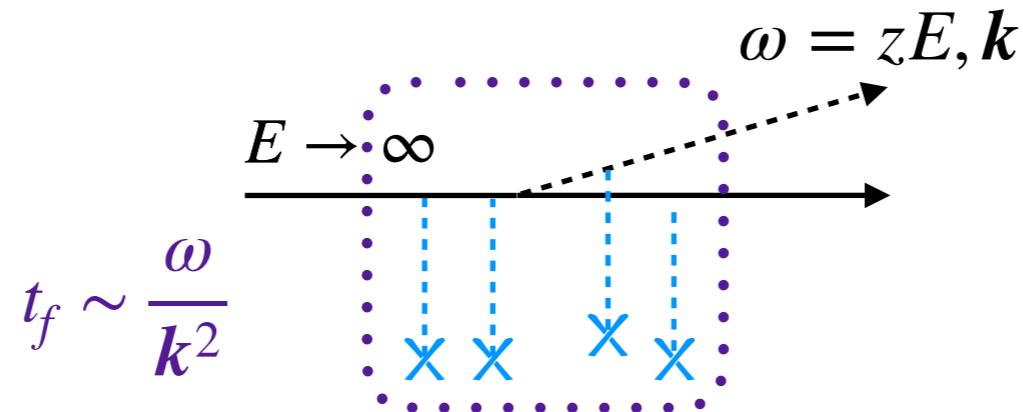
Baier, Dokshitzer, Mueller, Peigné, Schiff (96)

Zakharov (97)

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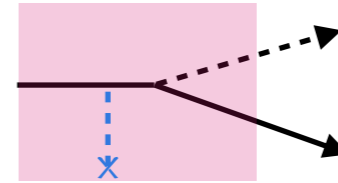
Zakharov (97)

Medium-induced radiation

- In practice, solved for some approximations

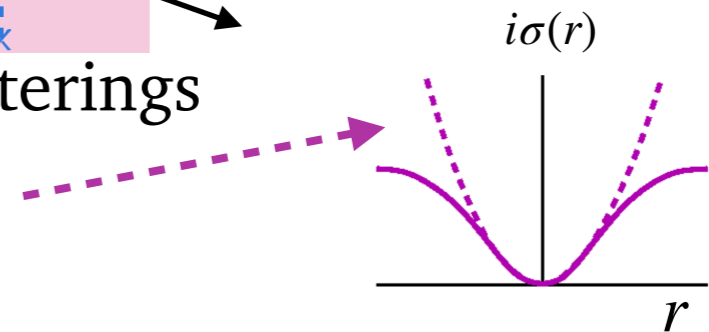
- **Opacity expansion** in the number of scatterings

$N = 1$: GLV Gyulassy, Levai, Vitev (2000)



- **Harmonic oscillator (HO)**: multiple soft scatterings (Gaussian approximation)

$$n\sigma(r) \approx \frac{1}{2}\hat{q}r^2$$



- **AMY**: infinite length medium Arnold, Moore, Yaffe (2002)

- Recent approaches going beyond these approximations

$$V(q) \propto \frac{1}{q^4}$$

- **Improved opacity expansion (IOE)**

Semi-analytical expansion around the HO

Mehtar-Tani, Barata, Soto-Ontoso, Tywoniuk,
[1903.00506](#), [2004.02323](#), [2106.07402](#)

- **Fully resummed spectrum** ($z \ll 1$)

Kernel as a time dependent Schrödinger equation

CA, Apolinario, Martinez, Dominguez,
[2002.01517](#), [2011.06522](#)

Beyond the soft limit but integrated in k_T :
Caron-Huot and Gale, [1006.2379](#)

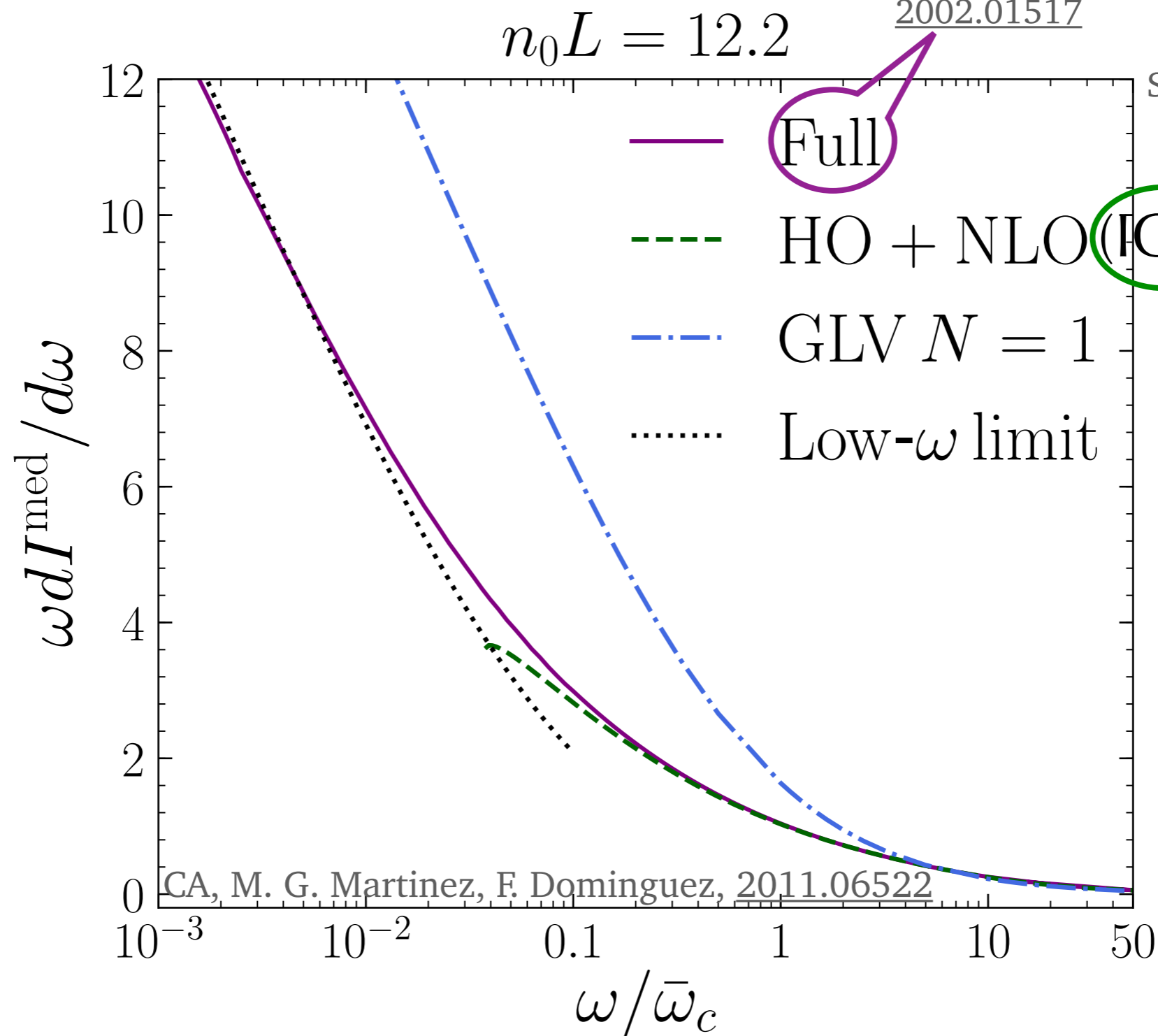
Medium-induced radiation

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Soto-Ontoso, Tywoniuk,

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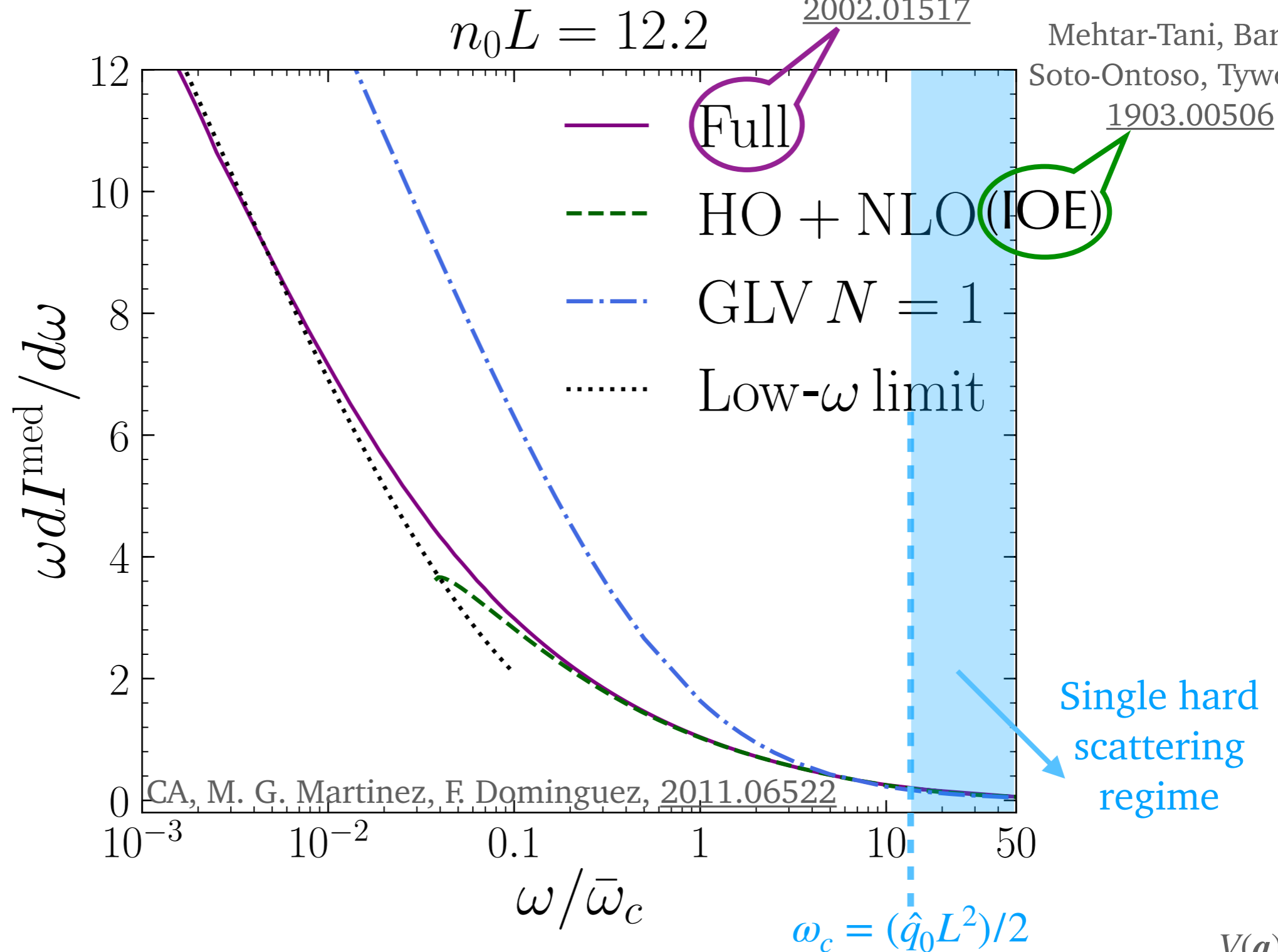
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Medium-induced radiation

