# Evolution of the initial state in the hotspot model of the proton structure

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## Abstract

The hotspot model has proven to be an efficient tool to study coherent and incoherent diffraction HERA data by modelling the initial state of the proton. The hotspot model in its original form is a non-perturbative model applicable for low momentum transfer and underestimates the incoherent cross section in orders of magnitude when extended for large momentum transfer studies for J/ψ photo-production at HERA. We present here a model of hotspot splittings based on the resolution for the evolution of the initial state of the proton. The incoherent diffraction at large momentum transfer probes the gluon wave function at smaller length scales as we increase the resolution which appears as hotspot splittings in our model. In addition to the geometrical fluctuations, we have additional sources of fluctuations in our model namely the hotspot width, number, and normalisation fluctuations which leads to a good agreement of our model's prediction with data.

## Diffractive Vector Meson production in the dipole picture

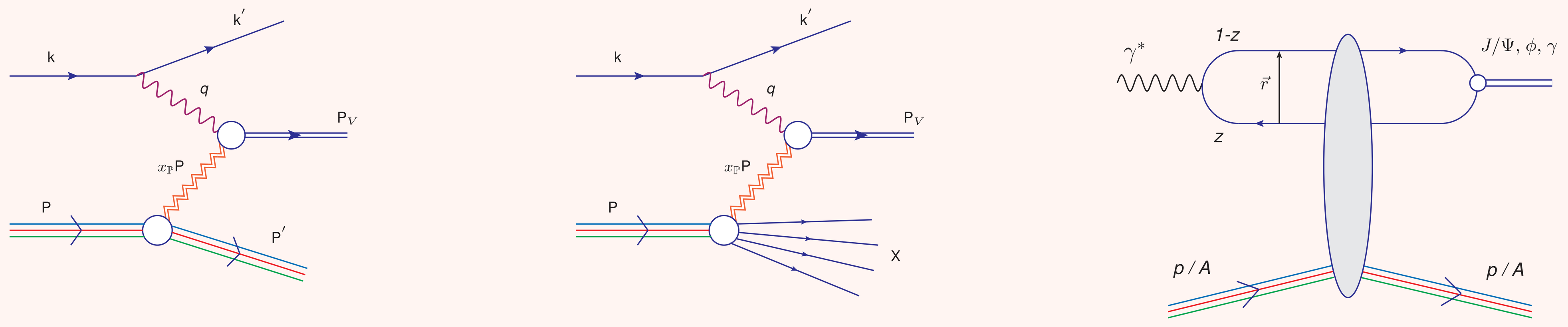


Figure 1. a) Coherent diffractive events b) Incoherent (proton dissociative) diffractive events in $ep$ collisions c) Exclusive vector meson production in the dipole picture

## The dipole model

High energy factorisation:

- $\gamma^* \to q\bar{q}$ splitting : QED
- Dipole-proton interaction (QCD)
- Dipole → vector meson (modelling)

Access to the geometry :

Impact parameter is Fourier conjugate to the momentum transfer Δ= (p'-p)$_\perp$

The scattering amplitude is given as:

$$A_{T,L}^{\gamma^* p\to Vp}(x,Q^2,\Delta) \simeq \int d^2r \int d^2b \int dz \times (\Psi^*\Psi_V)_{T,L}(Q^2,r,z) \times e^{-ib.\Delta} \times N(b,r,x,\Omega)$$

bSat : $N(\mathbf{b},\mathbf{r},x)=2\left[1-\exp\left(-\frac{\pi^2}{2N_C}\mathbf{r}^2\alpha_s(\mu^2)xg(x,\mu^2)T_p(\mathbf{b})\right)\right]$

bNonSat : *linearised version*

## Good-Walker picture

The total cross section is :

$$\sigma_{tot} \propto < \mathcal{A} >^2 + (< \mathcal{A}^2 > - < \mathcal{A} >^2)$$

