Universality of energy-momentum response in conformal kinetic theories

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Hydrodynamics

Hydrodynamic theory

- Macroscopic theory at long wave-length, low frequency limit

$$k l_{mfp} < 1, \qquad \omega t_{mfp} < 1$$

- Constructed as an order expansion of gradients close to equilibrium

$$\underbrace{\underbrace{\langle T^{\mu\nu}\rangle = T^{\mu\nu}_{(0)}}_{\text{Ideal hydro}} + T^{\mu\nu}_{(1)}(\nabla) + T^{\mu\nu}_{(2)}(\nabla^2)}_{\text{Viscous hydro}} + \cdots$$

Hydrodynamic modes

small gradients $\qquad \nabla \sim k$ small

vanishing frequency $\qquad \omega(k \to 0) = 0$

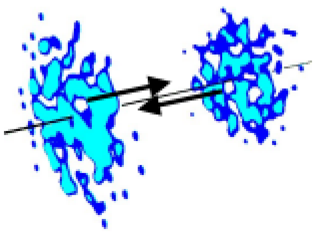
Hydrodynamics in relativistic heavy-ion collisions (HICs)

- Successful description of near-equilibrium quark-gluon plasma (QGP)
- Medium background for hard probes (jets, heavy quarks, quarkonium, etc..)
- Energy deposition to medium from hard probes as well

Non-hydrodynamics

Non-equilibrium dynamics beyond hydrodynamics in HICs

- Equilibration of jets in near-equilibrium QGP — Kinetic theory
- Pre-equilibrium stage of HICs



Kinetic theory	Viscous hydro	Ideal hydro
non-equilibrium	near-equilibrium	equilibrium

Non-hydrodynamic modes: anything not hydrodynamic …

large gradients $\nabla \sim k$ not small

non-vanishing frequency $\omega(k \to 0) \neq 0$

How to study non-hydrodynamic modes

- Any correlator in quantum field theory out of hydrodynamic region
- Specifically, effective kinetic theory with linear response

Effective kinetic theory

Linearized effective kinetic theory

- Consider an Effective Kinetic Theory (set of Boltzmann equations)

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{p^0} \cdot \nabla_x\right) f_a(t,x,p) = -C_a^{LO\ 2\leftrightarrow 2, 1\leftrightarrow 2}[f](t,x,p)$$

- Decompose distribution into spatially homogeneous background f(t,p) and inhomogeneous perturbation δf(t,x,p)

$$f_a(t,x,p) = \underbrace{\boxed{f_a(t,T,p)}}_{\text{Background}} + \underbrace{\boxed{\delta f_a(t,x,p)}}_{\text{Perturbation}}$$

- Fourier transform: **gradients → wavenumber k**

$$\delta f_a(t,\boldsymbol{k},p) = \int \frac{d^3x}{(2\pi)^3} e^{-ix\cdot k} \delta f_a(t,\boldsymbol{x},p)$$

- Results in a Linearized Effective Kinetic Theory with a wavenumber **k**

$$\left(\frac{\partial}{\partial t} + \frac{ip\cdot k}{p^0}\right) \delta f_a(t,k,p) = -\delta C_a^{LO\ 2\leftrightarrow 2, 1\leftrightarrow 2}[f](t,k,p)$$

Linear response of kinetic theory

Energy-momentum tensor

- Background

$$\boxed{T^{\mu\nu}(t)} = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{p} \sum_a \nu_a f_a(t,p)$$

- Perturbation

$$\boxed{\delta T^{\mu\nu}(t,k)} = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{p} \sum_a \nu_a \delta f_a(t,k,p)$$

Response function

- Response function in terms of time **t** and wavenumber **k**

$$G^{\mu\nu}_{\alpha\beta}(t,k) = \frac{\delta T^{\mu\nu}(t,k)}{\delta T^{\alpha\beta}(0,k)}$$

- Consider initial condition with scalar type perturbation

$\boxed{f_a(t_0,p)} = f_a^{eq}(T,p)$ Background $\boxed{\delta f_a(t_0,k,p)} = -\frac{\delta T}{T} \partial_p f_a^{eq}(T,p)$ Perturbation

- Sound channel $G^{00}_{00}(t,k) = \frac{\delta T^{00}(t,k)}{\delta T^{00}(0,k)}$ (not the only one, but they are related)

Universal scales

- Response functions are in terms of **t** and **k**
- Different interaction strengths give different relaxation times

Universal scales

- Relaxation time

$$\tau_R \propto \eta/s$$

- Rescaled & dimensionless time and wave-number

$$\bar{t} = t\frac{sT}{\eta} \qquad \bar{k} = k\frac{\eta}{sT}$$

Hydrodynamization time $\boxed{\bar{t}_H = 4\pi}$

Response function in universal scales

$$G(t,k) \rightarrow G\left(\bar{t},\bar{k}\right)$$

- 1st order hydrodynamics response function can be formulated in universal scales

$$G_{\text{hydro}}^{\text{1st}}(t,k) = \cos(c_s k t)\, e^{-\Gamma k^2 t} \quad \text{with} \quad \Gamma = \frac{2}{3}\frac{\eta}{sT}$$

