Analytic and Semi-Analytic Calculations for Color Glass in the Weak Field Limit

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Color Glass and a Gaussian Model

- The nucleus of an atom moves at a significant fraction of the speed of light in a heavy ion collision. As a result the nucleus experiences a large amount of Lorentz contraction and its longitudinal extent can no longer be resolved by a gluon.
  - The nucleus is said to be in an effective state of matter called color glass (Gelis 2010).
- Color glass fields behave classically because of the large occupation numbers of the field. This is most applicable at the center of a large nucleus.
  - The classical description is enforced through the use of event averages. The weight functional of the volume charge density is taken to be Gaussian.

$$\langle \rho \rangle = 0 \qquad \langle \rho_{\underline{a}}(x)\rho_{\underline{b}}(y)\rangle = \frac{g^2}{{N_c}^2-1}\delta_{\underline{ab}}h(x^-)\delta(x^- - y^-)\mu(\vec{R})\mathcal{D}(\vec{r}) \qquad \int d^2r\,\mathcal{D}(\vec{r}) = 0$$

- One common model of color glass is the McLerran Venugopalan model which takes $\mathcal{D}(\vec{r})$ to be a 2D delta function. It is easily seen that this choice fails the integral on the right.
  - This integral test is important because it was shown that a hadron must pass it in order to be color neutral (Lam 1999).
- The form of $\mathcal{D}(\vec{r})$ can be derived from the same picture as IP-Glasma. The result forms a Gaussian model. It is found that this Gaussian model is free from UV singularities and is less susceptible to IR divergences.

$$\mathcal{D}(\vec{r}) = f(\vec{r}) - C(\vec{r}) \qquad f(\vec{r}) = \frac{1}{4\pi B_q}\exp\left[-\frac{r^2}{4B_q}\right] \qquad C(\vec{r}) = \frac{1}{4\pi\left(B_q + B_{qc}\right)}\exp\left[-\frac{r^2}{4\left(B_q+B_{qc}\right)}\right]$$

Yang-Mills Equations in the Weak Field Limit

- The weak field limit entails looking for a solution to the Yang-Mills equations to first order in the strong coupling constant g. Keeping A and $A_\perp$ to order g means changing the covariant derivative into an ordinary derivative. Reducing the known recursive solution (Chen 2015) to order g allows it to be re-summed in Fourier space.

$$\frac{1}{\tau}\frac{\partial}{\partial\tau}\frac{1}{\tau}\frac{\partial}{\partial\tau}\tau^2 A - \left[D^i,\left[D^i,A\right]\right] = 0$$
$$ig\tau\left[A,\frac{\partial}{\partial\tau}A\right] - \frac{1}{\tau}\left[D^i,\frac{\partial}{\partial\tau}A^i_\perp\right] = 0$$
$$\frac{1}{\tau}\frac{\partial}{\partial\tau}\tau\frac{\partial}{\partial\tau}A^i_\perp - ig\tau^2\left[A,\left[D^i,A\right]\right] - \left[D^j,F^{ji}\right] = 0$$

$$A_{(n)} = \frac{1}{n(n+2)}\sum_{k+l+m=n-2}\left[D^i_{(k)},\left[D^i_{(l)},A_{(m)}\right]\right] = \frac{1}{n(n+2)}\left(\nabla^2\right)A_{(n-2)} + O(g^2)$$

$$A^i_{\perp(n)} = \frac{1}{n^2}\left(\sum_{k+l=n-2}\left[D^j_{(k)},F^{ji}_{(l)}\right] + ig\sum_{k+l+m=n-4}\left[A_{(k)},\left[D^i_{(l)},A_{(m)}\right]\right]\right) = \frac{1}{n^2}\partial^j F^{ji}_{(n-2)} + O(g^2)$$

- Converting the re-summed potentials into chromo-electric and chromo-magnetic fields allows simple expressions for the classical phase of the initial stage stress energy tensor to be derived.