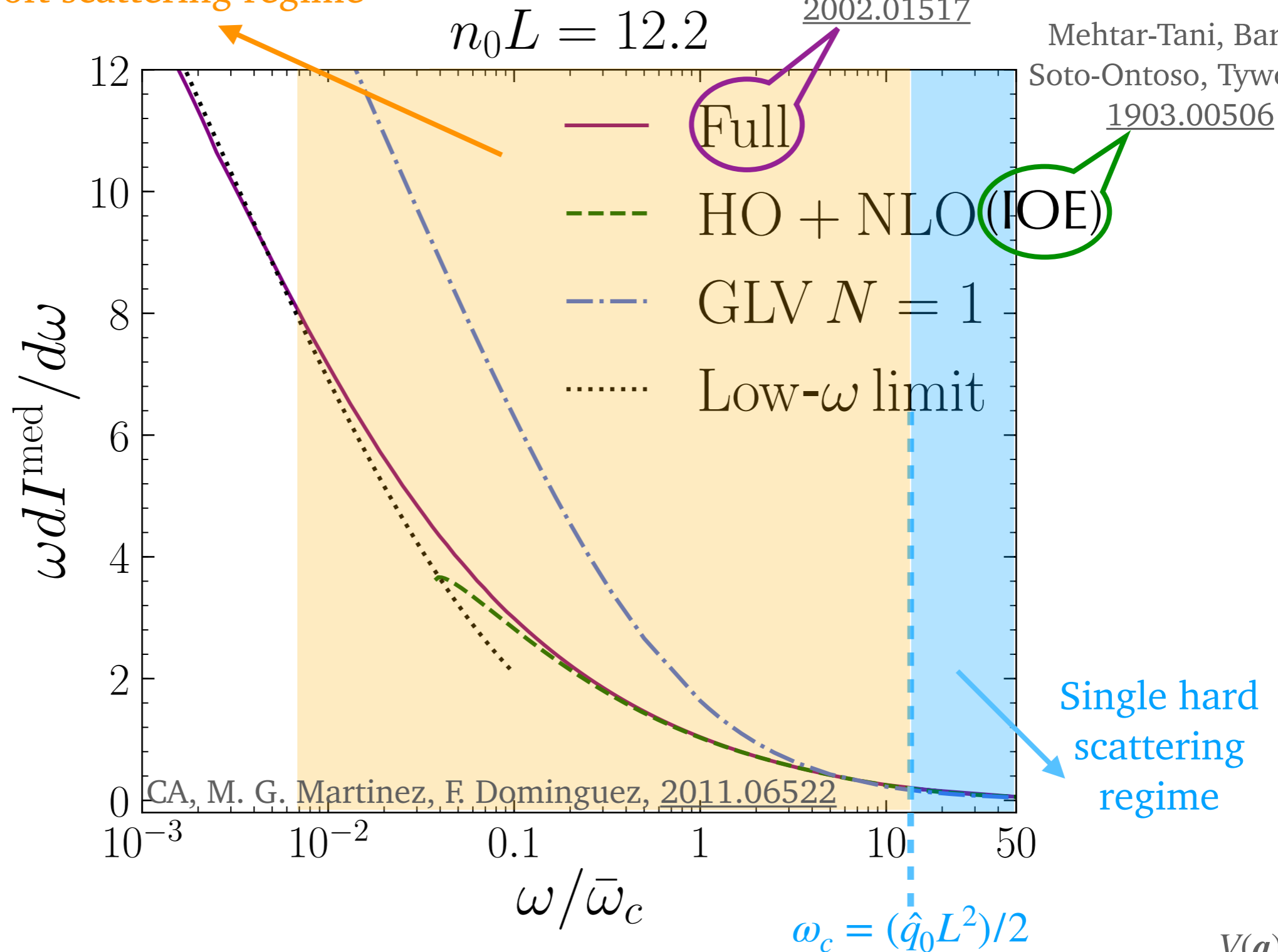
Multiple soft scattering regime

CA, Apolinário, Dominguez

2002.01517

Mehtar-Tani, Barata,
Soto-Ontoso, Tywoniuk,

1903.00506



$$V(q) \propto \frac{1}{q^4}$$

Medium-induced radiation

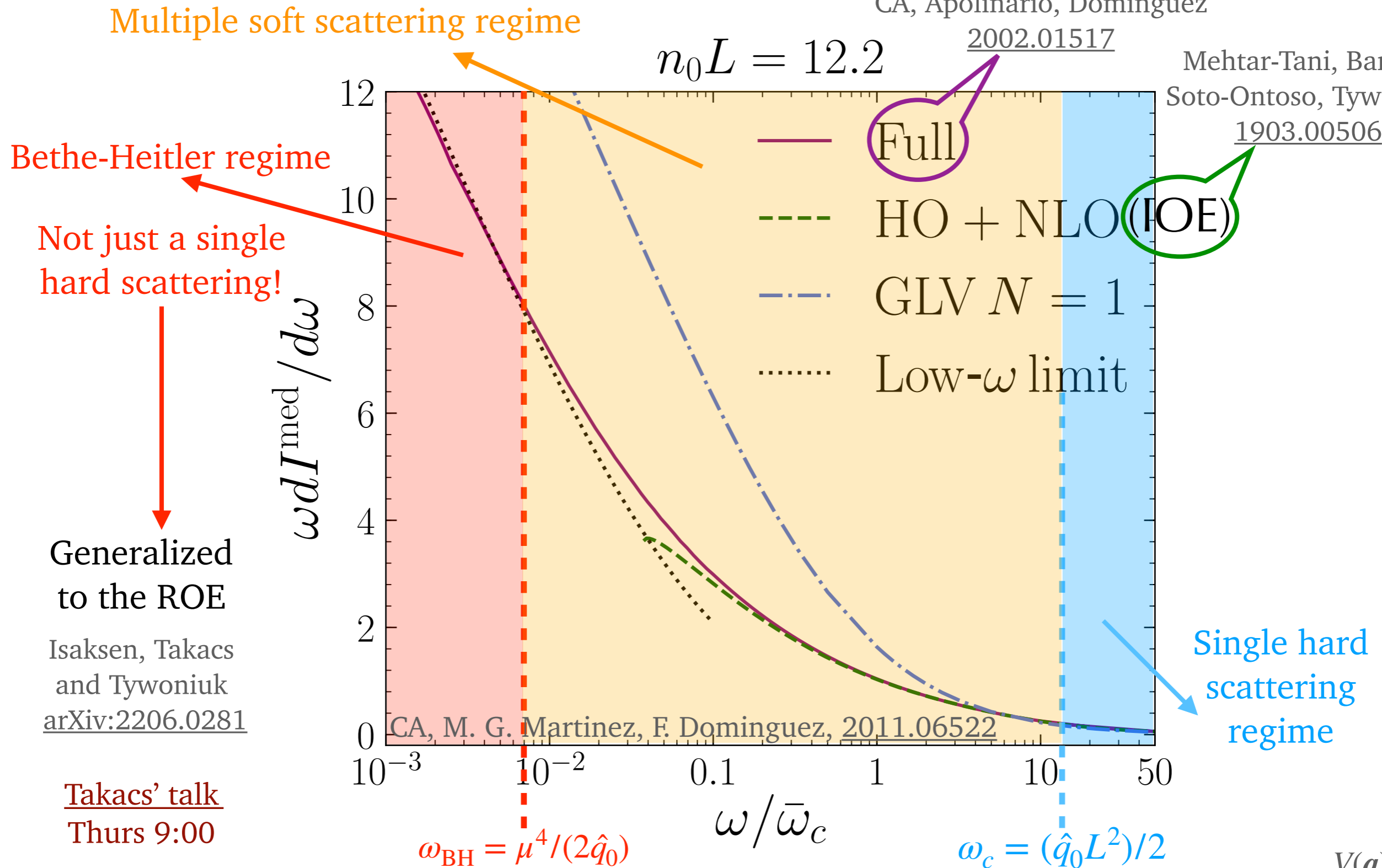
CA, Apolinário, Dominguez

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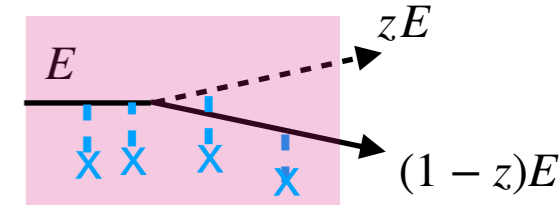
$n_0 L = 12.2$



$$V(\mathbf{q}) \propto \frac{1}{q^4}$$

Medium-induced radiation: non-perturbative potential

- Perturbative potential (*broadening kernel*) known at LO and NLO in pQCD Aurenche Gelis Zaraket (2002), Caron-Huot (2008)

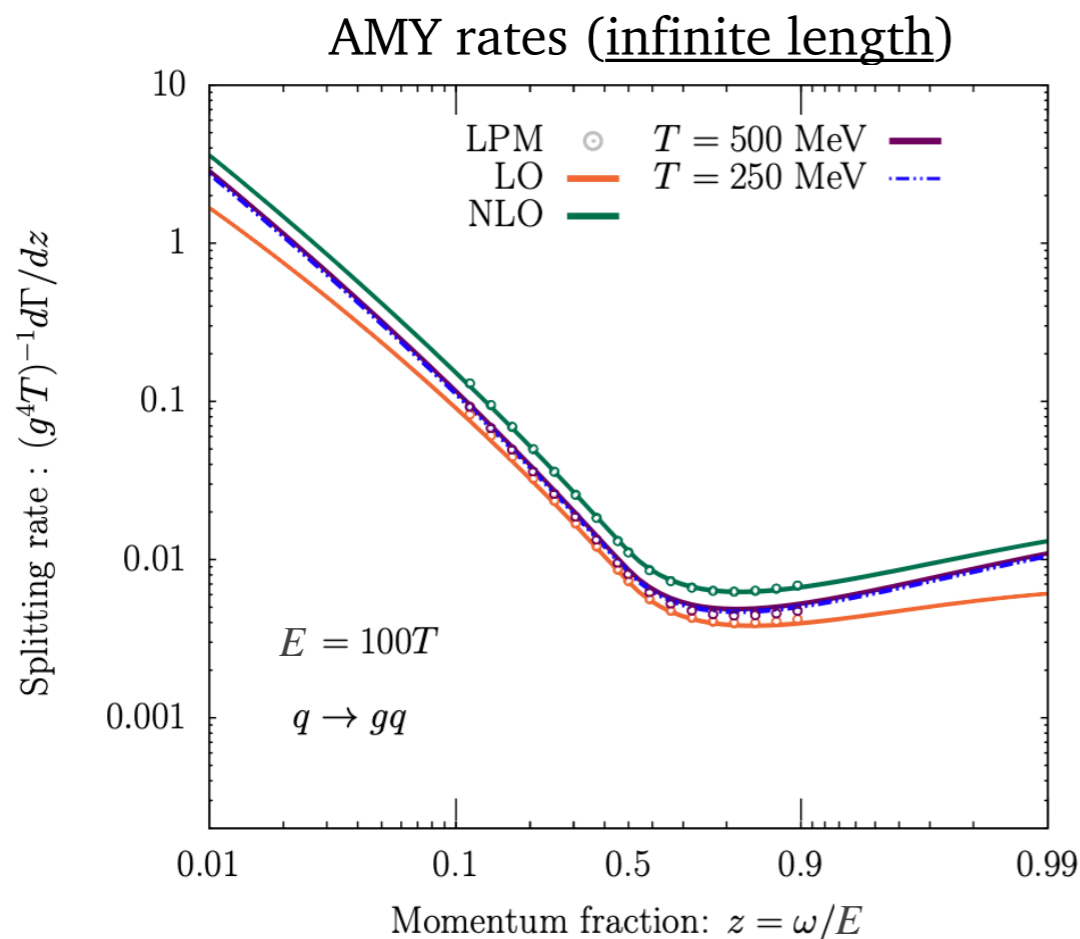


- **Non-perturbative result** from EQCD Moore, Schlusser, [2009.06614](#) [Ghiglieri's talk](#)

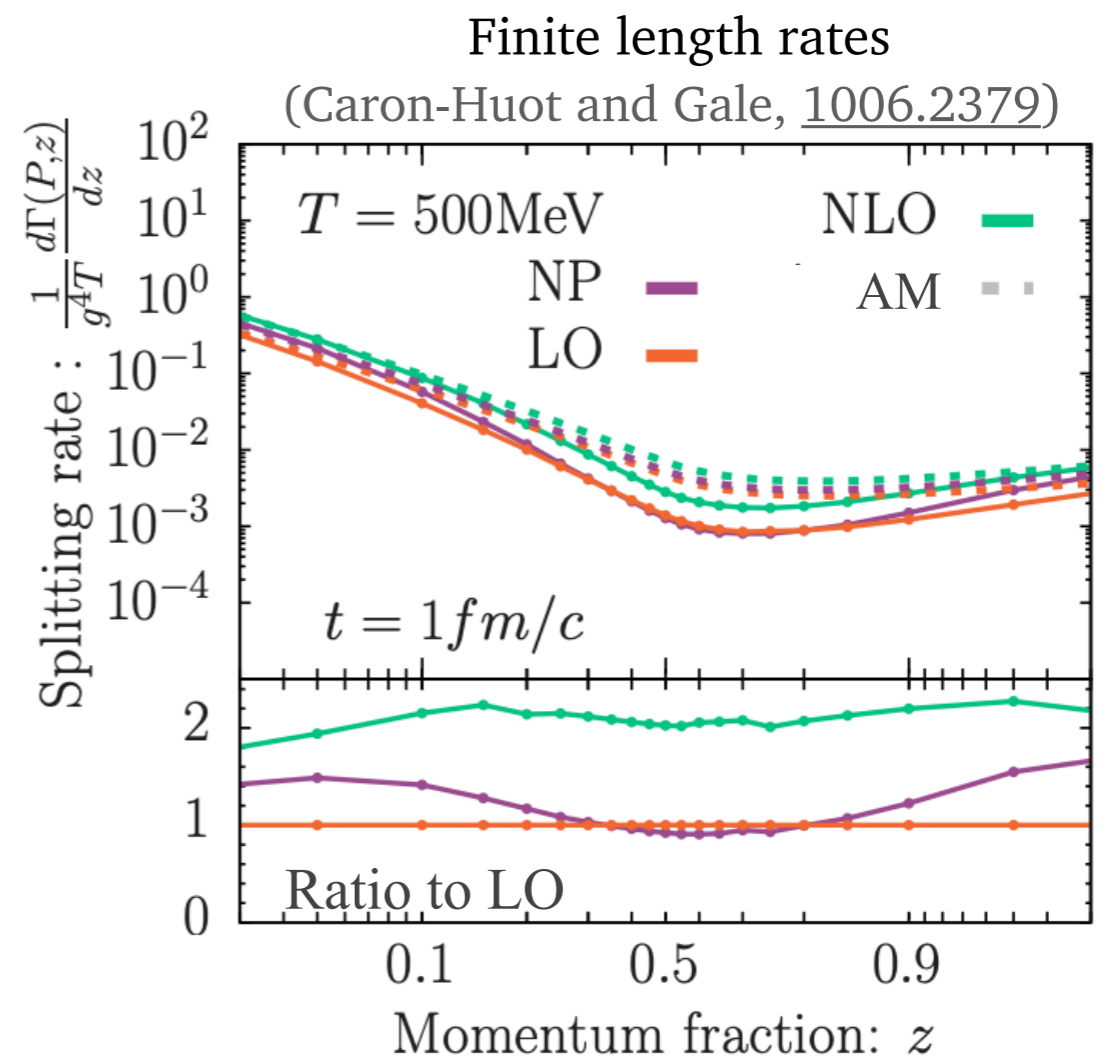
Tue 10:50

- Matching between the EQCD IR and the perturbative UV results

Moore, Schlichting, Schlusser, Soudi, [2105.01679](#)