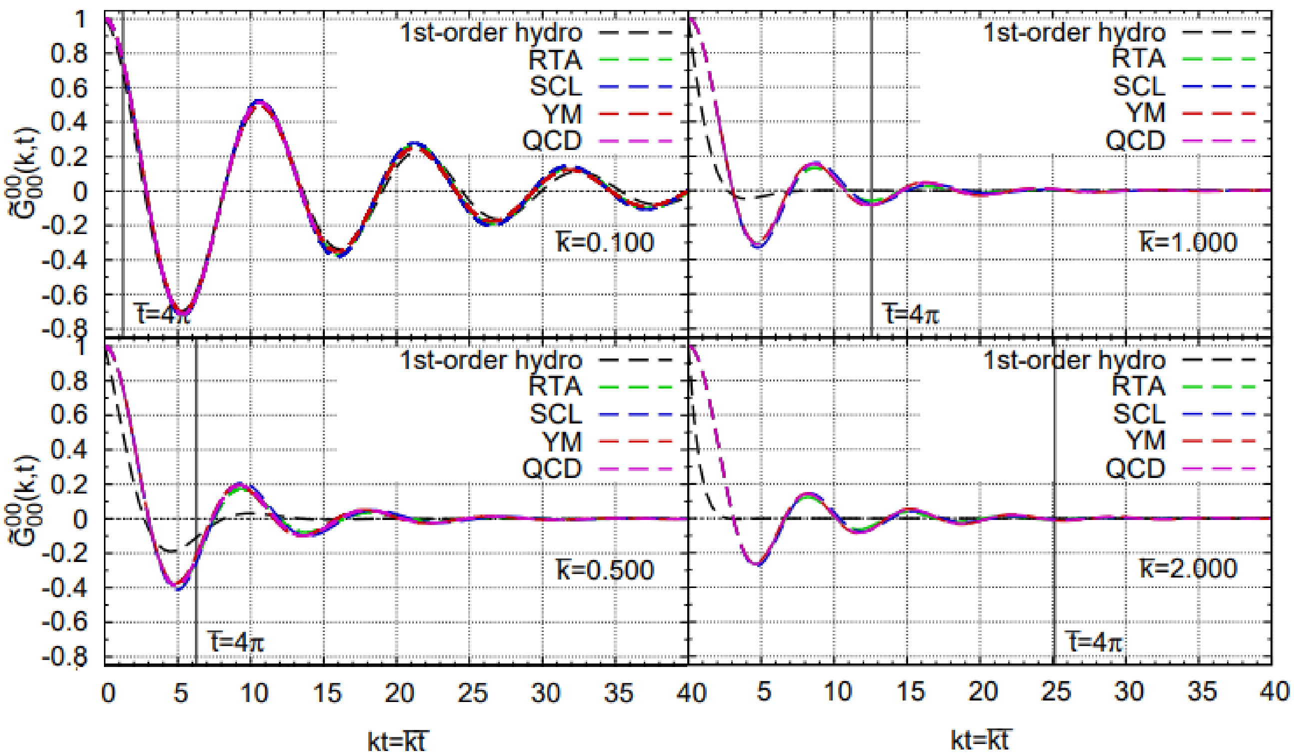
So that $G_{\text{hydro}}^{\text{1st}}(t,k) = \underbrace{\boxed{\cos\left(c_s \bar{k}\bar{t}\right)}}_{\text{Dispersion}}\, \underbrace{\boxed{e^{-\frac{2}{3}\bar{k}^2\bar{t}}}}_{\text{Damping}}$

Universality among kinetic theories

- Relaxation time approximation (RTA), φ-4 scalar theory (SCL)
- Yang-Mills theory (YM), Quantum chromodynamics (QCD)

Response functions from kinetic theories

- Expected to reproduce hydrodynamics at small k (long wave-length limit)
- Universality among different kinetic theories even at large k and early time

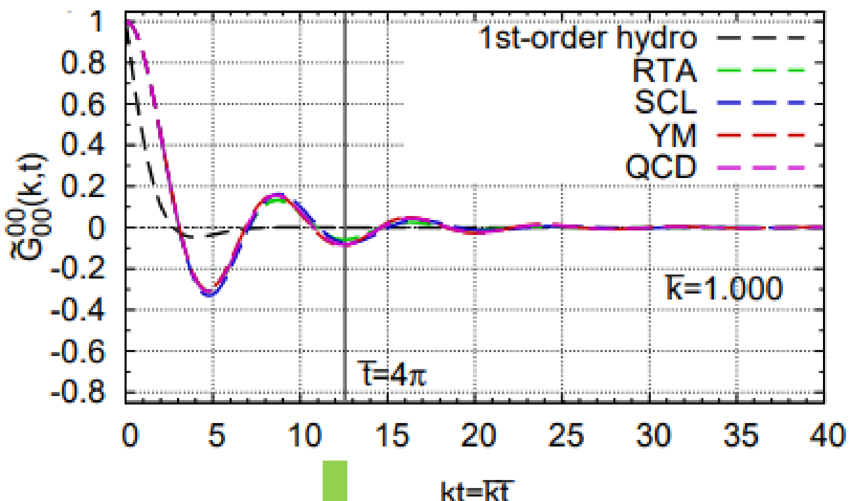


Sound and non-hydrodynamic modes

Fitting response functions in kinetic theory

- With real (oscillation) and imaginary (damping) frequencies

$$G_{s,n}(t,k) \sim Z_k \exp[-i(\boxed{\omega_k t} + \phi_k)] \qquad \omega_k = Re[\omega_k] + iIm[\omega_k]$$