$$\tilde{A}(\tau,\vec{k}) = 2\tilde{A}_0\frac{J_1(k\tau)}{k\tau} + O(g^2)$$

$$\tilde{A}^i_\perp(\tau,\vec{k}) = \tilde{A}^i_{\perp 0} + \epsilon^{ij}\frac{(i\vec{k})^j}{k^2}\tilde{B}_z(0,\vec{k})\left[J_0(k\tau) - 1\right] + O(g^2)$$

$$\tilde{E}_i(\tau,\vec{k}) = \left(t\epsilon^{ij}(i\vec{k})^j\tilde{B}_z + z(i\vec{k})^i\tilde{E}_z\right)\frac{J_1(k\tau)}{k\tau} + O(g^2)$$
$$\tilde{E}_z(\tau,\vec{k}) = \tilde{E}_z J_0(k\tau) + O(g^2)$$
$$\tilde{B}_i(\tau,\vec{k}) = \left(-t\epsilon^{ij}(i\vec{k})^j\tilde{E}_z + z(i\vec{k})^i\tilde{B}_z\right)\frac{J_1(k\tau)}{k\tau} + O(g^2)$$
$$\tilde{B}_z(\tau,\vec{k}) = \tilde{B}_z J_0(k\tau) + O(g^2)$$

Analytic Expressions for the Stress Tensor Components

- Form functions f_i and φ_j are known series expansions of r'. Solutions taken to second order gradients in the color charge area density μ. Second order terms of ε_T, S_i, and T_{iz} are omitted for brevity. Full analytic solutions of the final integrals are obtainable but complex. Only terms with second order gradients of μ have a dependence on the IR cutoff m. These integral forms can also be used in the MV model to obtain analytic expressions.
- Longitudinal energy density:

$$\langle\epsilon_L\rangle = \frac{1}{2\pi}\int_0^{2\tau} dr'\left(4\tau^2 - r'^2\right)^{-1/2}\left[2\mu_1\mu_2(\vec{R})f_1(r') + \vec{\nabla}\mu_1\cdot\vec{\nabla}\mu_2(\vec{R})\left\{\frac{1}{4}\left(4\tau^2 - r'^2\right)f_1(r') + r'^2 f_2(r')\right\} + \left(\mu_1\nabla^2\mu_2 + \mu_2\nabla^2\mu_1\right)(\vec{R})\left\{\frac{4\tau^2 - r'^2}{8}f_1(r') + 2f_3(r') + f_4(r')\right\}\right.$$
$$+ \nabla^2\mu_1\nabla^2\mu_2(\vec{R})\left\{\frac{(4\tau^2 - r'^2)^2}{256}f_1(r') + \frac{4\tau^2 r'^2 - r'^4}{16}f_2(r') + \frac{4\tau^2 - r'^2}{4}f_3(r') + \frac{12\tau^2 - 3r'^2}{16}f_4(r') + 2f_5(r') + 2f_6(r') + \frac{1}{4}f_9(r')\right\}$$
$$\left.+\nabla^{ij}\mu_1\nabla^{ij}\mu_2(\vec{R})\left\{\frac{(4\tau^2 - r'^2)^2}{128}f_1(r') + \frac{4\tau^2 r'^2 - r'^4}{8}f_2(r') - \frac{4\tau^2 - r'^2}{8}f_4(r') + 2f_7(r') + f_8(r') + \frac{1}{2}f_9(r')\right\}\right]$$

- Transverse energy density:

$$\langle\epsilon_T\rangle = \frac{1}{4\pi\tau^2}\int_0^{2\tau} dr'\frac{2\tau^2 - r'^2}{\left(4\tau^2 - r'^2\right)^{1/2}}\left[2\mu_1\mu_2(\vec{R})f_1(r') + \vec{\nabla}\mu_1\cdot\vec{\nabla}\mu_2(\vec{R})\left\{\frac{1}{4}\left(4\tau^2 - r'^2\right)f_1(r') + r'^2 f_2(r')\right\}\right.$$

- Transverse Poynting vector:

$$\langle S_i\rangle = \frac{1}{\pi\tau}\int_0^{2\tau} dr'\left(4\tau^2 - r'^2\right)^{1/2}\left[-\nabla^i(\mu_1\mu_2)(\vec{R})\frac{f_1(r')}{4}\right]$$