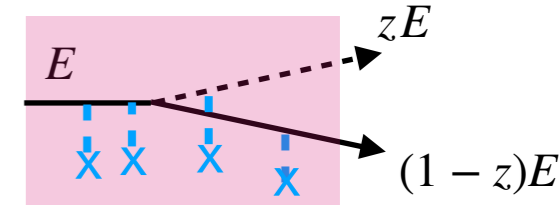
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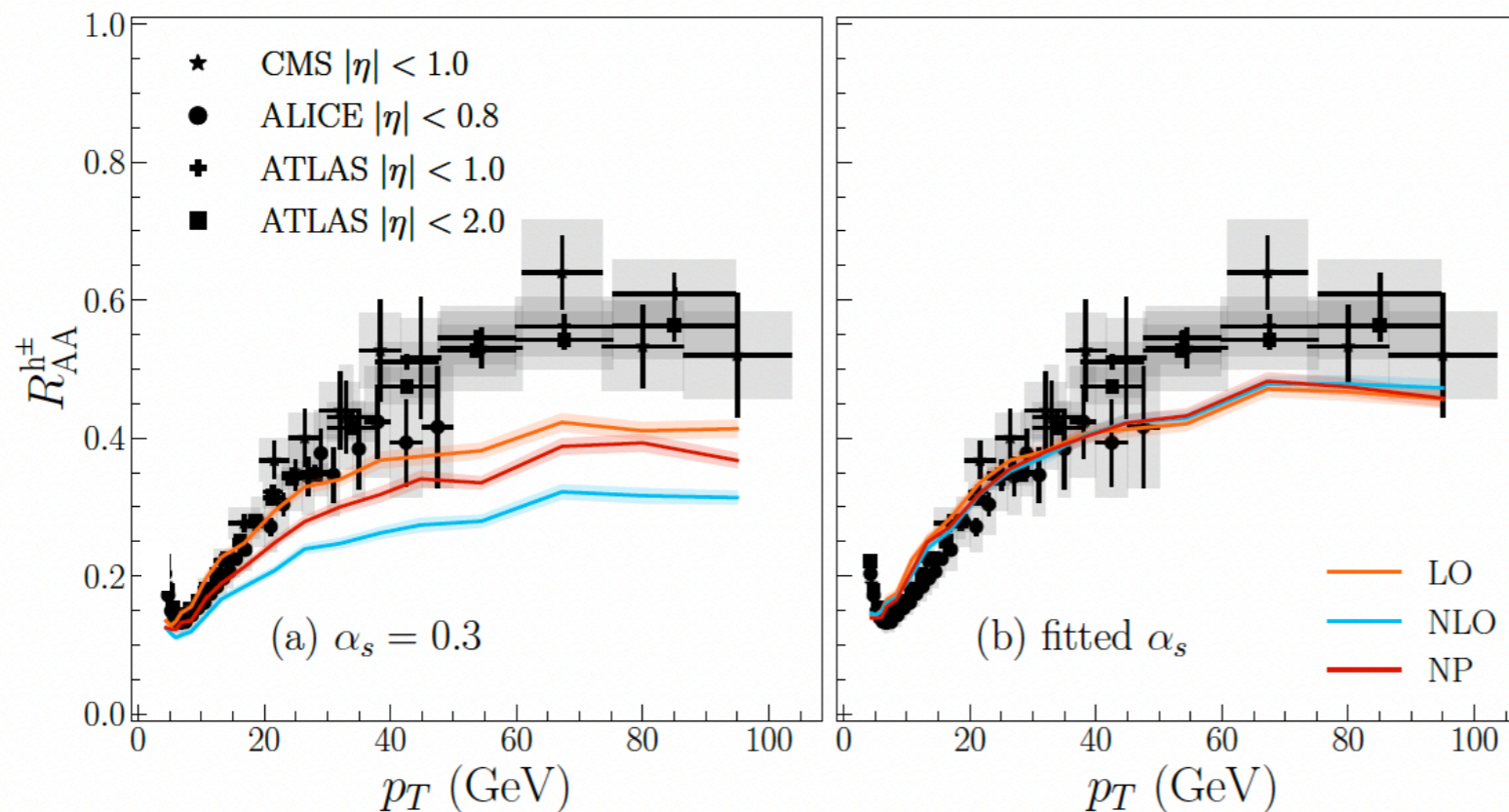
Schlichting, Soudi [2111.13731](#)

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AMY rates (infinite length)



Rate Set	α_s
LO	0.280
NLO	0.242
NP	0.260

Yazdi, Shi, Jeon, Gale, [2206.05855](#)

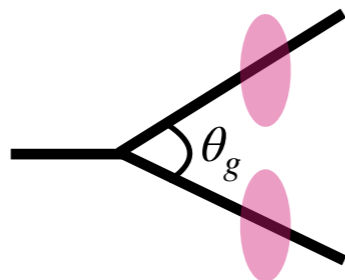
From energy loss to jet substructure

- Grooming techniques to isolate prongs corresponding to a hard splitting

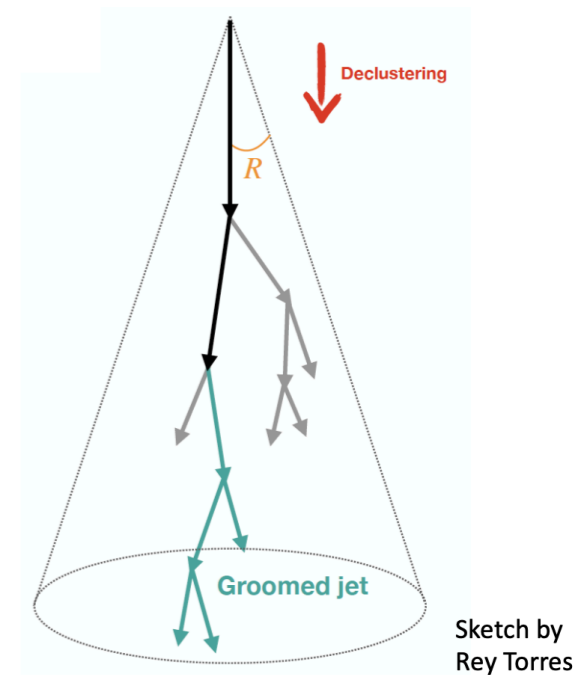
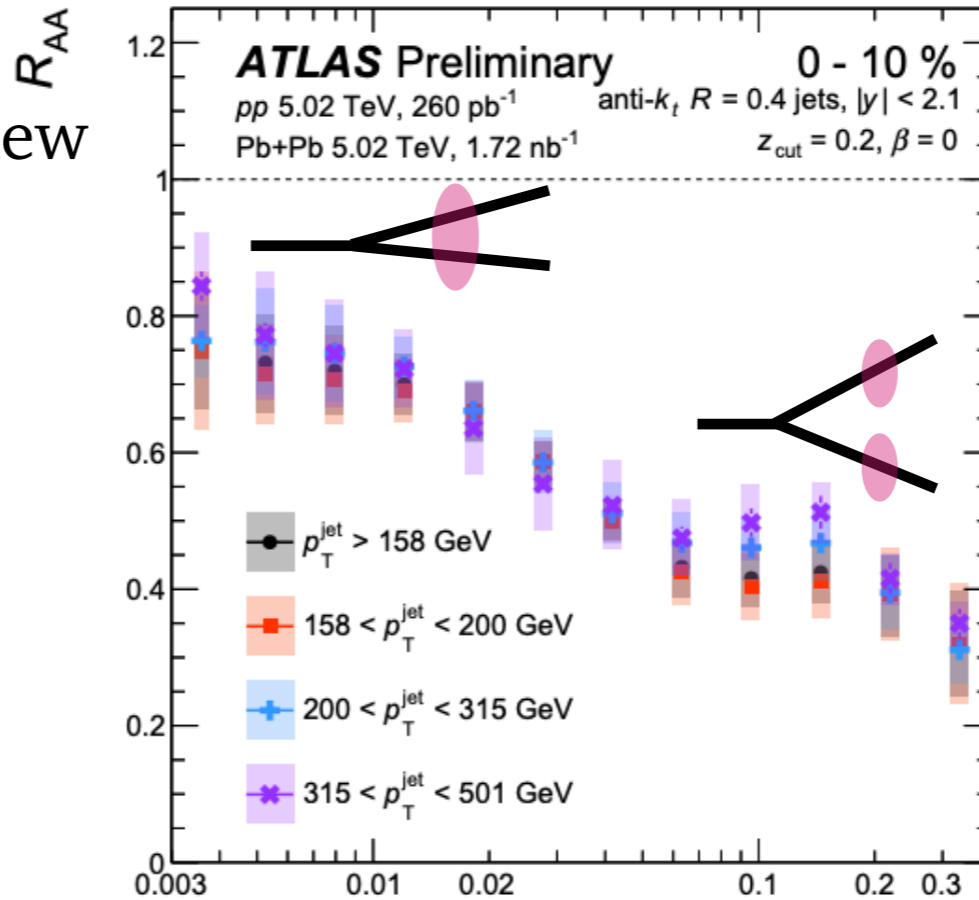
[ATLAS-CONF-2022-026]

- The medium introduces a new resolution scale: θ_c

$$\theta_g > \theta_c$$



$$\theta_g < \theta_c$$



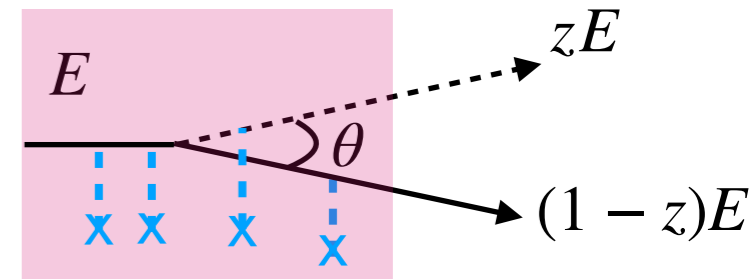
Sketch by Rey Torres

Soto-Ontoso's talk
Mon 17:05

- Obtain θ_g and z_g distributions

- Soft limit ($z \ll 1$) not that useful: $\frac{dI^{\text{med}}}{dzd\theta} ??$

Beyond the soft limit



- Complete (multiple scatterings) medium-induced emission spectrum **keeping** z and θ not available

until last Thursday! (See Isaksen's talk Tuesday 14:40) and slide 12 of this talk

- Two available approaches:

- **Opacity expansion:**

- $N = 1$ result Ovanesyan, Vitev [1103.1074](#), [1109.5619](#)

- Highly complicated recursive relations to go to all orders

Sievert, Vitev, [1807.03799](#)

- Some numerical results at $N = 1$ and $N = 2$ Sievert, Vitev, Yoon, [1903.06170](#)

- Unitarity issues can lead to negative cross sections

- **Tilted Wilson lines:**

- Assumes semi-hard splittings (z not too small)

Dominguez, Milhano, Salgado, Tywoniuk, Vila, [1907.03653](#)

- All partons propagate along straight line trajectories

Isaksen, Tywoniuk [2107.02542](#)

See also

Djordjevic's talk

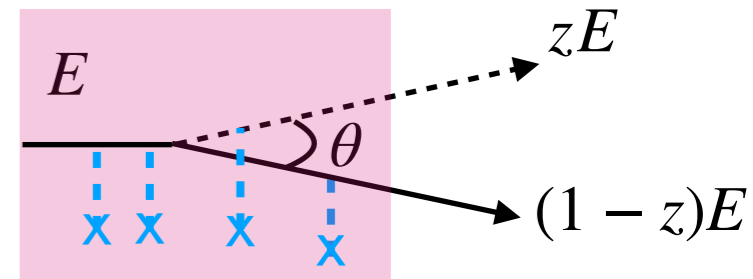
Tues 09:20

Energy correlators

- **Two-point energy correlator** of a heavy-ion quark jet:

