Thermal photon production rate from Transverse-Longitudinal (T-L) mesonic correlator on the lattice

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Outline

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    - Pade Ansatz
    - Backus-Gilbert Method
6. Effective Diffusion coefficent
7. Summary

- Photons and dileptons produced from QGP are important probes to study Quark-Gluon-Plasma.
- Photons and dileptons will directly come out of the plasma without further interaction with the plasma.
- The photon production rate (Γ_γ) and di-lepton production rate ($\Gamma_{l^+l^-}$) from a thermalized QGP can be calculated in terms of the spectral function L.D. McLerran and T. Toimela, Phys. Rev. D 31 (1985) 545.

$$\frac{d\Gamma_\gamma}{d^3\vec{k}} = \frac{\alpha_{em} n_b(\omega)}{2\pi^2 k} g^{\mu\nu} \rho_{\mu\nu}(\omega = |\vec{k}|, \vec{k})$$

$$\frac{d\Gamma_{l^+l^-}}{d\omega d^3\vec{k}} = \frac{\alpha_{em}^2 n_b(\omega)}{3\pi^2(\omega^2 - k^2)} g^{\mu\nu} \rho_{\mu\nu}(\omega, \vec{k})$$

- Need to estimate $\rho_{\mu\nu}$ from lattice.
- We need correlation function of the current operator $J_\mu(\vec{x},\tau) = \bar{\psi}(\vec{x},\tau)\gamma_\mu\psi(\vec{x},\tau)$ on the lattice.

$$G^E_{\mu\nu}(\tau,\vec{k}) = \int d^3\vec{x}\,\exp\Big(i\vec{k}.\vec{x}\Big)\langle J_\mu(\vec{x},\tau)J_\nu(\vec{0},0)\rangle$$

- Relation with spectral function,

$$G^E_{\mu\nu}(\tau,\vec{k}) = \int_0^\infty \frac{d\omega}{\pi}\rho_{\mu\nu}(\omega,\vec{k})\frac{\cosh[\omega(\tau-\frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

- Numerically unstable problem.
  1) Difference in the number of degrees of freedom.
  2) Small error in G^E become very large error in ρ.

- $\rho_{\mu\nu}$ can be decomposed,

$$\rho_{\mu\nu}(\omega,\vec{k}) = P^T_{\mu\nu}\rho_T(\omega,\vec{k}) + P^L_{\mu\nu}\rho_L(\omega,\vec{k})$$

T/L=Transverse/Longitudinal component of the spectral function.

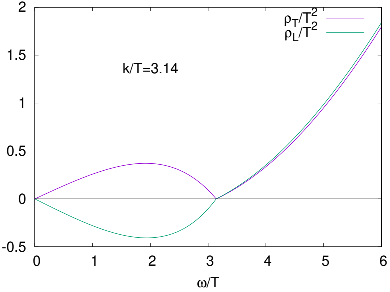
$$\rho_V(\omega,\vec{k}) = \rho^\mu_\mu(\omega,\vec{k}) = 2\rho_T(\omega,\vec{k}) + \rho_L(\omega,\vec{k})$$

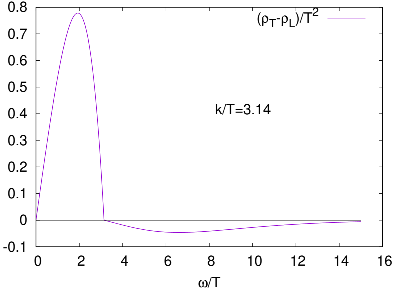
- At the photon point $\rho_L(|\vec{k}|,\vec{k}) = 0$.

$$\frac{d\Gamma_\gamma}{d^3\vec{k}} \propto 2\rho_T(|\vec{k}|,\vec{k})$$

$$\frac{d\Gamma_\gamma}{d^3\vec{k}} \propto 2(\rho_T(|\vec{k}|,\vec{k}) - \rho_L(|\vec{k}|,\vec{k}))$$

- Free result for ρ_T and ρ_L, G. Aarts and J.M. Resco,Nucl. Phys. B 726 (2005) 93