non-hydro mode

$G_n(\bar{t} < \bar{t}_H, k)$

sound mode

$G_s(\bar{t} > \bar{t}_H, k)$

Hydrodynamization time

$\bar{t}_H = 4\pi$

Remarks

- Negative frequency gives the same mode as positive frequency

Sound modes appear in pair

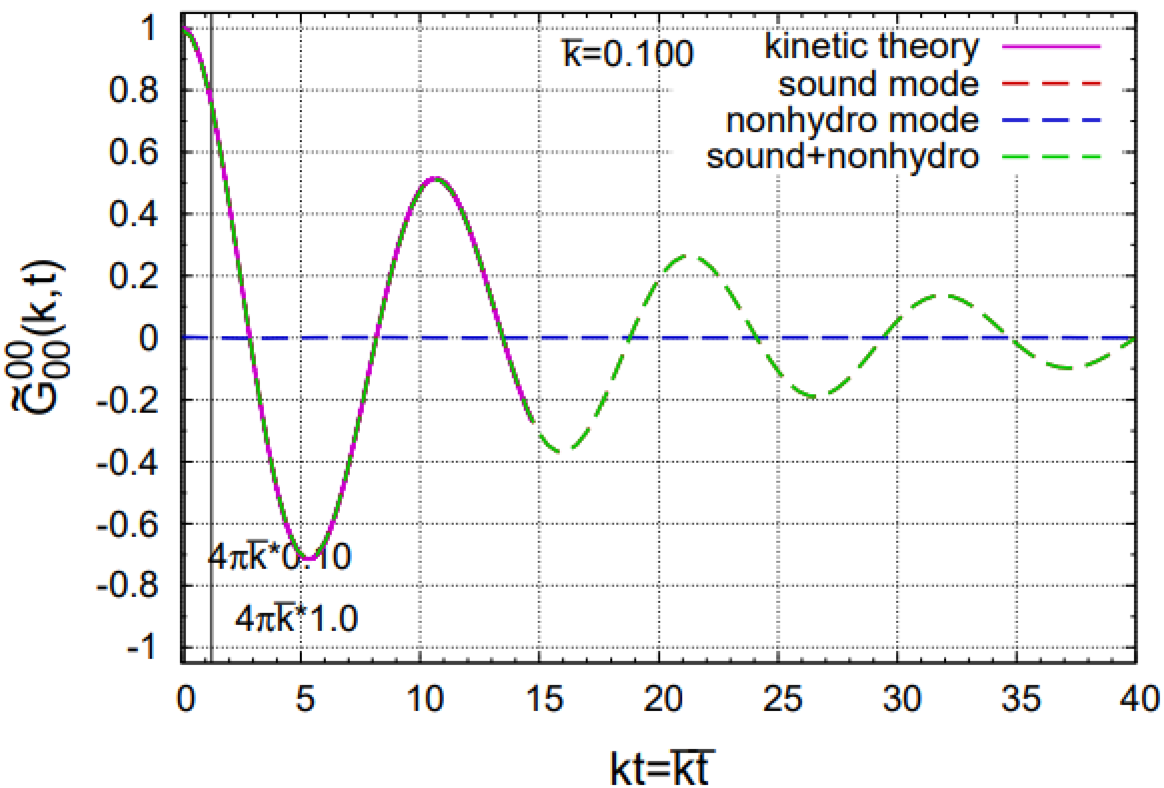
- Expected many/infinite number of non-hydrodynamic modes

We represent them with a single non-hydrodynamic mode

Sound and non-hydrodynamic modes

Fitting QCD response functions ($\bar{k}$=0.1)