Proposed by CA, Dominguez, Elayavalli, Holguin, Marquet, Moul

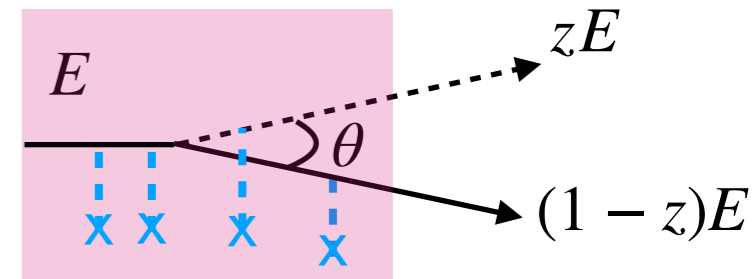
$$\frac{d\Sigma^{(n)}}{d\theta} = \frac{1}{\sigma_{qg}} \int dz \frac{d\sigma_{qg}}{dzd\theta} z^n (1-z)^n + \mathcal{O}\left(\frac{\mu_s}{E}\right)$$



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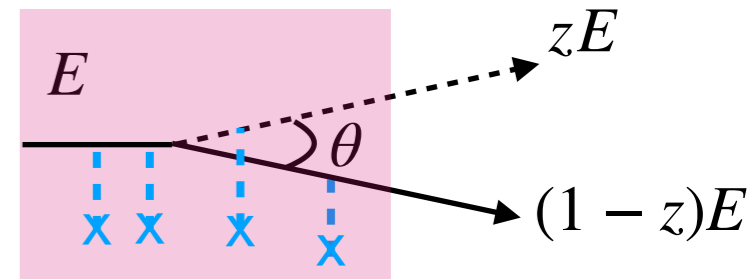
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Inclusive

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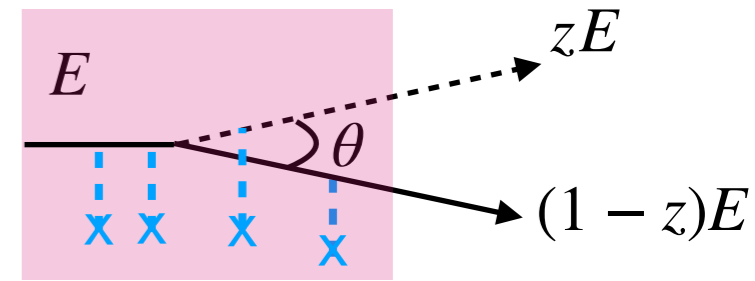
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$$\frac{d\Sigma^{(n)}}{d\theta} = \left(\frac{1}{\sigma_{qg}} \int dz \left(\frac{d\sigma_{qg}^{\text{vac}}}{dzd\theta} + \frac{d\sigma_{qg}^{\text{med}}}{dzd\theta} \right) z^n (1-z)^n \right) \left(1 + \mathcal{O}\left(\frac{\bar{\mu}_s}{Q}\right) \right) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{\theta Q}\right)$$

$$\frac{1}{\sigma_{qg}} \frac{d\sigma_{qg}^{\text{vac}}}{dzd\theta} = \frac{\alpha_s(\theta Q)}{\pi} C_F \frac{1 + (1-z)^2}{z\theta} + \mathcal{O}(\alpha_s^2, \theta)$$

Inclusive medium-induced cross section
differential in z and θ

Energy correlators



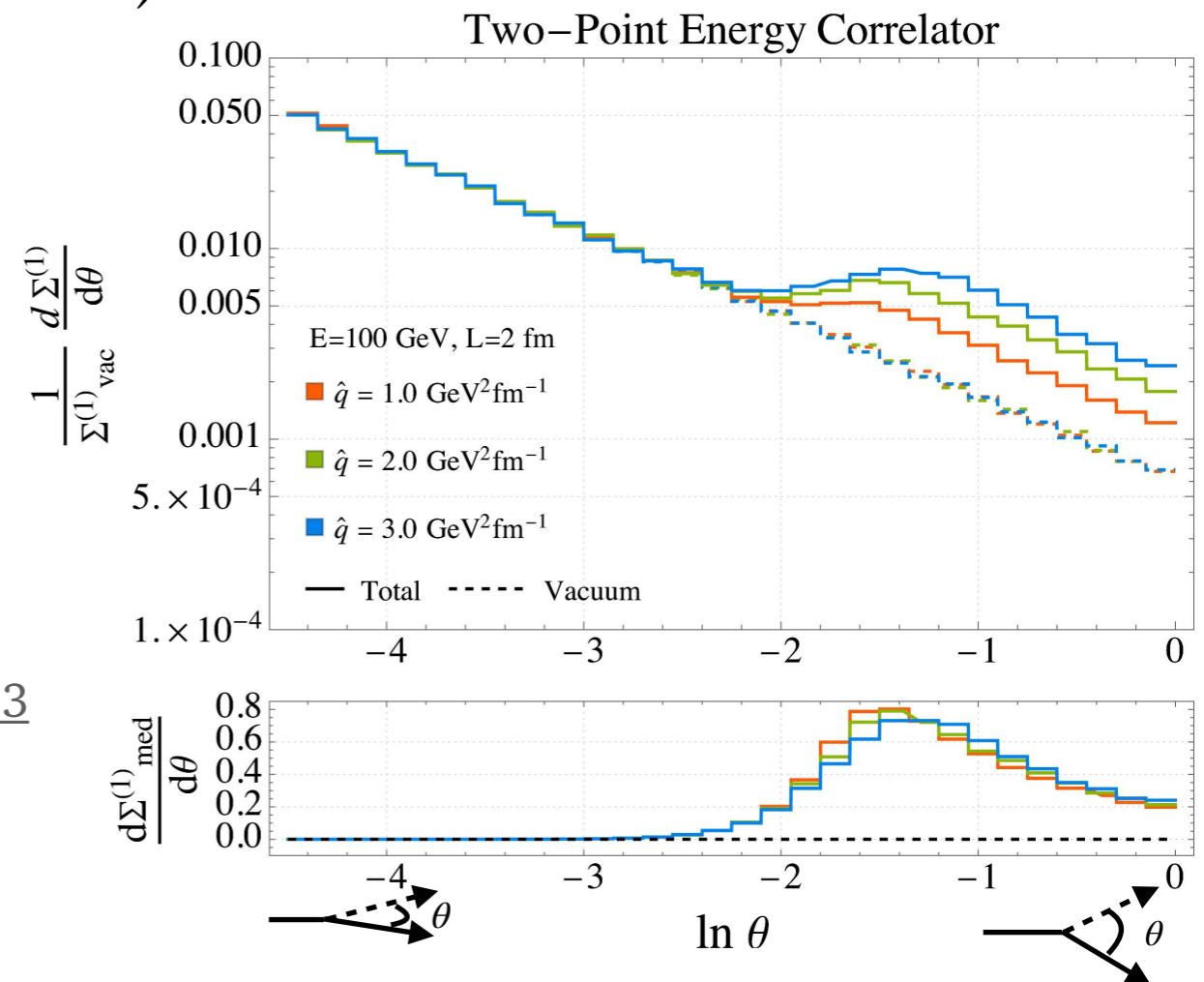
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CA, Dominguez, Elayavalli, Holguin, CA, Dominguez, Holguin,
Marquet, Moul, [2209.11236](#) Marquet, Moul, [2303.03413](#)

[Dominguez's talk](#)
Wed 15:20

[Holguin's talk](#)
Wed 11:50

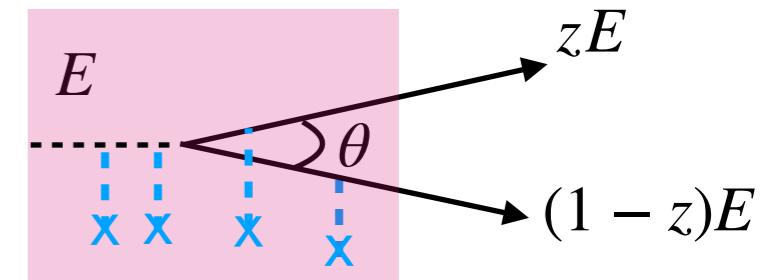
[Barata's talk](#)
Tues 12:10

Also experimental talks: [Cruz-Torres](#) for ALICE Tues 17:30, and [Tamis](#) for STAR Wed 11:30

Beyond the soft limit: $g \rightarrow c\bar{c}$

- **Not divergent** for $z \rightarrow 0$. Need to go beyond the soft limit

Results with multiple scatterings but integrated in transverse momentum: Caron-Huot and Gale, [1006.2379](#)



- **Keeping both z and θ :**

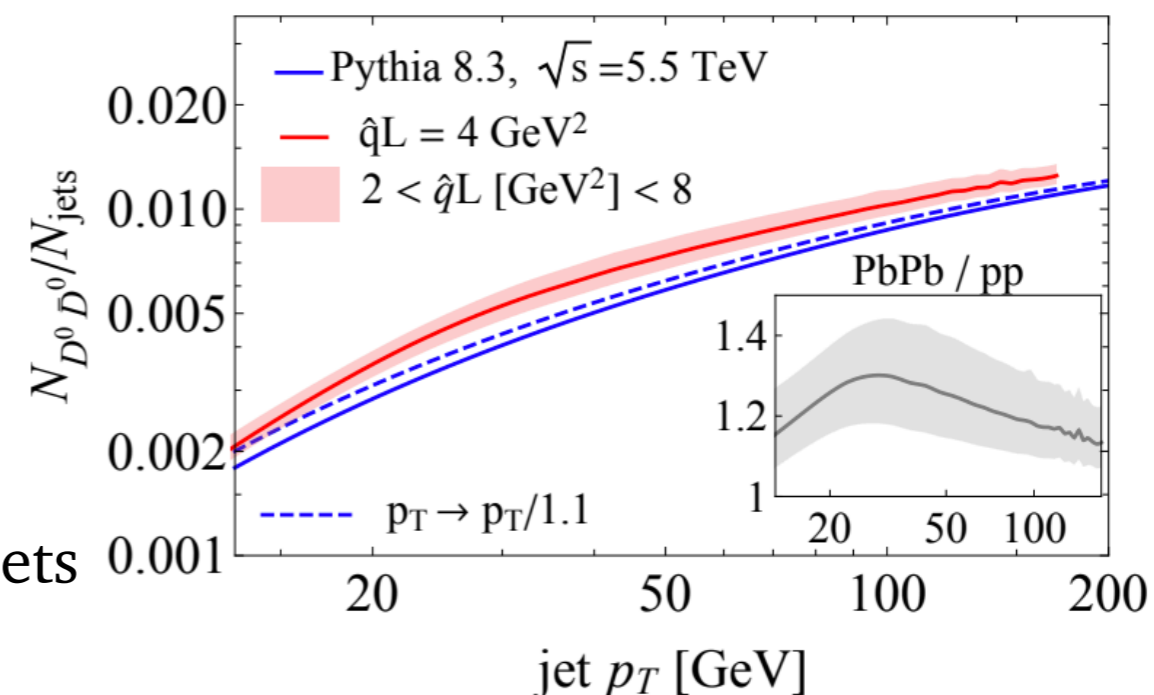
- Opacity expansion results at $N = 1$ and $N = 2$ Sievert, Vitev, Yoon, [1903.06170](#)

- Results with **resummation of multiple scatterings** recently obtained in the **large N_c** limit and HO approximation

(No need of the semi-hard approximation)

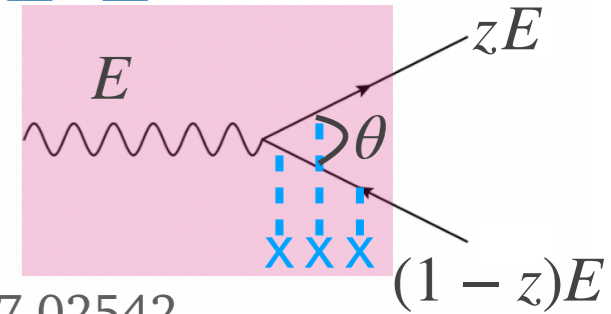
Attems, Brewer, Innocenti,
Mazeliauskas, Park, van der Schee,
Wiedemann, [2203.11241](#), [2209.13600](#)