In Good-Walker paradigm:

$$\begin{aligned}\sigma_{incoherent} &\sim \sum_{f\neq i} |< f|\mathcal{A}|i >|^2\\ &= \sum_f < i|\mathcal{A}^\dagger|f >< f|\mathcal{A}|i > - < i|\mathcal{A}|i >^\dagger < i|\mathcal{A}|i >\\ &= \left\langle|\mathcal{A}|^2\right\rangle_\Omega - \left|\langle\mathcal{A}\rangle_\Omega\right|^2\end{aligned}$$

Coherent Cross section: probes average geometry

$$\frac{d\sigma_{coherent}}{dt} = \frac{1}{16\pi}\left|\langle\mathcal{A}\rangle_\Omega\right|^2$$

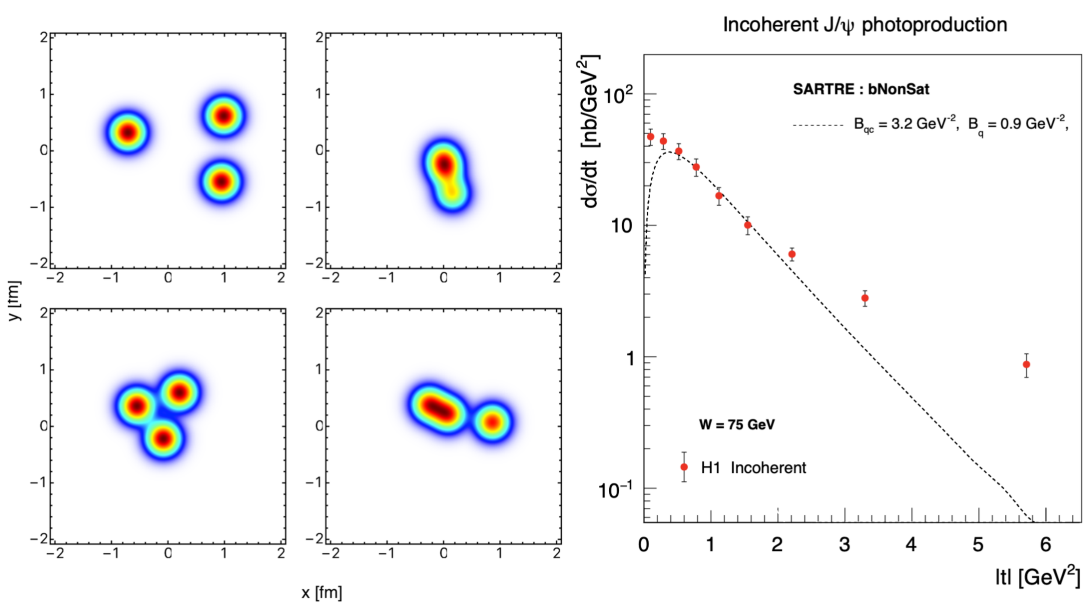
Incoherent Cross section: probes fluctuations

$$\frac{d\sigma_{incoh}}{dt} = \frac{1}{16\pi}\left\langle|\mathcal{A}|^2\right\rangle_\Omega - \left|\langle\mathcal{A}\rangle_\Omega\right|^2$$

## The spatial proton profile

Hotspot model : Lumpy proton [1, 2]

$T_p(b) \to \sum_{i=1}^{N_q} T_q(b-b_i)$ and $T_q(\mathbf{b}) = \frac{1}{2\pi B_q}\exp\left[-\frac{\mathbf{b}^2}{2B_q}\right]$