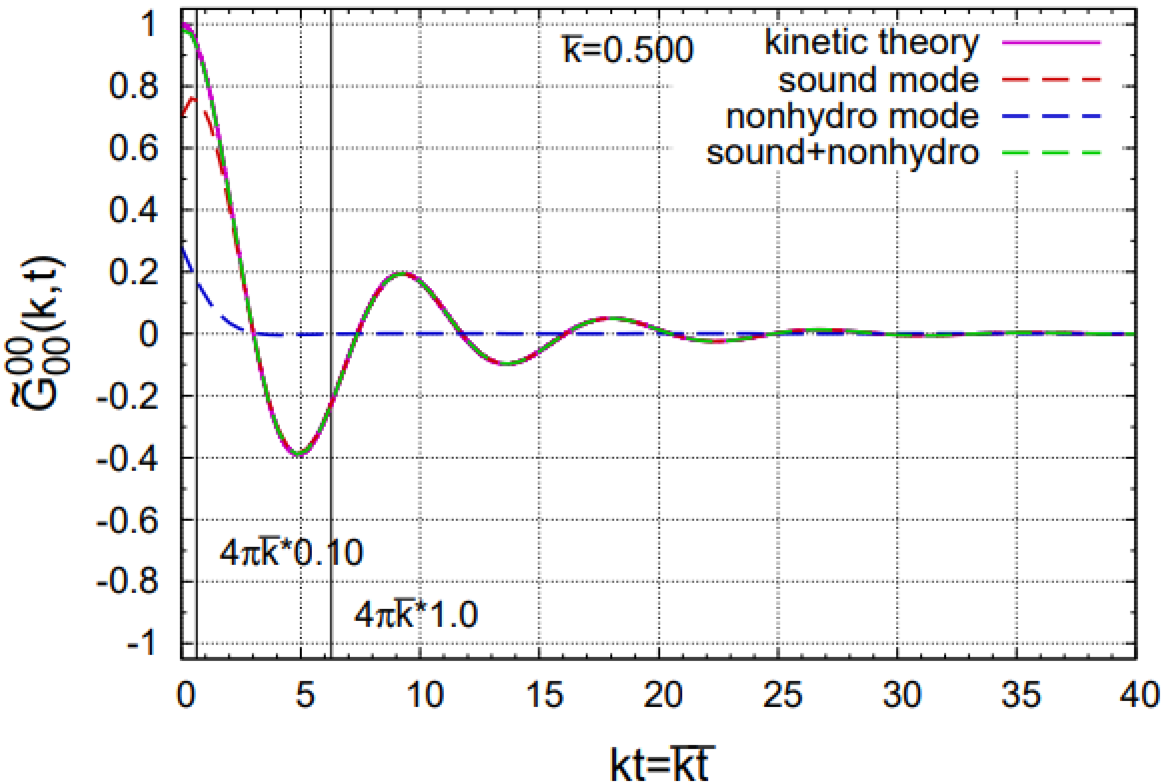
- Sound mode dominates



Sound and non-hydrodynamic modes

Fitting QCD response functions ($\bar{k}$=0.5)

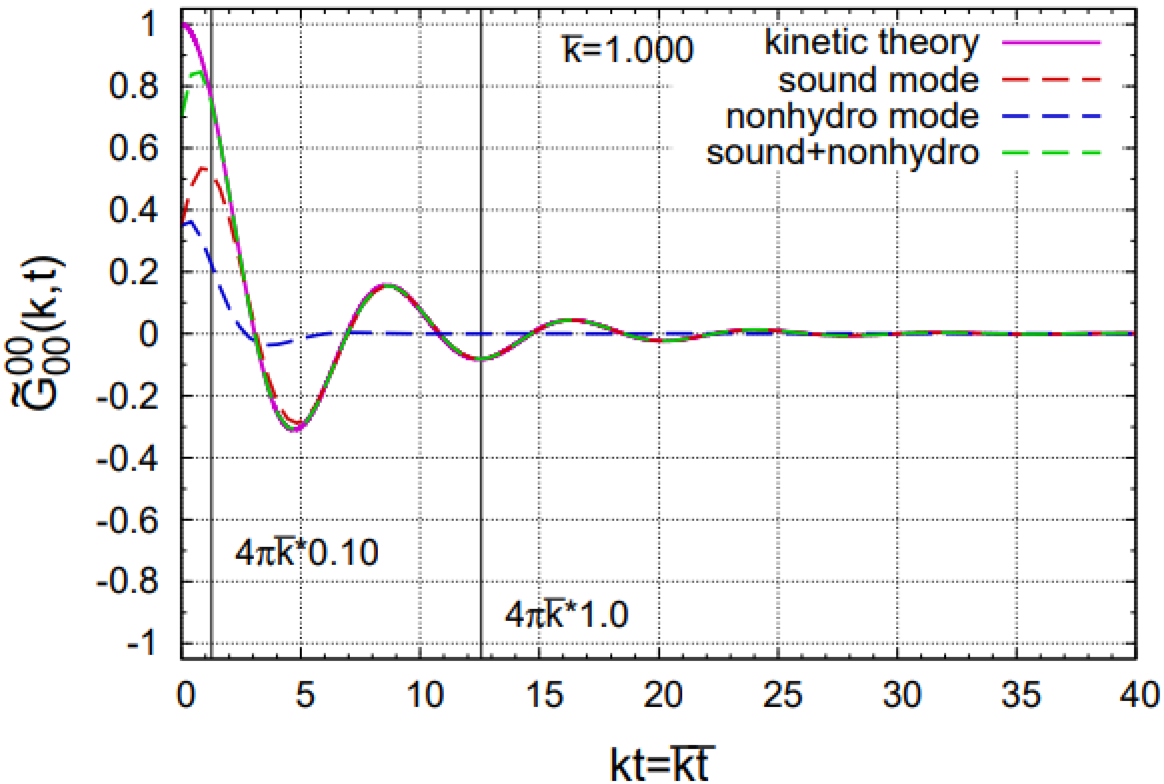
- Non-hydrodynamic mode appears



Sound and non-hydrodynamic modes

Fitting QCD response functions ($\bar{k}$=1.0)