Brewer's talk
Wed 12:10



Medium-enhanced production of $c\bar{c}$ pairs with jets

Beyond the soft limit: $\gamma \rightarrow q\bar{q}$

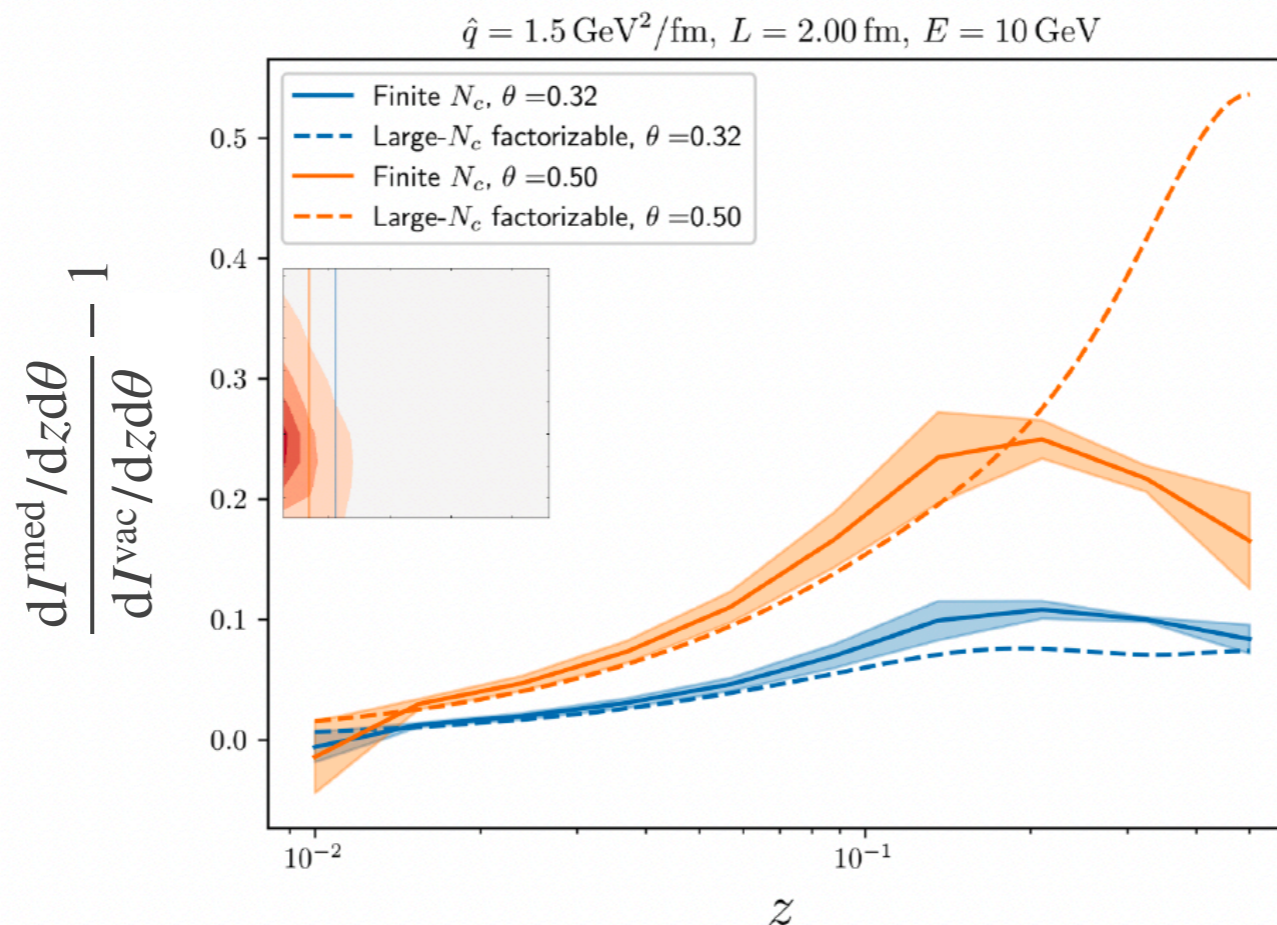


- Results with multiple scatterings in the *semi-hard approximation* available at large and finite N_c

Dominguez, Milhano, Salgado, Tywoniuk, Vila, [1907.03653](#) Isaksen, Tywoniuk [2107.02542](#)

- **Complete** (multiple scatterings) medium-induced emission spectrum **keeping z and θ** (Without using of the *semi-hard* approximation!!) [Isaksen, Tywoniuk, 2303.12119](#) (HO approximation)

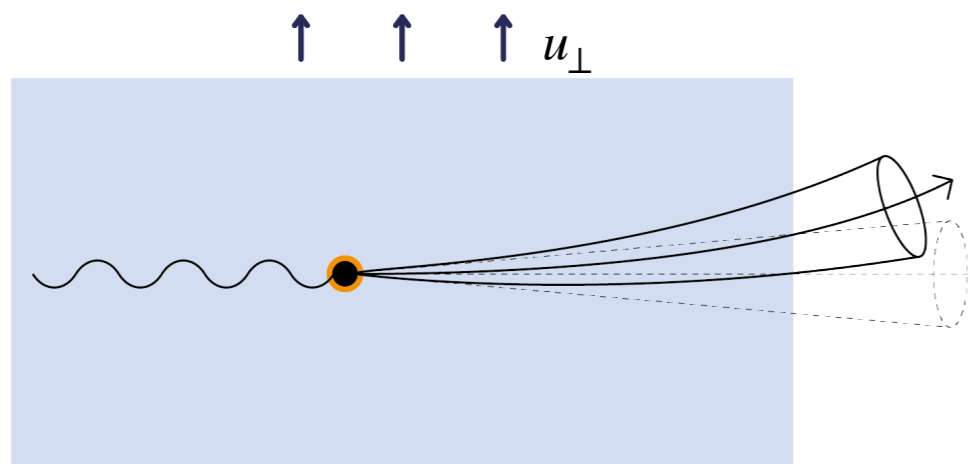
[Isaksen's talk](#)
Wed 14:40



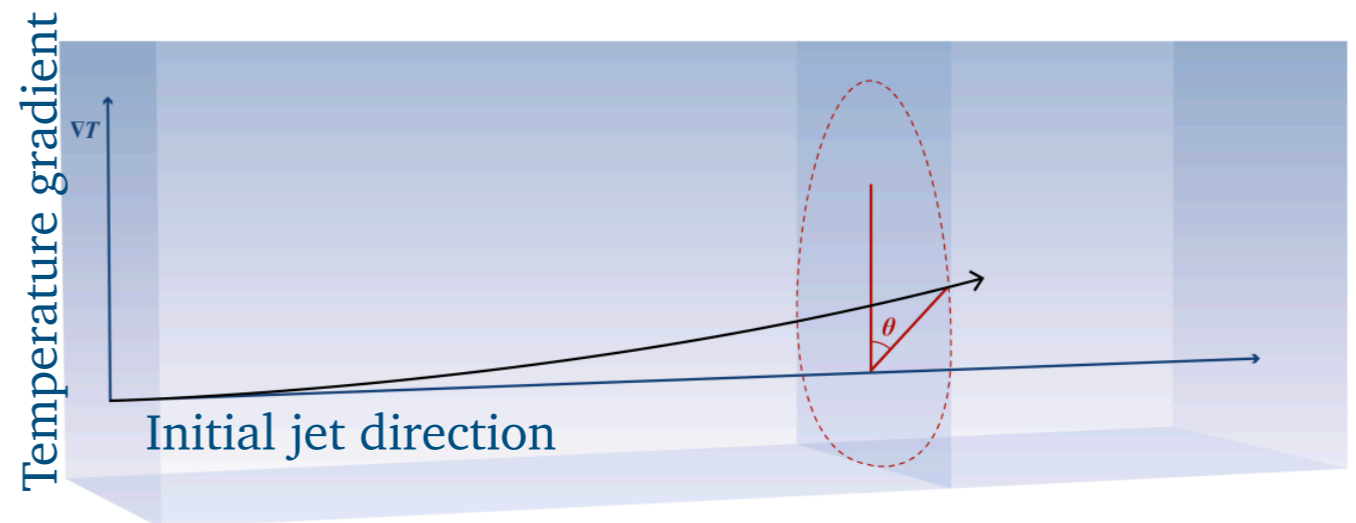
Isaksen, Tywoniuk,
[2303.12119](#)

Medium-induced radiation and transverse dynamics

- **Jets decouple** from the medium **transverse** dynamics in the **usual** (*eikonal*) **medium-induced approaches**



Uniform transverse flow



Transverse temperature gradients

- Need of **generalizing** the medium-induced formalisms to account for $\mathcal{O}(1/E)$ (*subeikonal*) terms
- **GLV emission spectrum** (and broadening) obtained: Sadofyev, Sievert, Vitev, [2104.09513](#)
 - for transversely **flowing** matter
 - for transversely **inhomogeneous** matter

Sadofyev's talk
Wed 10:00

Medium-induced radiation and transverse dynamics

- **Broadening with multiple scatterings** obtained:

Sadofyev's talk
Wed 10:00

- for transversely **flowing** matter CA, Dominguez, Sadofyev, Salgado, [2207.07141](#)

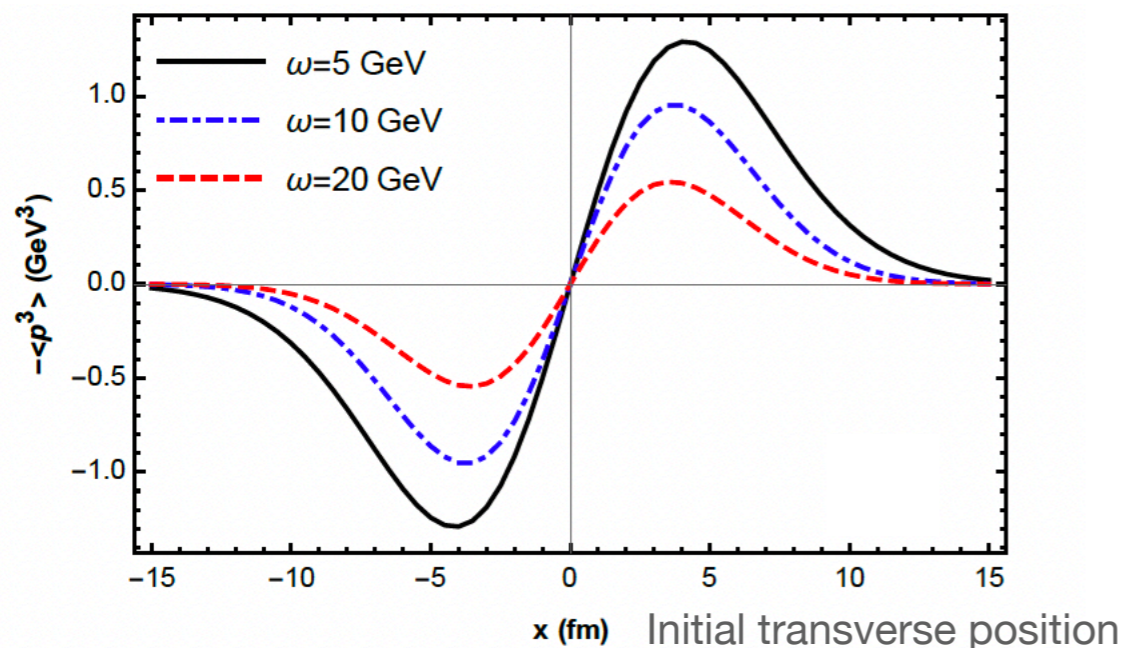
- for transversely **inhomogeneous** matter Barata, Sadofyev, Salgado, [2202.08847](#)
Fu, Casalderrey-Solana, Wang, [2204.05323](#)

See also Barata, Sadofyev, Wang [2210.06519](#)

See also Mayo López's poster

Transverse inhomogeneities

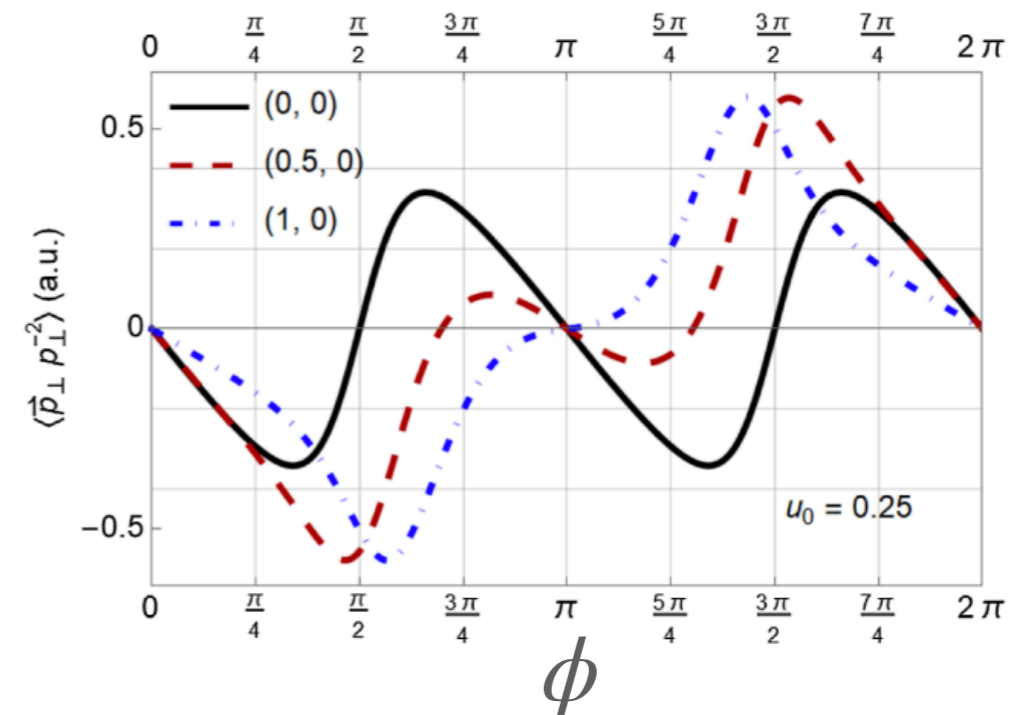
Fu, Casalderrey-Solana, Wang, [2204.05323](#)