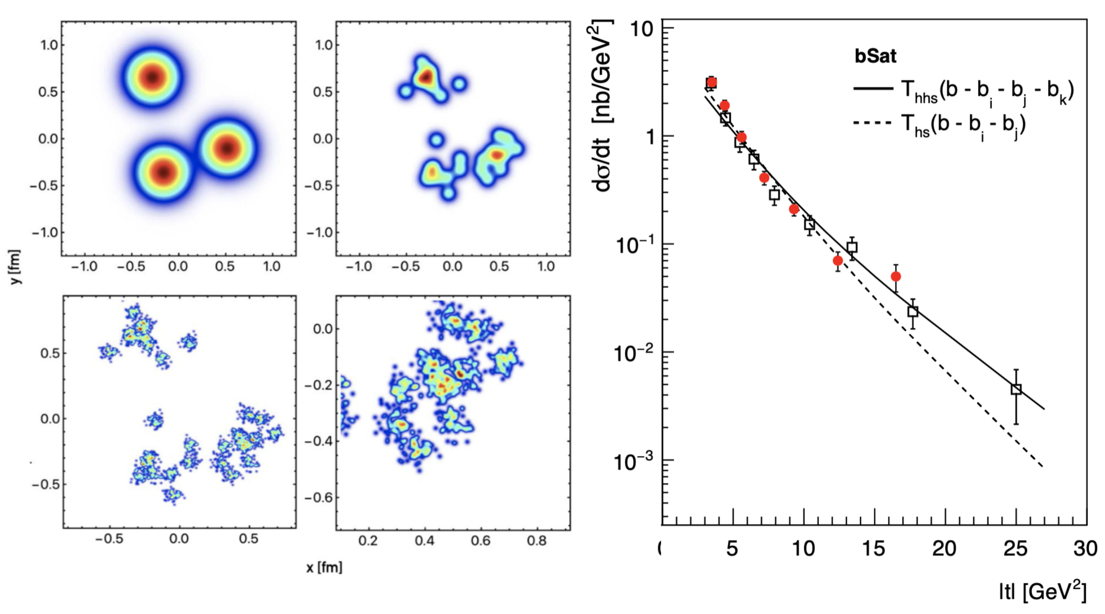



- Underestimates the cross section at large |t|

## Hotspots within hotspots . . .

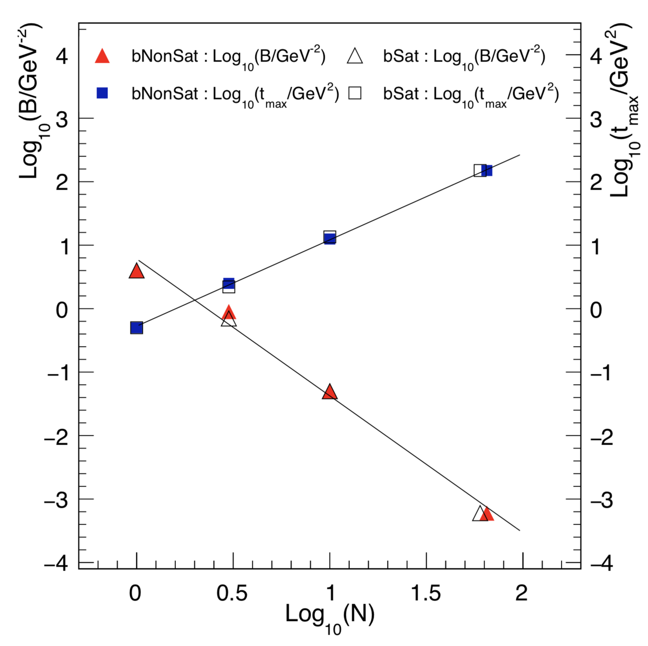
More complicated structure [3]:

$$T_P(b) \to \frac{1}{N_q N_{hs} N_{hhs}} \sum_{i=1}^{N_q}\sum_{j=1}^{N_{hs}}\sum_{k=1}^{N_{hhs}} T_{hhs}(\mathbf{b}-\mathbf{b_i}-\mathbf{b_j}-\mathbf{b_k})$$




| Model | $B_{qc}$ | $B_q$ | $N_q$ | $B_{hs}$ | $N_{hs}$ | $B_{hhs}$ | $N_{hhs}$ |
|---|---|---|---|---|---|---|---|
| bSat | 3.2 | 1.15 | 3 | 0.05 | 10 | 0.0006 | 65 |

- Additional event-by-event small size fluctuations contributing at different scales in the |t| spectrum