- Transverse longitudinal shear stress:

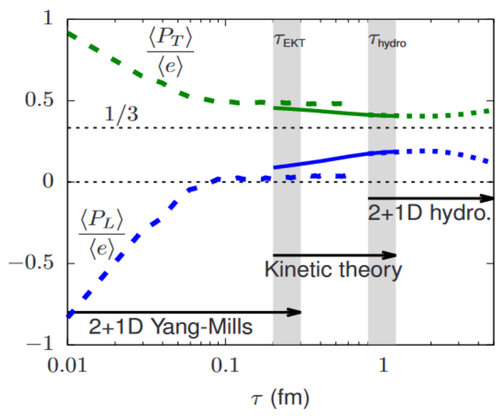
$$\langle T_{iz}\rangle = \frac{1}{2\pi\tau}\int_0^{2\tau} dr'\frac{r'^2}{\left(4\tau^2 - r'^2\right)^{1/2}}\left[\left(\mu_1\nabla^i\mu_2 - \mu_2\nabla^i\mu_1\right)(\vec{R})\phi_1(r')\right]$$

- Longitudinal Poynting vector:

$$\langle S_z\rangle = \frac{1}{2\pi\tau^2}\int_0^{2\tau} dr'\left(4\tau^2 - r'^2\right)^{-1/2} r'^2\left(\mu_1\nabla^2\mu_2 - \mu_2\nabla^2\mu_1\right)(\vec{R})\left\{-\frac{4\tau^2 - r'^2}{4}\phi_1(r')\right\}$$

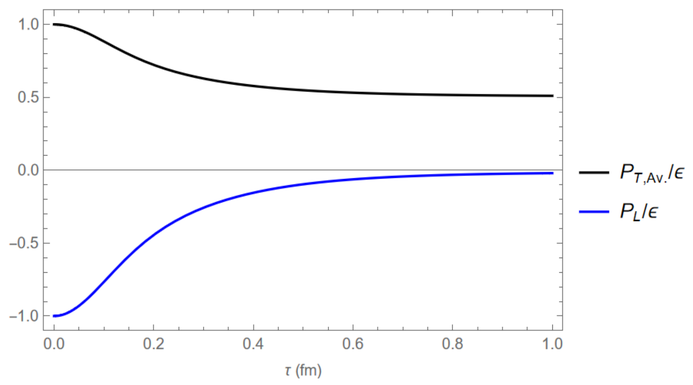
Applications

- One direct application of this work is to calculate the initial conditions needed for later phases of a heavy ion collision. Some authors (Kurkela et al. 2019) use a second phase to the initial stage, consisting of an effective kinetic theory, to smooth the transition to thermodynamic equilibrium. A rough sketch on when to start their effective kinetic theory shows it may start ~.2fm/c after the collision.



- Fig. 3 in (Kurkela et al. 2019) shows the numerical IP-glasma simulation of the first phase which makes p_L vanish and brings p_T to half the energy density ~.2 fm/c after the collision. This is when their effective kinetic theory may start.

- Our work provides a more detailed picture of the first phase in the weak field limit. The pressures approach the same limits as in IP-glasma. Here, ~0.7fm/c after the collision seems a more appropriate time to begin the second phase, due to the Abelianization of the Yang-Mills equations.



- The angular momentum can be directly calculated from the stress tensor relations. To second order gradients in μ, it can be written in terms of the first time evolution functions in the transverse longitudinal shear stress and the longitudinal Poynting vector. It can be seen this quantity eventually becomes constant.

$$\frac{dL_x}{d\eta} = \frac{dL_z}{d\eta} = 0$$

$$\frac{dL_y}{d\eta} = \tau\cosh\eta\left(\tau\sinh^2\eta T_{xz,1}(\tau) + \cosh(2\eta)S_{z,1}(\tau)\right)$$
$$\int d^2R\left(\mu_1\nabla^x\mu_2 - \mu_2\nabla^x\mu_1\right)(\vec{R})$$

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