Xin-Nian Wang's
talk Tue 10:00

Transverse flow (single scattering)

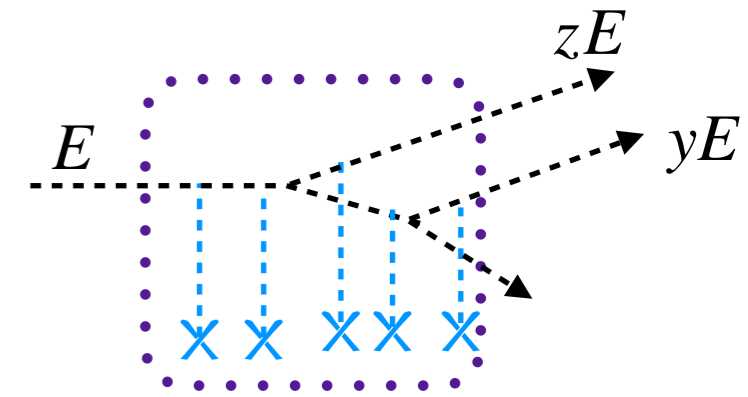
Antiporda, Bahder, Rahman, Sievert, [2110.03590](#)



Multiple splittings

- **Overlapping effects among multiple splittings?**

Extremely hard calculation performed for $g \rightarrow ggg$
(on-shell emitter, infinite, static medium, large N_c , HO)

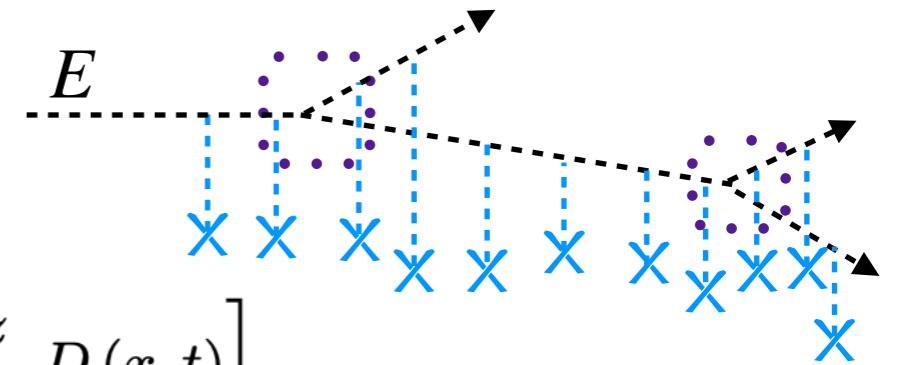


Seems to be a **small effect for gluon showers**. **Parton showers** seem to be **safe!**

Arnold, Iqbal, Chang, Gorda, Rase, Elgeadwy (2015-2022) [2212.08086](#), [2302.10215](#)

- **Independent medium-induced splittings**

Jeon, Moore (2003) Blaizot, Dominguez, Iancu, Mehtar-Tani (2014)



$$\frac{\partial}{\partial t} D(x, t) = \frac{1}{t_*} \int dz \mathcal{K}(z) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z}, t\right) - \frac{z}{\sqrt{x}} D(x, t) \right] \text{BDIM}$$

$$t_* = \alpha \sqrt{\frac{E}{\hat{q}}}$$

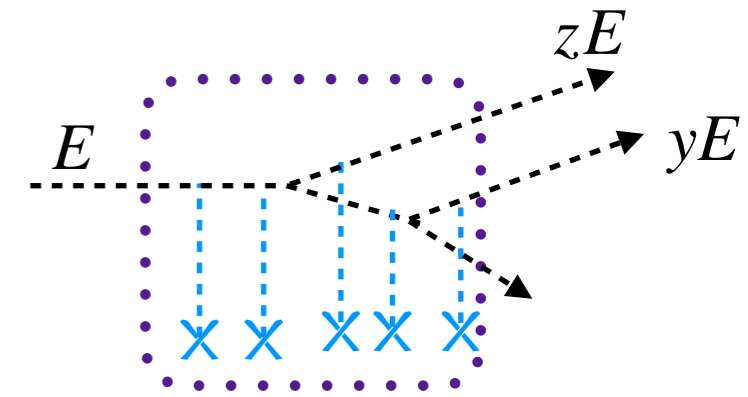
$$\frac{\partial}{\partial t} D(x, \mathbf{k}, t) = \frac{1}{t_*} \int dz \mathcal{K}(z) \left[\frac{1}{z^2} \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \frac{\mathbf{k}}{z}, t\right) - \frac{z}{\sqrt{x}} D(x, \mathbf{k}, t) \right] + \int_{\mathbf{q}} \mathcal{C}(\mathbf{q}) D(x, \mathbf{k} - \mathbf{q}, t)$$

*Neglecting any momentum transfer during the formation time of the splittings

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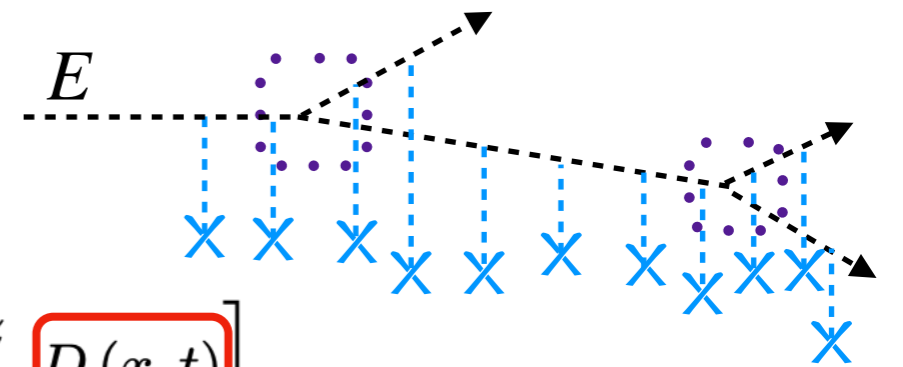


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- **Independent medium-induced splittings**

Jeon, Moore (2003) Blaizot, Dominguez, Iancu, Mehtar-Tani (2014)



$$\frac{\partial}{\partial t} D(x, t) = \frac{1}{t_*} \int dz \mathcal{K}(z) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z}, t\right) - \frac{z}{\sqrt{x}} D(x, t) \right] \text{BDIM}$$

Inclusive gluon distribution
Gain term
Loss term

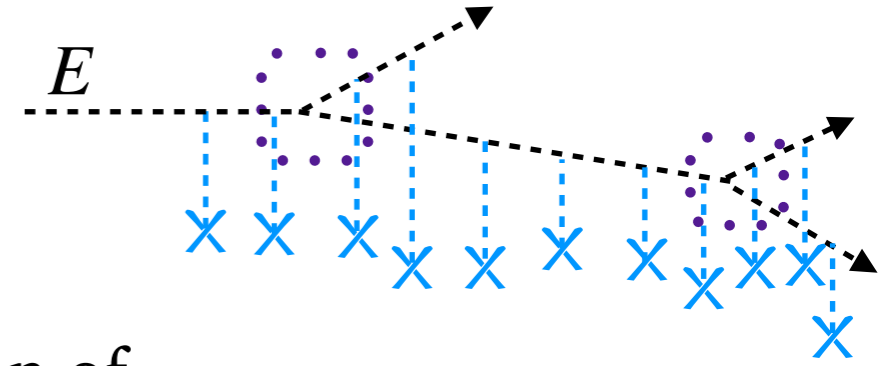
$$t_* = \alpha \sqrt{\frac{E}{\hat{q}}}$$

$$\frac{\partial}{\partial t} D(x, \mathbf{k}, t) = \frac{1}{t_*} \int dz \mathcal{K}(z) \left[\frac{1}{z^2} \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \frac{\mathbf{k}}{z}, t\right) - \frac{z}{\sqrt{x}} D(x, \mathbf{k}, t) \right] + \int_{\mathbf{q}} \mathcal{C}(\mathbf{q}) D(x, \mathbf{k} - \mathbf{q}, t)$$

Broadening

*Neglecting any momentum transfer during the formation time of the splittings

Multiple splittings



- Generalization of BDMI to account for the transmission of **polarization** in an **anisotropic** medium (Static medium, HO, \hat{q} constant but $\hat{q}_y \neq \hat{q}_z$)

Polarization is found to be created and washed out at each step of the branching

Hauksson, Iancu [2303.03914](#)

- **Energy** and **angular profile** of the **medium-induced** cascade

- **Kinetic theory** approach: linearized Boltzmann equation

Energy distribution: Soudi, Schlichting [2008.04928](#)

(Static medium, AMY radiation rates)

Angular distribution: Mehtar-Tani, Soudi, Schlichting, [2209.10569](#)

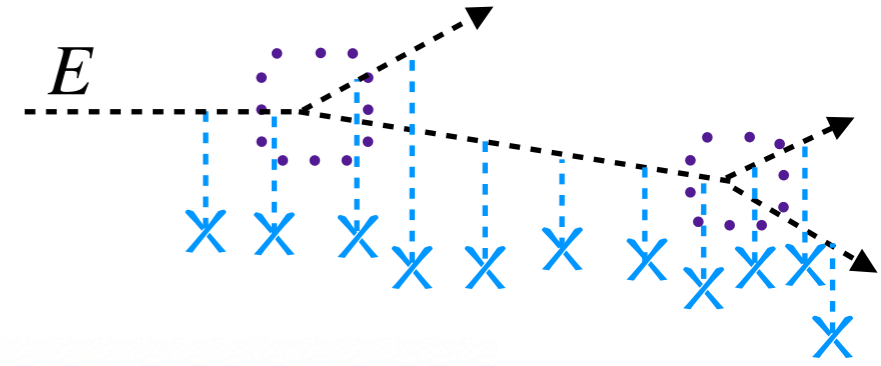
- Solving **BDMI** equations for **longitudinally expanding media** (HO)

Energy distribution: Adhya, Spousta, Salgado, Tywoniuk, [2106.02592](#)

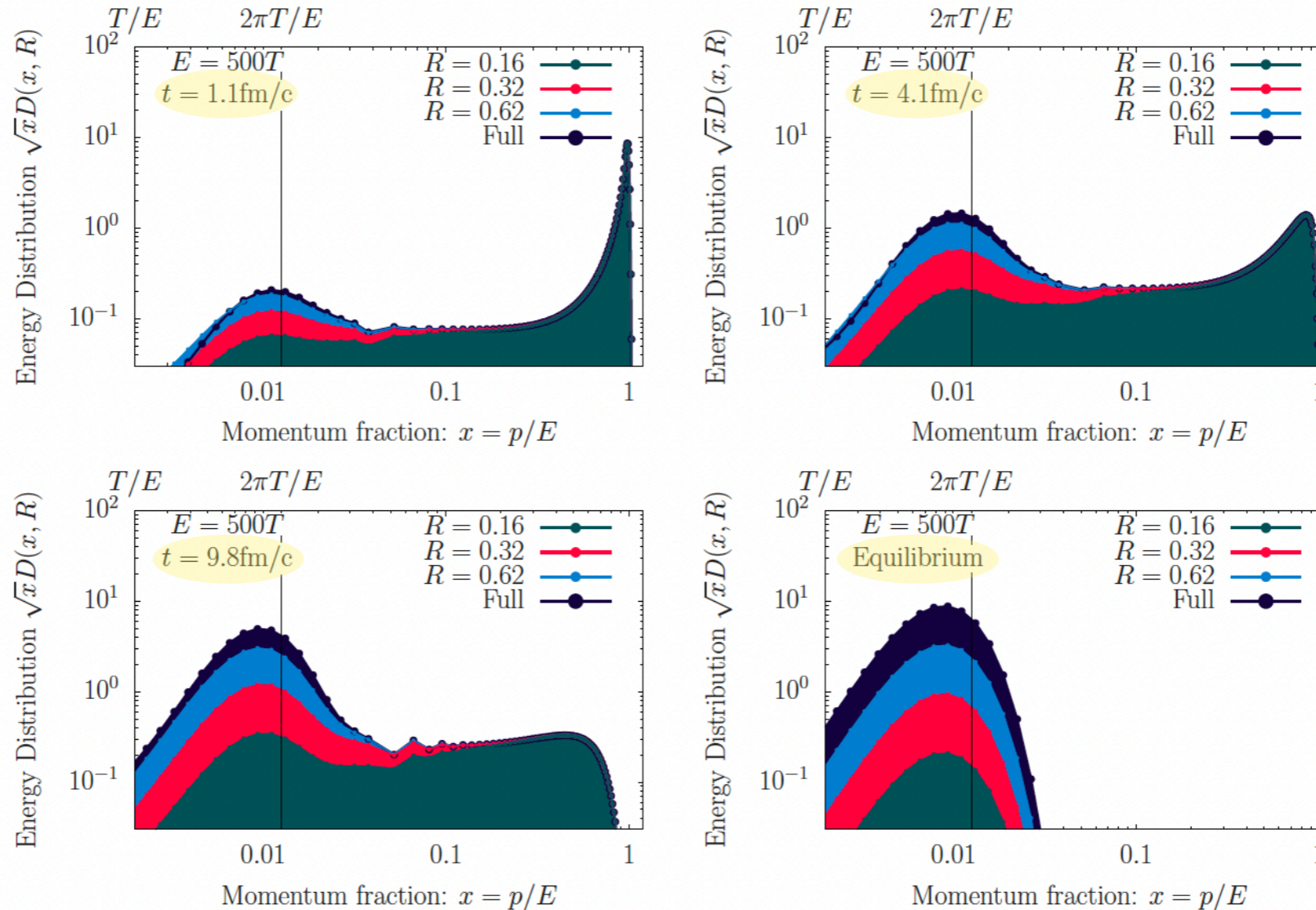
[Adhya's poster](#)

Angular distribution: Adhya, Kutak, Iaczek, Rohrmoser, Tywoniuk, [2211.15803](#)

Multiple splittings



Mehtar-Tani, Soudi, Schlichting, [2209.10569](#)



Out-of-cone energy loss via **medium-induced radiation**, followed by **elastic scatterings** of soft fragments pushing the distribution to **large angles** and thermalization

Jet quenching in the initial stages?