Scaling behaviour in the parameters of the model

Insights:

- Proton becomes dilute at large |t|
- Scaling behaviour suggests that we can describe the t-spectrum with a linear scale independent (in log |t|) evolution for the increasing number of hotspots

## The Hotspot Evolution model

Idea : Transverse part of gluon wavefunction probed with areal resolution $\delta b^2 \sim \frac{1}{|t|}$, increased resolution appears as hotspot splitting

Analogy : Momentum transfer ↔ Resolution in optics


- *Hotspot model as the initial state for $t = t_0$*
- *Evolution of the initial state as splitting of the hotspots based on the resolution for $t > t_0$*

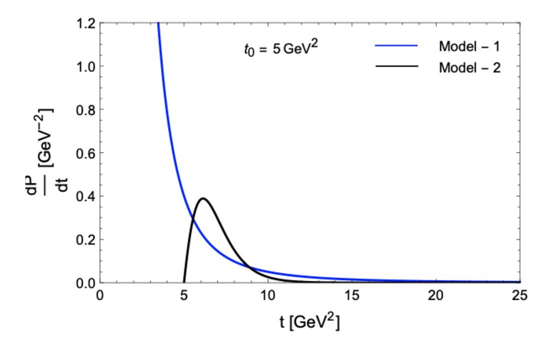
Different models for evolution:

Model 1 : $\frac{d\mathcal{P}_{split}}{dt} = \frac{\alpha}{|t|}$, $\frac{d\mathcal{P}_{no-split}}{dt} = \exp\left(-\int_{t_0}^{t} dt' \frac{d\mathcal{P}_{split}}{dt'}\right)$

$$\frac{d\mathcal{P}_a}{dt} = \frac{\alpha}{t}\left(\frac{t_0}{t}\right)^\alpha$$

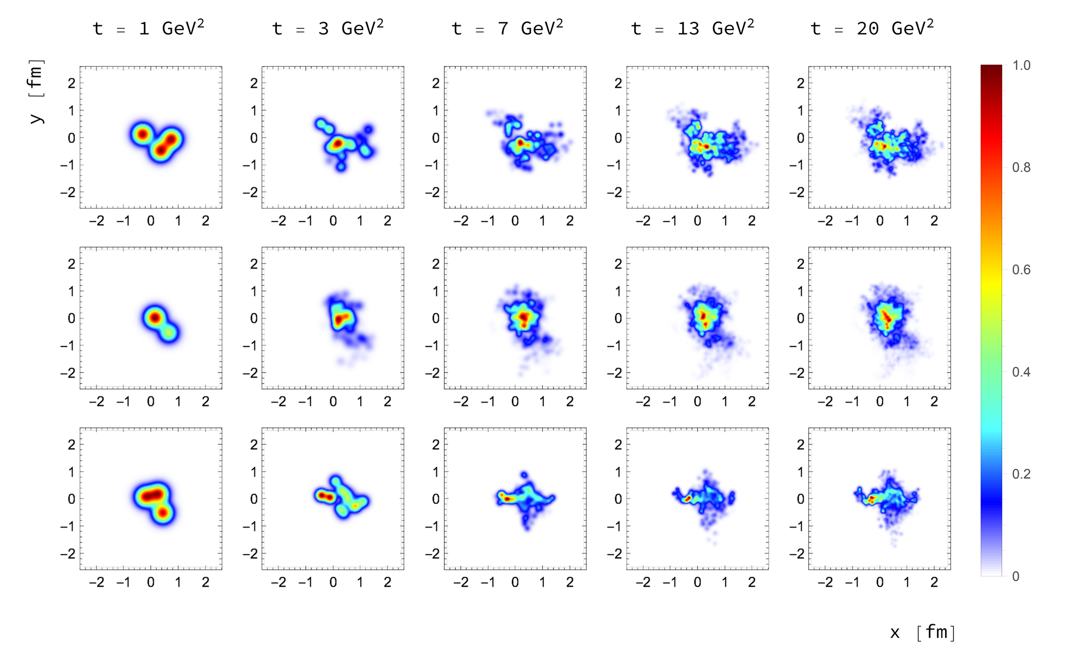
Model 2 : $\frac{d\mathcal{P}_{split}}{dt} = \frac{\alpha}{|t|}\frac{t-t_0}{t}$, $\frac{d\mathcal{P}_{no-split}}{dt} = \exp\left(-\int_{t_0}^{t} dt' \frac{d\mathcal{P}_{split}}{dt'}\right)$