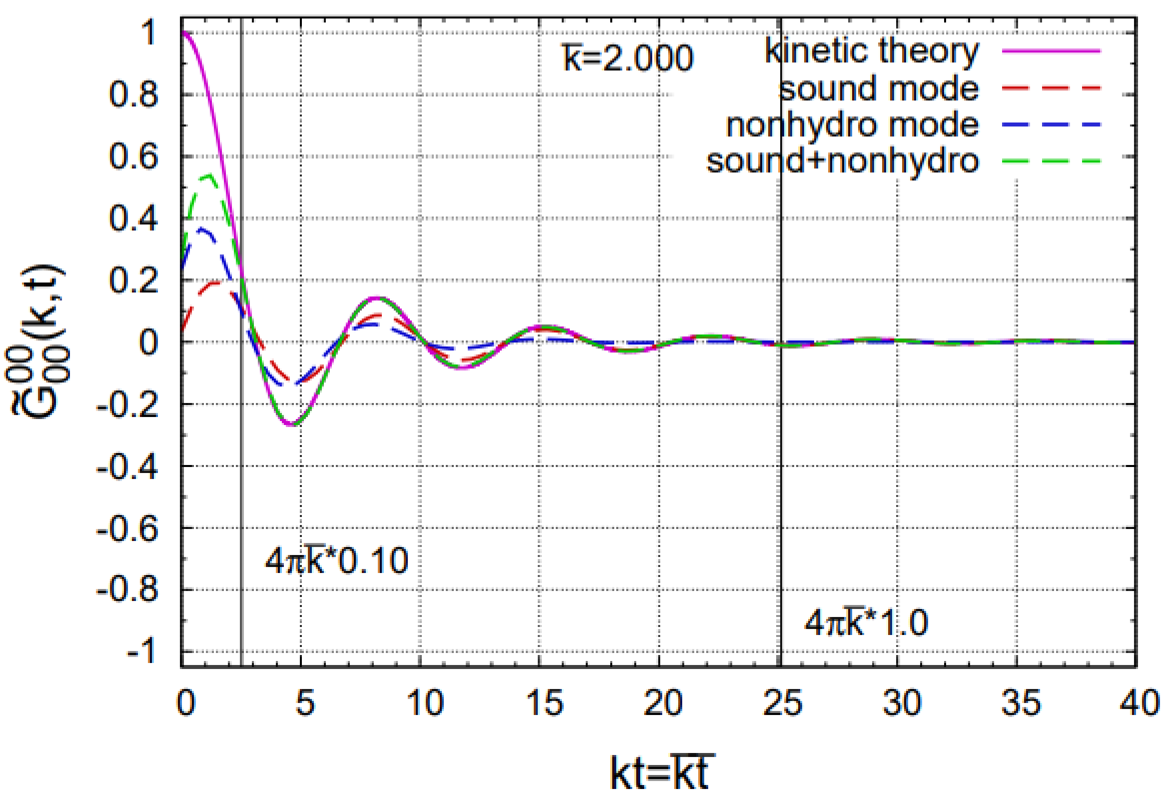
- More non-hydrodynamic modes appear at early time



Sound and non-hydrodynamic modes

Fitting QCD response functions $(\bar{k}=2.0)$

- Non-hydrodynamic mode takes over sound mode

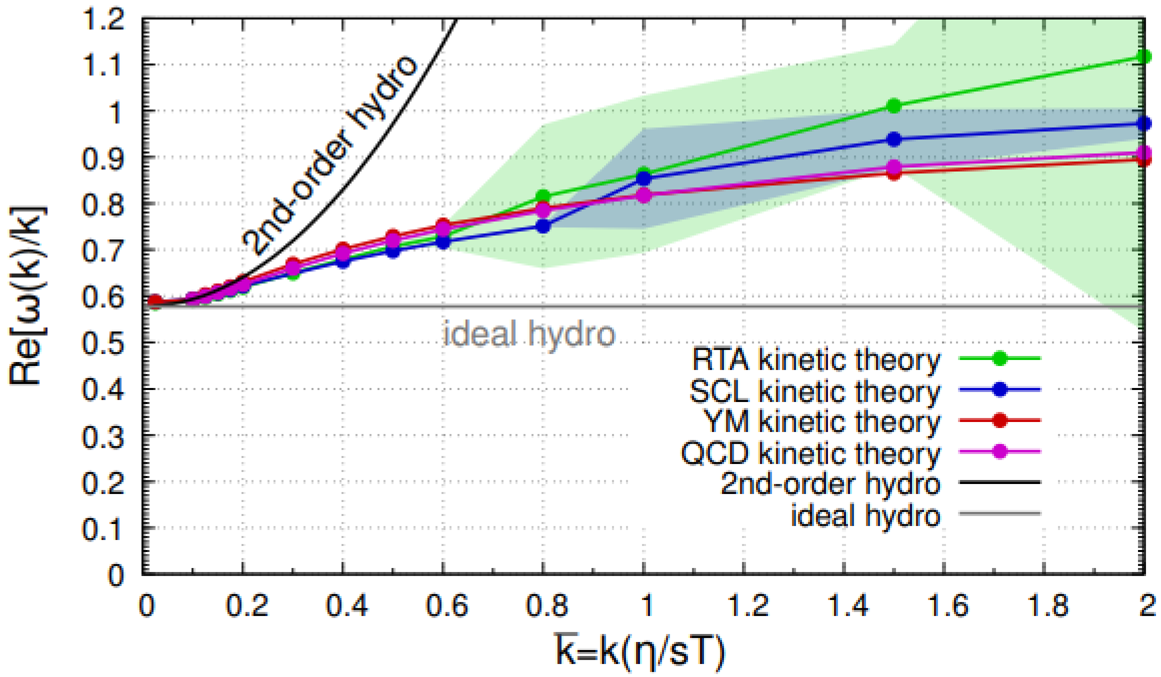


Dispersion relation

Dispersion relations among kinetic theories

- With real frequencies

$$G_{s,n}(t,k) \sim Z_k \exp[-i(\omega_k t + \phi_k)] \qquad \omega_k = Re[\omega_k] + iIm[\omega_k]$$



- Universality of sound modes among kinetic theories at various k
- Kinetic theories converge to 2[nd]-order hydrodynamics at small k