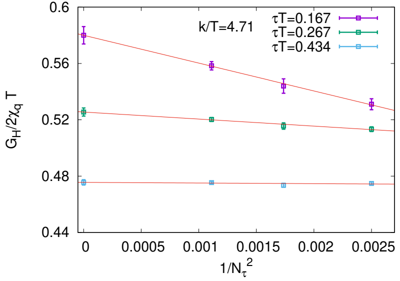


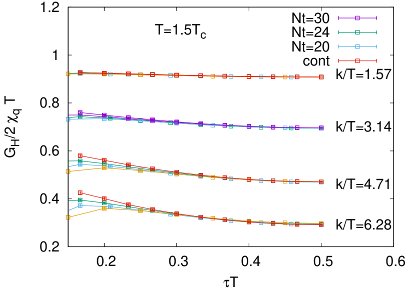
- $\rho_V = 2\rho_T + \rho_L$ has large UV part. G_V^E has large UV contribution.
- $\rho_H = 2(\rho_T - \rho_L)$ has small UV part. G_H^E has less UV contribution.
- Sum rule $\int_0^\infty d\omega \omega \rho_H(\omega) = 0$ and from OPE at large ω, $\rho_H(\omega) \sim \frac{1}{\omega^4}$
  M. Ce, T. Harris, H. B. Meyer, A. Steinberg, and A. Toniato , Phys. Rev. D 102, 091501(R)

- We calculated the $T - L$ correlator in pure gluonic theory at $T = 470 MeV$ and in $N_f = 2 + 1$ flavor QCD at $T = 220 MeV$.
- Lattice size:
  Gluonic theory: $120^3 \times 30$, $96^3 \times 24$ and $80^3 \times 20$
  Full QCD: $96^3 \times 32$
- The available momentum for gluonic theory $\frac{k}{T} = \frac{\pi n}{2}$ and for full QCD $\frac{k}{T} = \frac{2\pi n}{3}$.
- We use clover improved Wilson fermion for the calculation of these correlation functions.
- The quark masses $<<$ T.

- Lattice data has cut-off effects and need continuum extrapolation.

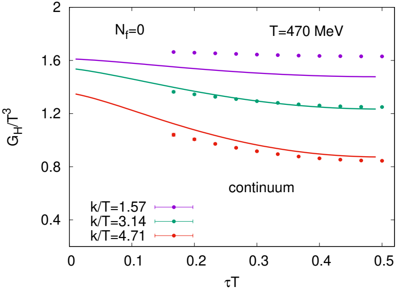
$$\frac{G_H}{2\chi_q T}(a) = \frac{G_H}{2\chi_q T}(a=0) + \frac{b}{N_\tau^2}$$

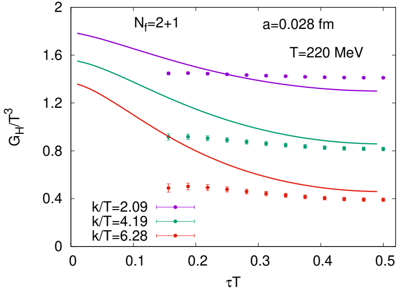




- Smaller cut-off dependence for G_H.
- The infrared part is the dominant contribution to G_H.
- For full QCD, we have results from finite lattice spacing.

- The perturbative estimate is at NLO.
  Leading order LPM resummation has been incorporated at the light cone.
  G. Jackson and M. Laine, JHEP 11, (2019) 144





- Non-pertubative effects are important.

- The polynomial fit ansatz for spectral function,

$$G_H^E(\tau, \vec{k}) = \int_0^\infty \frac{d\omega}{\pi} \rho_H(\omega, \vec{k}) \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

- For $\omega \leq \omega_0$

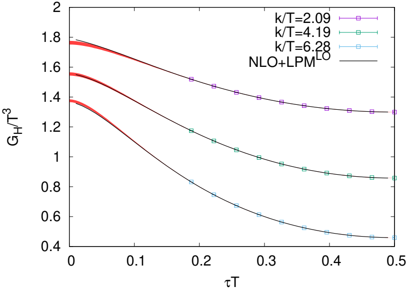
$$\rho_H(\omega) = \frac{\beta\omega^3}{2\omega_0^3}\left(5 - 3\frac{\omega^2}{\omega_0^2}\right) - \frac{\gamma\omega^3}{2\omega_0^2}\left(1 - \frac{\omega^2}{\omega_0^2}\right) + \delta_0\left(\frac{\omega}{\omega_0}\right)\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2$$

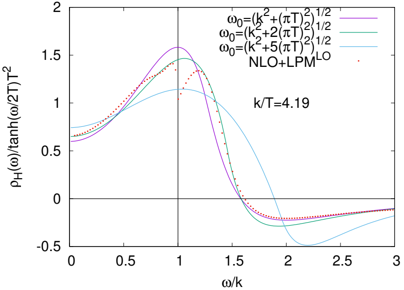
J. Ghiglieri, O. Kaczmarek, M. Laine, and F. Meyer, Phys. Rev. D 94, 016005. and $\omega \geq \omega_0$

$$\rho_H(\omega) = \frac{a}{\omega^4} + \frac{b}{\omega^6} + \frac{c}{\omega^8}$$