- **Jet quenching** is the only QGP signature **not** (yet?) **observed** in **small systems**
- In small systems the **initial stages** (IS) are specially important

Understanding jet quenching in the initial stages becomes crucial!

- Usually, jet quenching analyses neglect energy loss in the IS, but some observables have been found sensitive to the IS

Gonzalez-Martinez's

Wed 14:20

CA, Armesto, Niemi, Paatelainen, Salgado, [arXiv:1902.03231](#)

Stojku, Auvinen, Djordjevic, Huovinen, Djordjevic, [arXiv:2008.08987](#)

- Many new developments in the computation of the **broadening in the pre-hydro stages**

- In the Glasma phase: Ipp, Müller, Schuh, [arXiv:2001.10001](#), [arXiv:2009.14206](#)

Carrington, Czajka, Mrówczyński, [arXiv:2112.06812](#), [arXiv:2202.00357](#)

Avramescu, Baran, Greco, Ipp, Müller, Ruggieri, [2303.05599](#) Avramescu's talk

Tues 12:10

- Between the Glasma and hydro phases with Kinetic theory

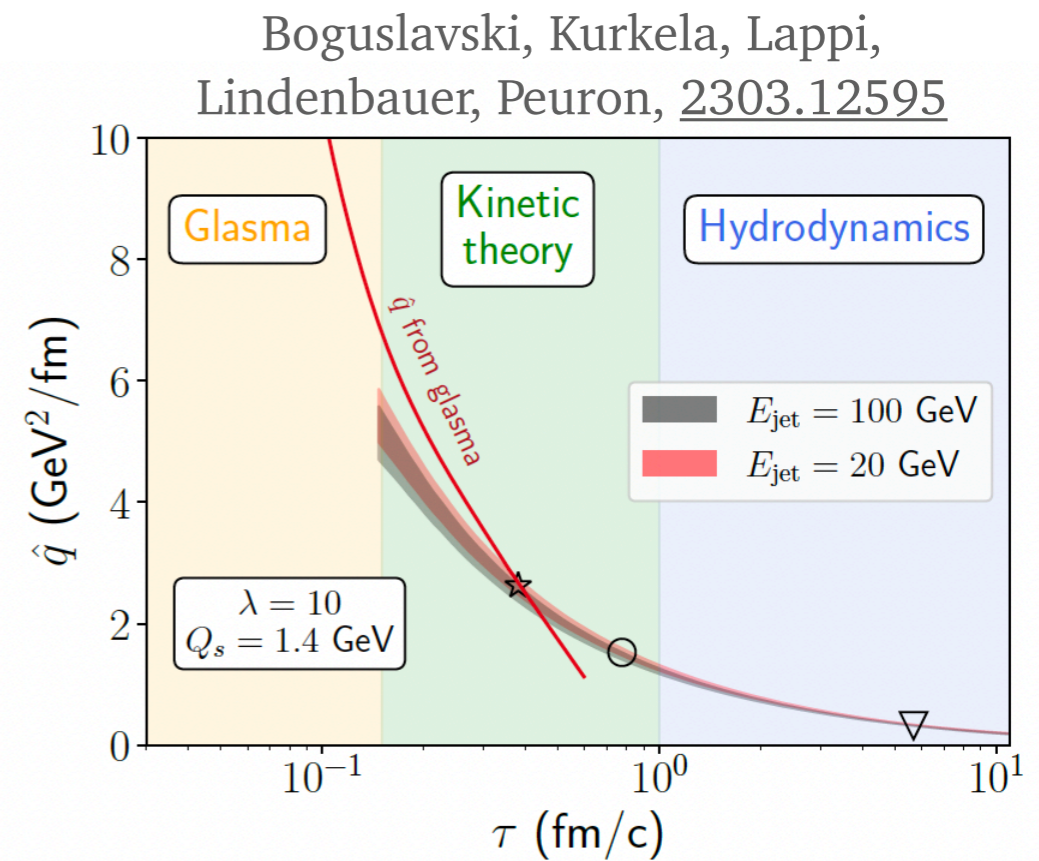
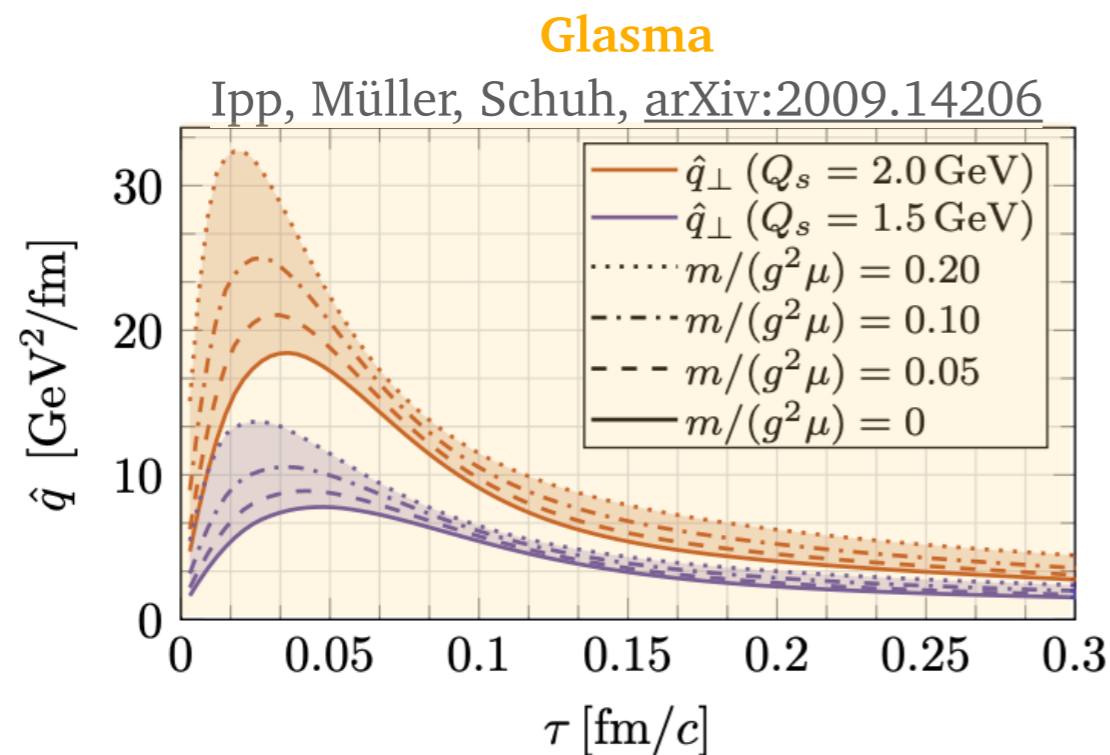
Boguslavski, Kurkela, Lappi,

Lindenbauer, Peuron, [2303.12595](#)

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\hat{q} relatively large!

Summary

- Many new developments in the theory of **medium-induced radiation**
 - **Improved analytical** approximations and **numerical solutions** for the emission spectrum in the soft limit ($z \ll 1$)
 - **Non-perturbative** determination of the potential (broadening kernel)
 - Results **beyond the soft limit keeping the angle**:
 - Opacity expansion
 - Semi-hard approximation for multiple scatterings
 - **First complete** results with **multiple scatterings**
 - Calculations coupling the jet to the **medium's transverse dynamics**
- **Overlapping effects** between multiple emissions in gluon showers found to be **small** (with caveats)
- Calculations of the **broadening** in the **pre-hydrodynamics** stages

Thank you!

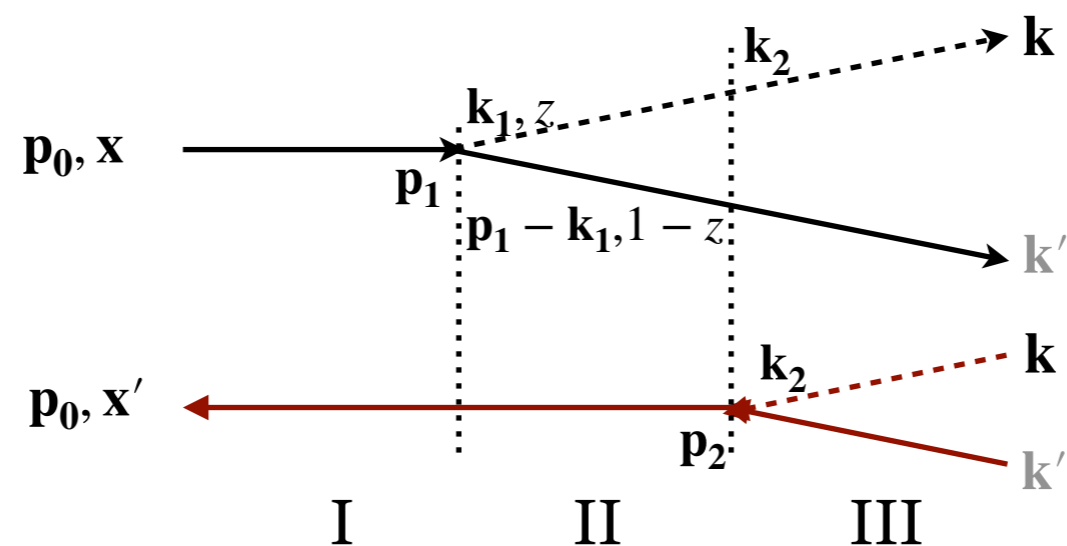
Beyond the soft approximation

- The emitted gluon carries a fraction of energy z of the energy of the parent quark

$$z \frac{dI}{dz d^2\mathbf{k}} = \frac{\alpha_s P_{g \leftarrow q}(z)}{(2\pi)^2 z(1-z)^2 E^2} \text{Re} \int_0^\infty dt' \int_0^{t'} dt \int_{\mathbf{p}_1 \mathbf{p}_2 \mathbf{k}_1 \mathbf{k}_2} \mathcal{P}_q(t, \mathbf{p}_1; 0, \mathbf{p}_0) \\ \times (\mathbf{k}_1 - z\mathbf{p}_1) \cdot (\mathbf{k}_2 - z\mathbf{p}_2) \tilde{\mathcal{K}}^{(3)}(t', \mathbf{k}_2 - z\mathbf{p}_2; t, \mathbf{k}_1 - z\mathbf{p}_1; \mathbf{p}_2 - \mathbf{p}_1) \\ \times \mathcal{P}_g(\infty, \mathbf{k}; t', \mathbf{k}_2)$$