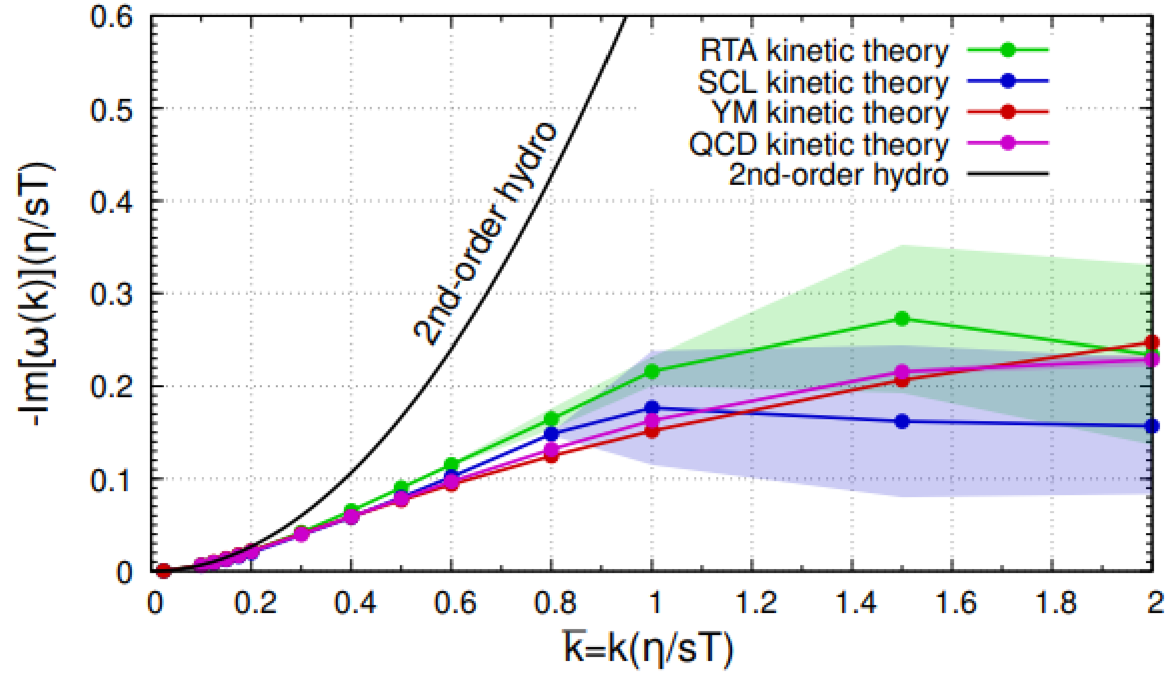
$$\omega^{2nd}_{hydro}(k) = c_s k - i\Gamma k^2 + \frac{\Gamma}{c_s}\left(c_s^2\tau_\pi - \frac{\Gamma}{2}\right)k^3 \qquad \text{with} \qquad \Gamma = \frac{2}{3}\frac{\eta}{sT}$$

Damping relation

Damping relations among kinetic theories

- With imaginary frequencies

$$G_{s,n}(t,k) \sim Z_k \exp[-i(\omega_k t + \phi_k)] \qquad \omega_k = Re[\omega_k] + iIm[\omega_k]$$



- Universality of sound modes among kinetic theories at various k
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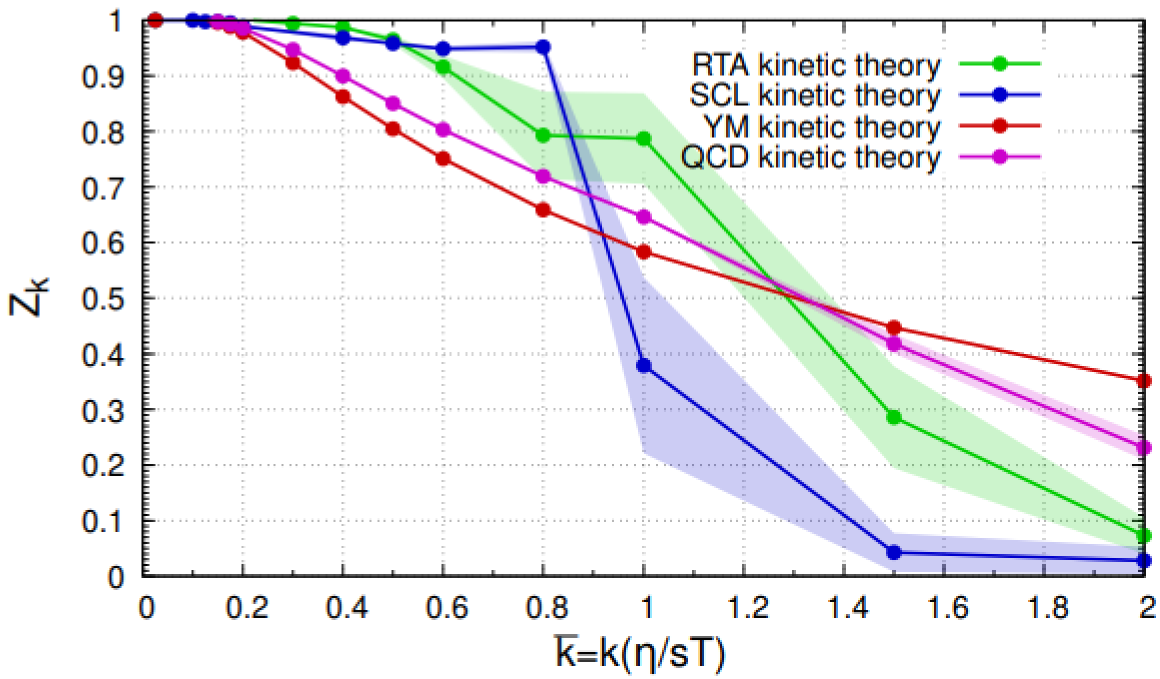
$$\omega^{2nd}_{hydro}(k) = c_s k - i\Gamma k^2 + \frac{\Gamma}{c_s}(c_s^2 \tau_\pi - \frac{\Gamma}{2})k^3 \qquad \text{with} \qquad \Gamma = \frac{2}{3}\frac{\eta}{sT}$$