$$\frac{d\mathcal{P}_a}{dt} = \frac{\alpha}{t}\frac{t-t_0}{t}\exp\left[-\alpha\left(\frac{t_0}{t} - \ln\frac{t_0}{t} - 1\right)\right]$$

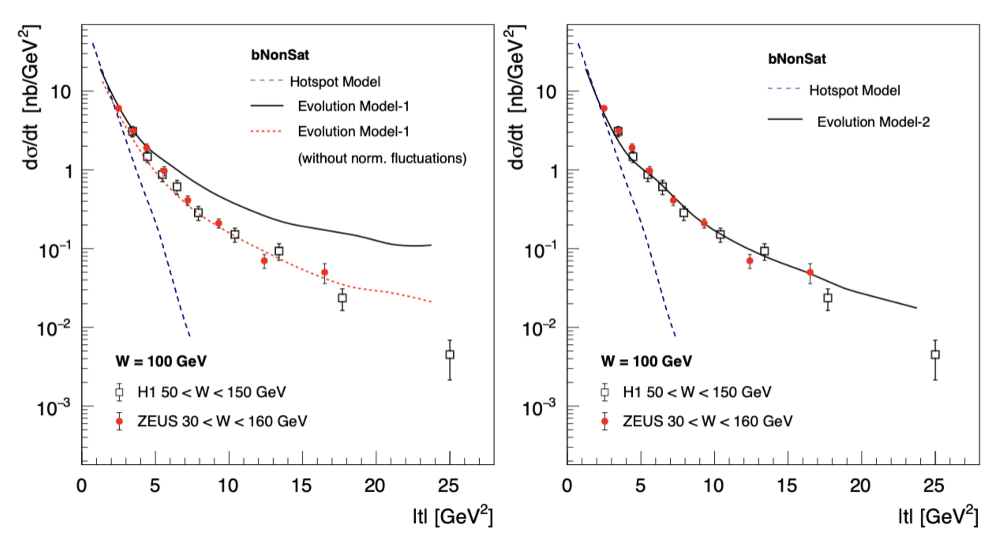



Steps:

- offspring hotspots i, j created at distance $d_{ij} = |b_i - b_j|$ sampled from parent hotspot with widths $B_{i,j} = \frac{1}{|t|}\mathrm{GeV}^{-2}$
- Probe & geometry resolution criterion : $d_{ij} > 2\sqrt{B_{i,j}}$ Reject if not resolved (effective hotspot repulsion)




## Results




- Additional event-by-event fluctuations at large |t| when we divide normalisation in each splitting
- Various sources of fluctuations in evolution models: Hotspot width, number, normalisation [4]

## Outlook & References

- Currently investigating several models, promising results for the whole t-spectrum with the addition of only 2 parameters
- Our model has size evolution from original geometry to hotspots becoming point like (as point-like sources in CGC)
- For eA or AA the hotspots could be very hot leading to large saturation scale fluctuations and become sensitive to the non-linear effects

[1] H. Mäntysaari, B. Schenke, Phys. Rev. D94 (3) (2016) 034042. doi:10.1103/PhysRevD.94.034042.

[2] H. Mäntysaari, B. Schenke, Phys. Rev. Lett. 117 (5) (2016) 052301. doi:10.1103/PhysRevLett.117.052301.

[3] A. Kumar, T. Toll, Eur. Phys. J. C 82 (9) (2022) 837. doi:10.1140/epjc/s10052-022-10774-3.

[4] A. Kumar, T. Tolldoi:in-preparation.