- $\beta = \rho_H(\omega_0)$ and $\gamma = \rho'_H(\omega_0)$
- a, b, c are determined from the smoothness condition at $\omega = \omega_0$ along with the sum rule $\int_0^\infty d\omega \omega \rho_H(\omega) = 0$.
- Constrained fit with $\delta_0 \geq 0, \rho_H(k, \vec{k}) \geq 0$ and $\frac{\partial G_H}{\partial \tau} \leq 0$

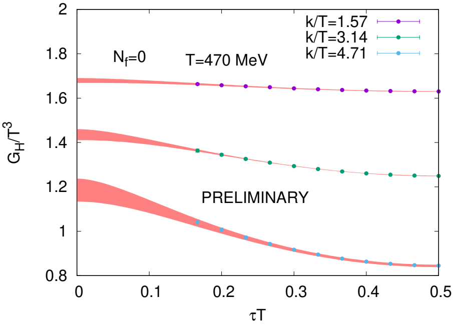
- Ten perturbative data points between 0.1875 to 0.5 in τT.
- An artificial error was introduced to the order $\delta G/G = 0.001$ and tried to reconstruct the spectral function.

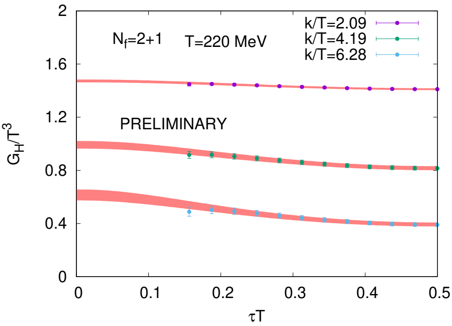




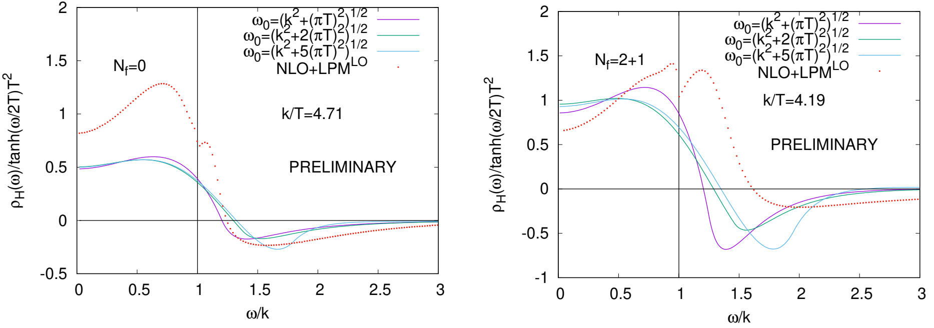
- The exact spectral function can be captured within a systematic uncertainty between $\omega_0 = \sqrt{k^2 + \pi^2 T^2}$ and $\omega_0 = \sqrt{k^2 + 5\pi^2 T^2}$

- The fit of lattice data.



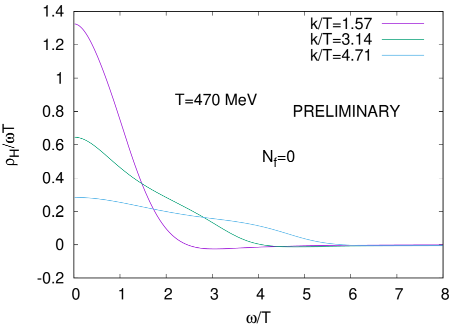


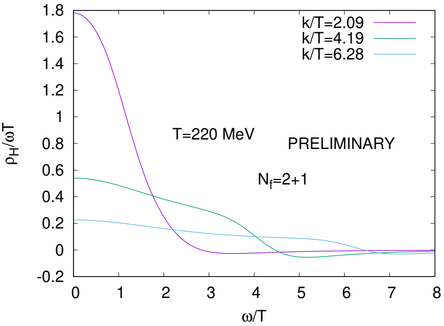
- Non-perturbative spectral function is very different from the perturbative spectral function.