Residue

Residue for sound & non-hydro modes among kinetic theories

- Non-equilibrium plasma described by sound mode + non-hydrodynamic mode

$$G_{s,n}(t,k) \sim Z_k \exp[-i(\omega_k t + \phi_k)]$$

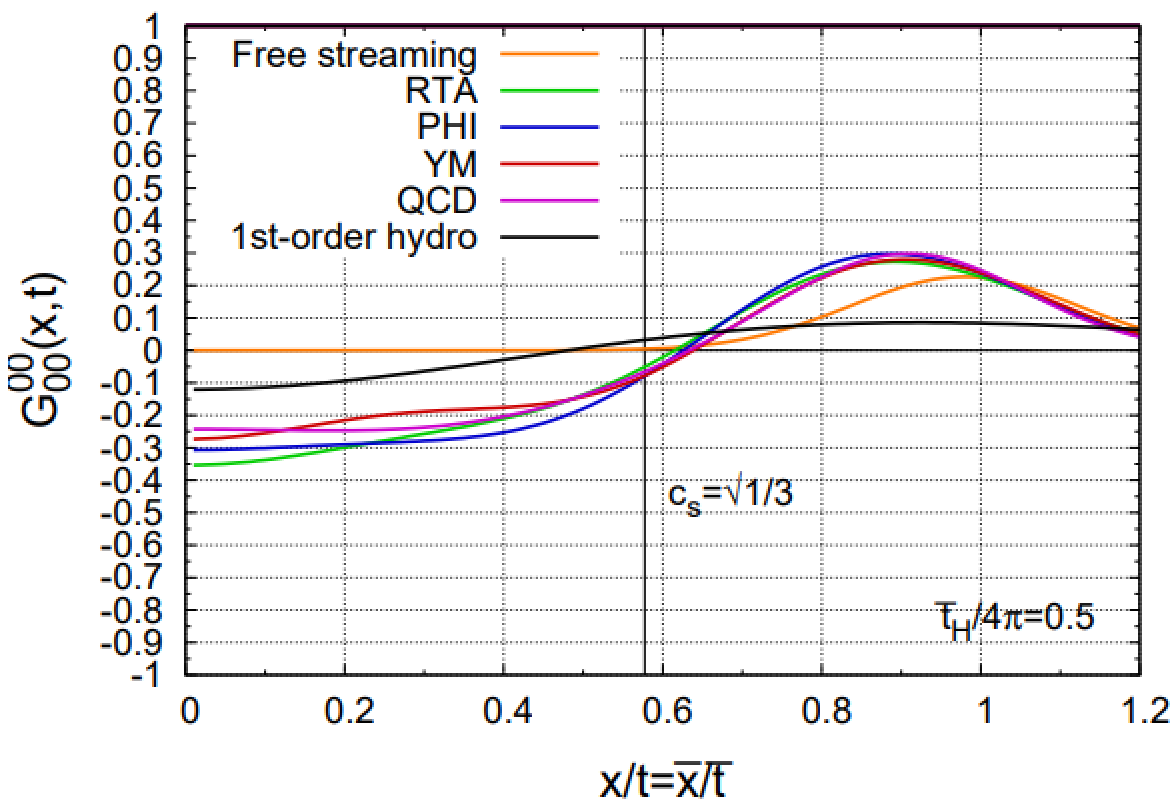


- Sound mode dominates at small k (non-hydro mode dominates at large k)
- Universality of residue at some degree