Blaizot, Dominguez, Iancu, Mehtar-Tani arXiv:1209.4585

Apolinario, Armesto, Milhano, Salgado arXiv:1407.0599



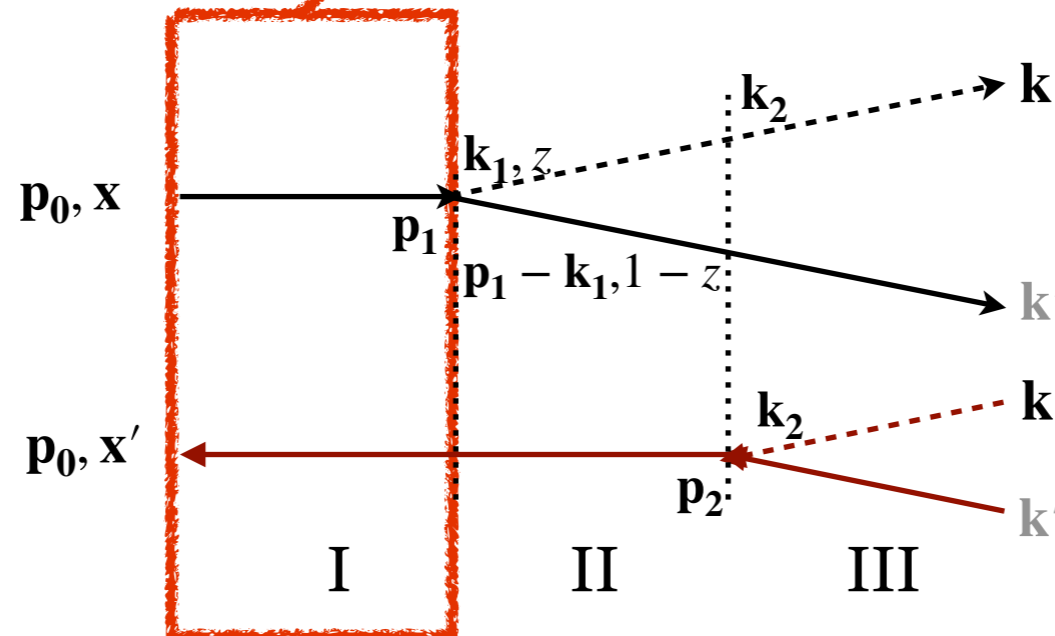
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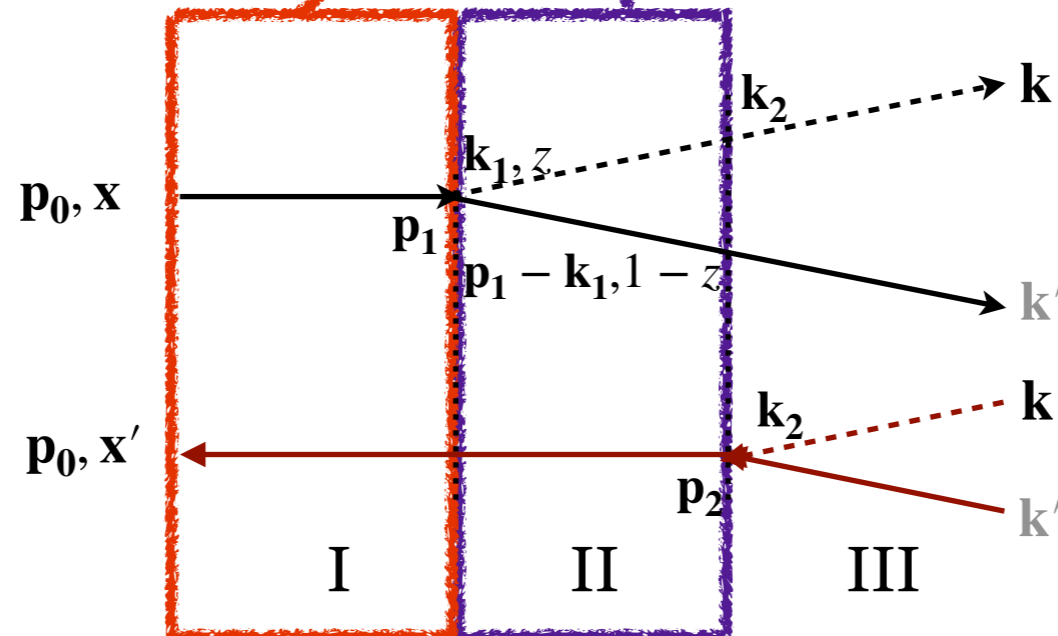
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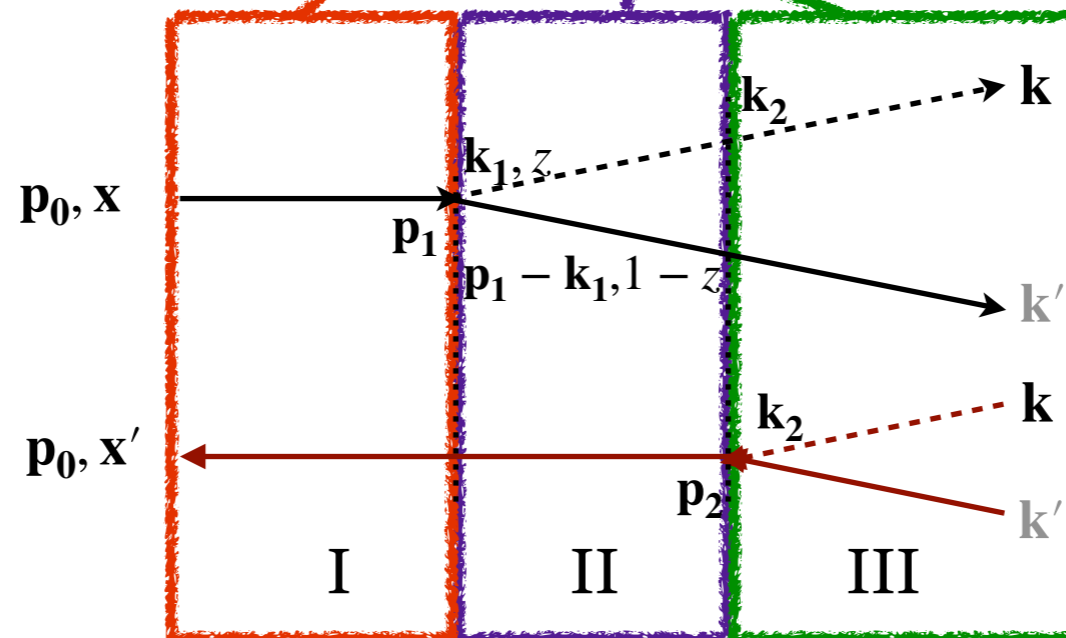
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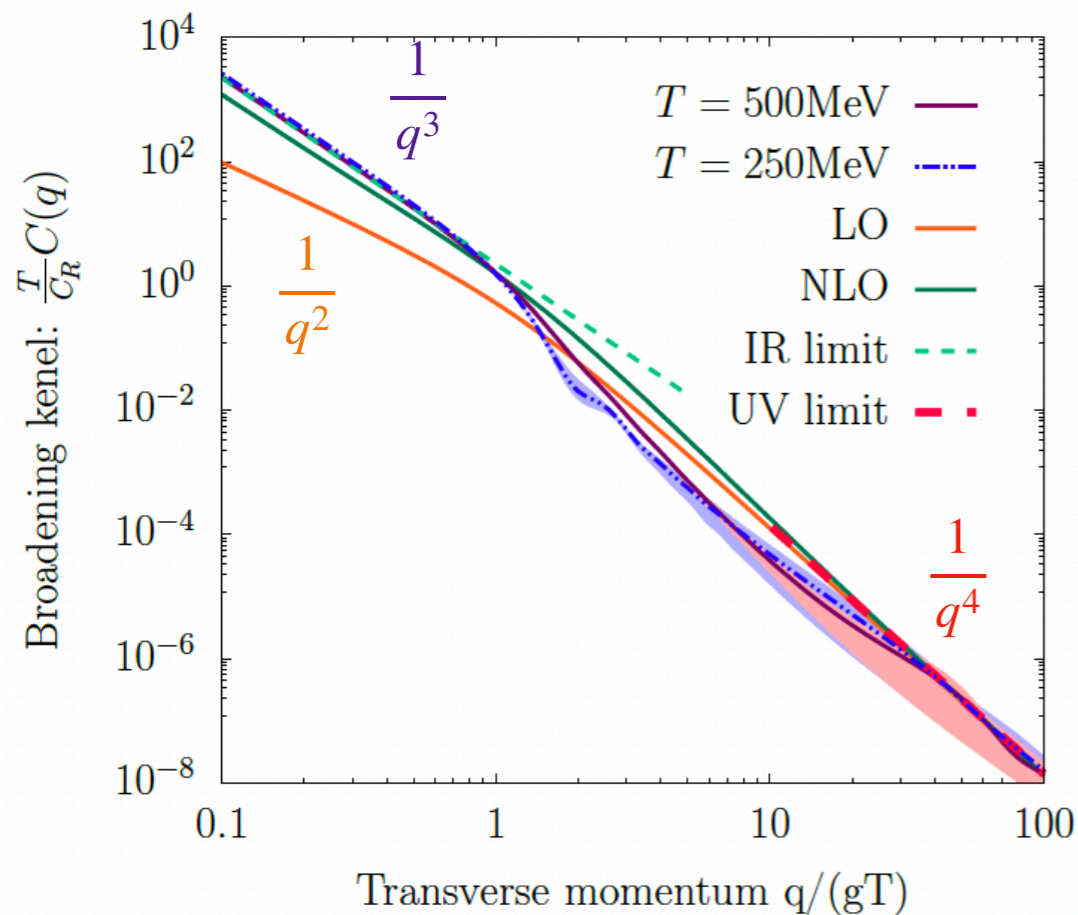
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Blaizot, Dominguez, Iancu, Mehtar-Tani arXiv:1209.4585

Apolinario, Armesto, Milhano, Salgado arXiv:1407.0599

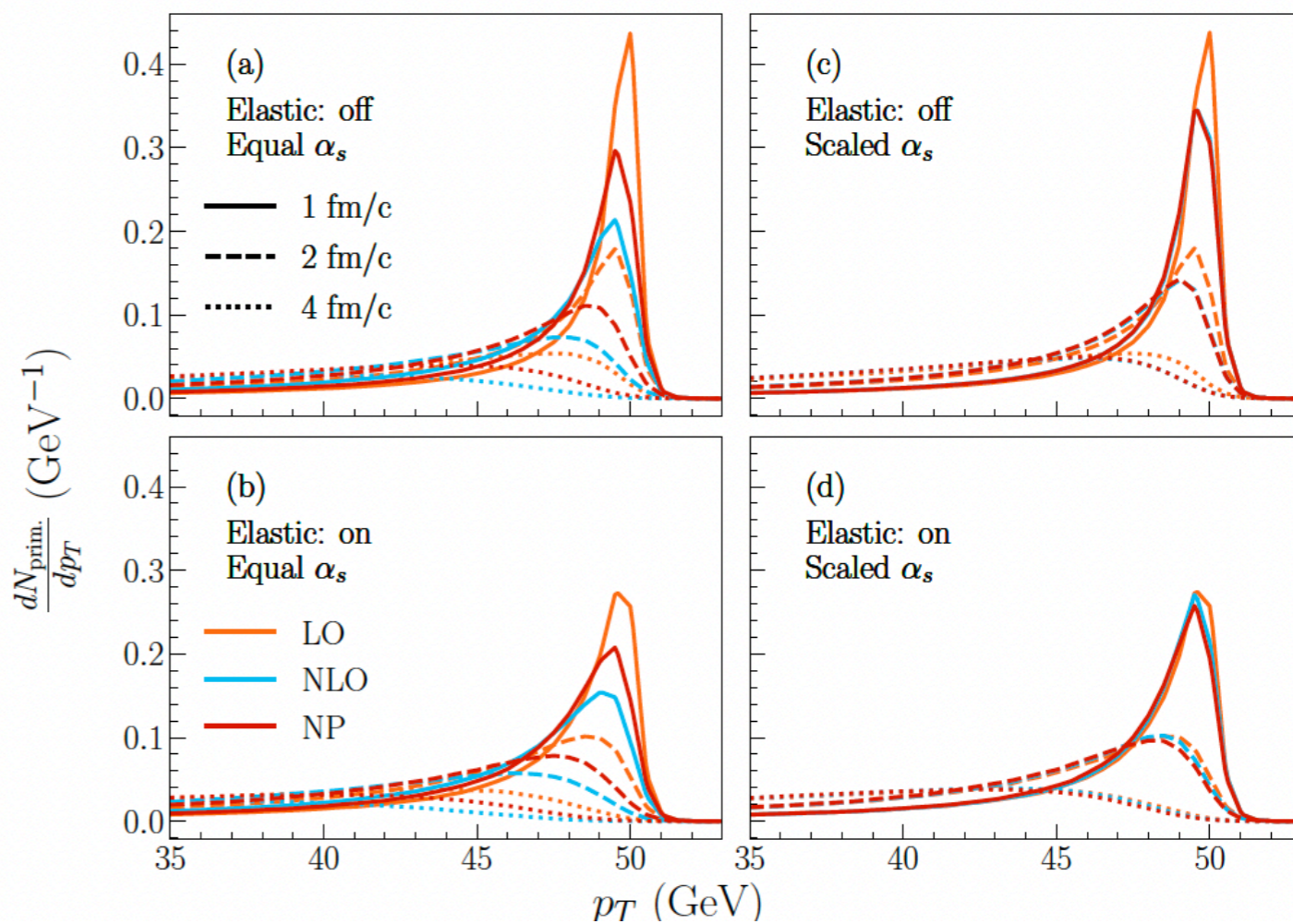


Non-perturbative potential



Schlichting, Soudi [2111.13731](#)

$E = 50\text{ GeV}, T = 300\text{ MeV}$



Yazdi, Shi, Jeon, Gale, [2206.05855](#)