- $$\rho_H^{PADE}(\omega,\vec{k}) = A\frac{\tanh(\omega/2T)\ (1+B\omega^2)}{(a^2+\omega^2)((\omega-\omega_0)^2+b^2)((\omega+\omega_0)^2+b^2)}$$

  M. Ce, T. Harris, H. B. Meyer, A. Steinberg, and A. Toniato , Phys. Rev. D 102, 091501(R)
- The sum rull relate B with a, ω_0 and b.
- The fit has been performed on A,a,ω_0, and b.
- Consistent with OPE prediction $\frac{1}{\omega^4}$.





- Backus-Gilbert estimate of the spectral function,

  G. Backus, F. Gilbert, Geophysical Journal of the Royal Astronomical Society 16, 169 (1968)

- $G_H(\tau) = \int_0^\infty \frac{d\omega}{\pi} \frac{\rho_H(\omega)}{f(\omega)} f(\omega) \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$
-

$$\frac{\rho^{BG}(\omega)}{f(\omega)} = \sum_i q_i(\omega) G(\tau_i) = \int_0^\infty d\bar{\omega} \delta(\omega, \bar{\omega}) \frac{\rho(\bar{\omega})}{f(\bar{\omega})}.$$

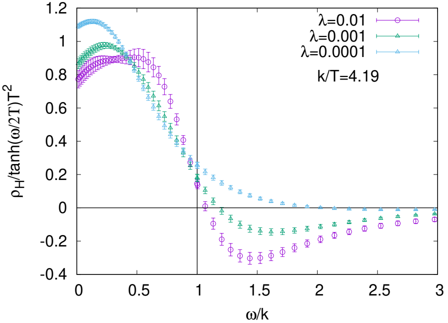
$\delta(\omega, \bar{\omega}) = \sum_i q_i(\omega) K(\bar{\omega}, \tau_i) f(\bar{\omega})$.

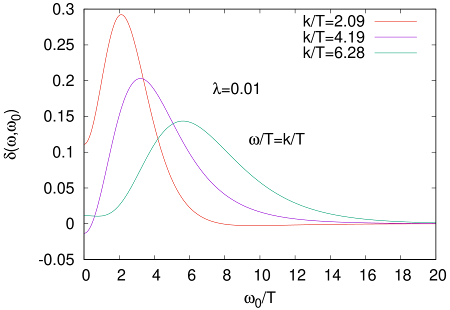
- Minimize $F(\omega) = \lambda \, Width[\delta(\omega, \bar{\omega})] + (1 - \lambda) \, var[\rho_{BG}(\omega)]$

$$f(\omega) = \frac{\tanh(\omega/T)}{(1 + (\omega/\omega_0)^2 + (\omega/\omega_0)^4)}$$

where, $\omega_0 = \sqrt{k^2 + \pi^2 T^2}$.

- $N_f = 2 + 1$





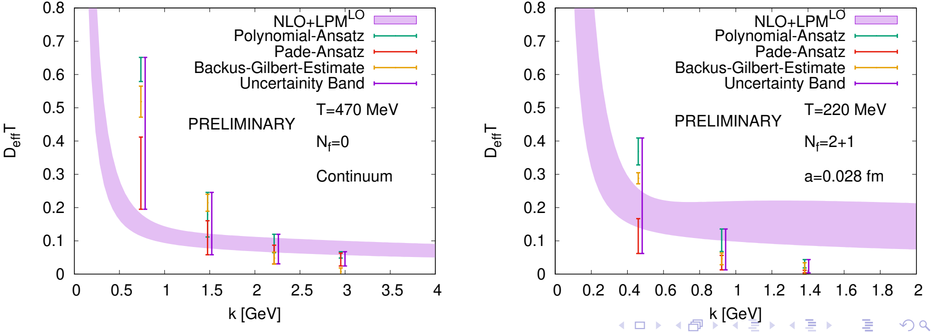
- The resolution function peaked around $\omega_0 \sim k$

- Photon production rate,

$$\frac{d\Gamma_\gamma}{d^3\vec{k}} = \frac{\alpha_{em} n_b(\omega)\chi_q}{\pi^2} Q_i^2 D_{eff}(k)$$

- The effective diffusion coefficient,

$$D_{eff}(k) = \frac{\rho_H(|\vec{k}|, \vec{k})}{2\chi_q|\vec{k}|}$$



- We calculated the T-L correlator in Full QCD.
- Polynomial spectral function with smoothly connected OPE expected expansion at large ω fit the lattice correlator.
- We have used the PADE ansatz of the spectral function.
- The Backus-Gilbert method has also been used.
- Photon production rate estimated from all these methods has been compared.