Response in position space

Response in position among kinetic theories

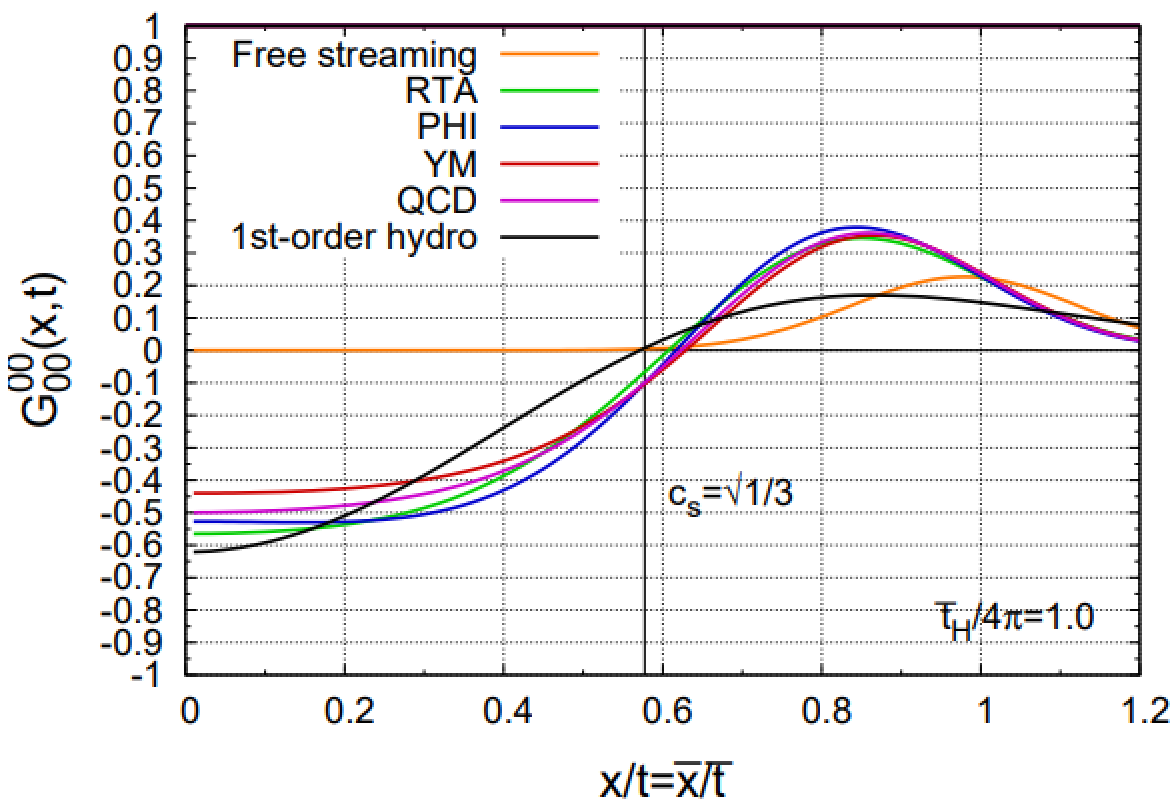
- Universality of position response



Response in position space

Response in position among kinetic theories

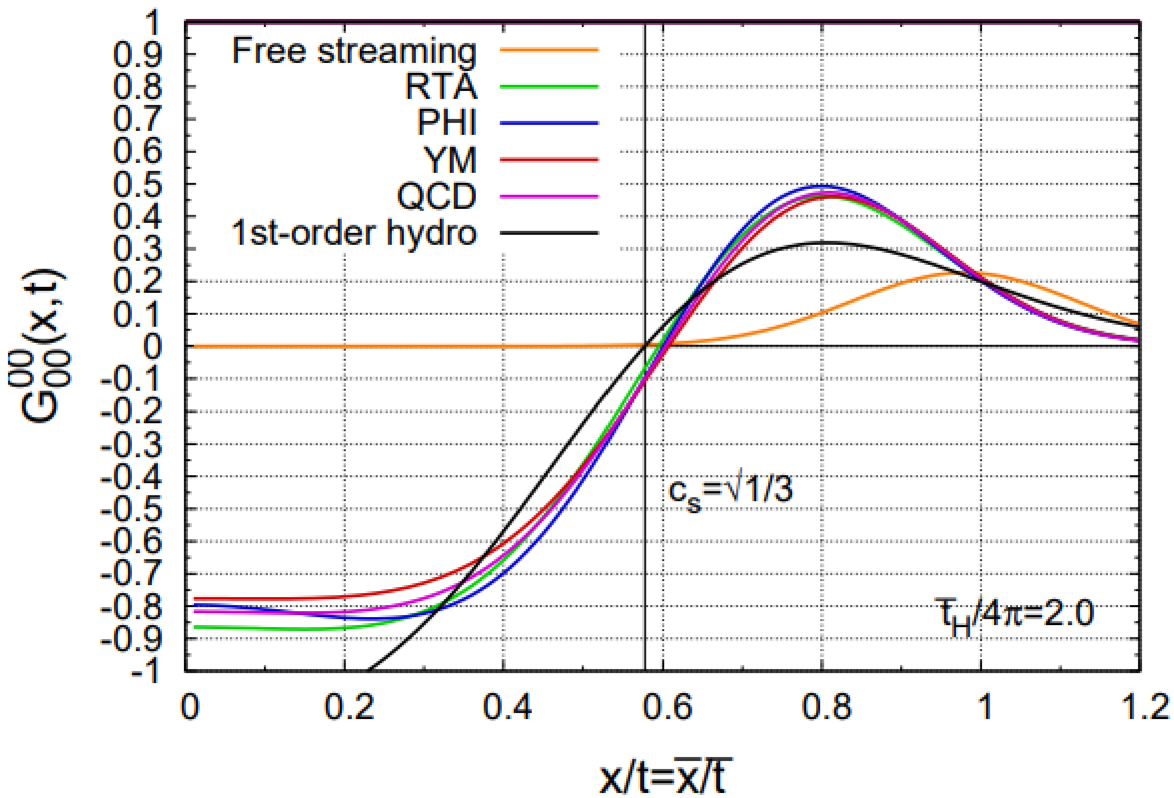
- Universality of position response



Response in position space

Response in position among kinetic theories

- Kinetic theories get closer to hydrodynamics at later time



Summary

What are expected and verified

- Kinetic theories converge to hydrodynamics at small k
- Linear response of kinetic theories can be described by sound+non-hydro modes

What are strikingly new

- Universality of energy-momentum response functions among kinetic theories

  (Even at early time and large k)

What are future perspectives

- Possible to construct hydrodynamic description to non-equilibrium region

Thanks!