A unified picture of medium-induced radiation

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Based on: arXiv:2206.02811

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Introduction to medium-induced emissions

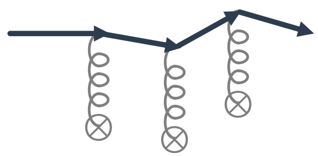
arXiv:2206.02811

QCD in a background medium

[Zakharov, BDMPS]
[Blaizot, Dominiguez, Iancu, Mehtar-Tani]

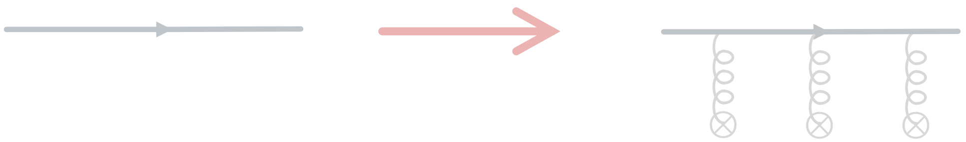
QCD with color bkg: $\mathcal{A}(t, \boldsymbol{x}) + \mathcal{A}_0(t, \boldsymbol{x})$

- Multiple scatterings

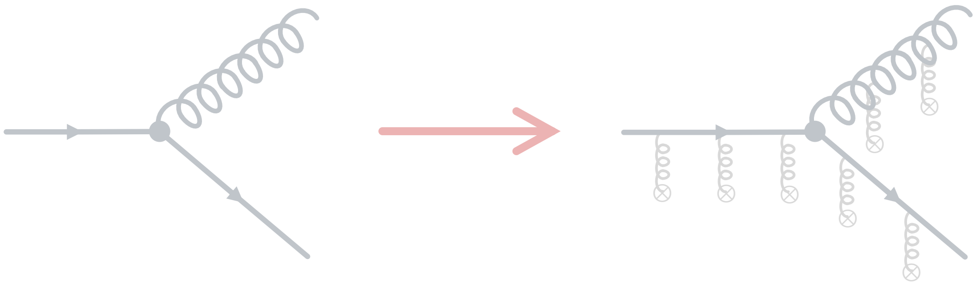


Medium Feynman rules:

- medium propagator:



- medium vertex:



- Medium average:

$\langle \mathcal{A}_0^-(t, \boldsymbol{x}) \mathcal{A}_0^-(t', \boldsymbol{x}') \rangle_{med}$

  - weakly coupled, thermal plasma
  - random fields
  - "idk, evaluate later"

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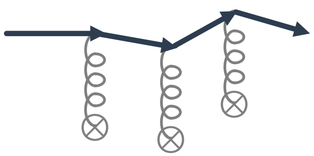

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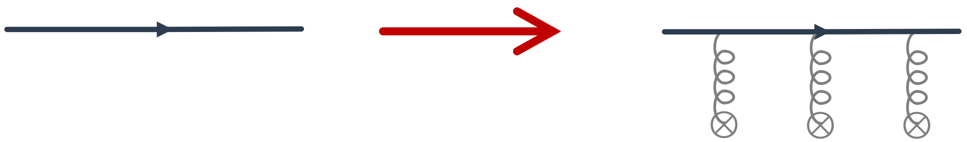

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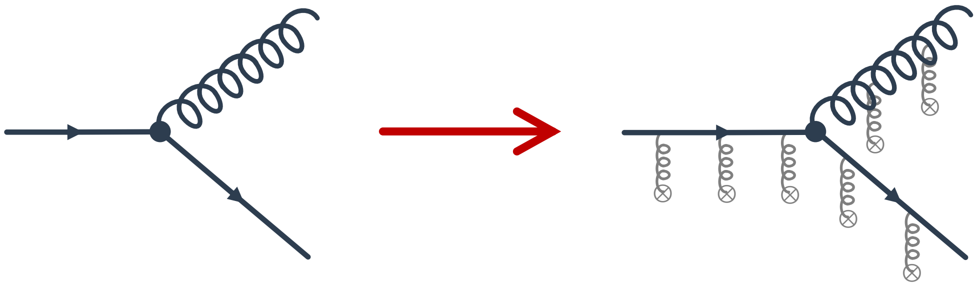

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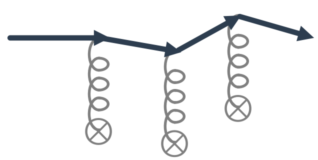

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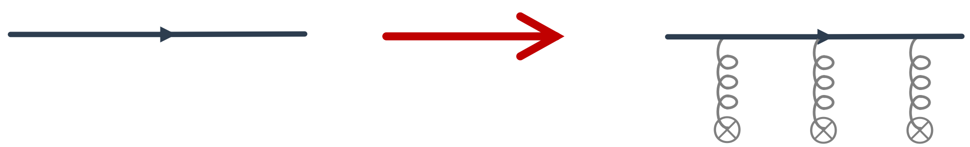

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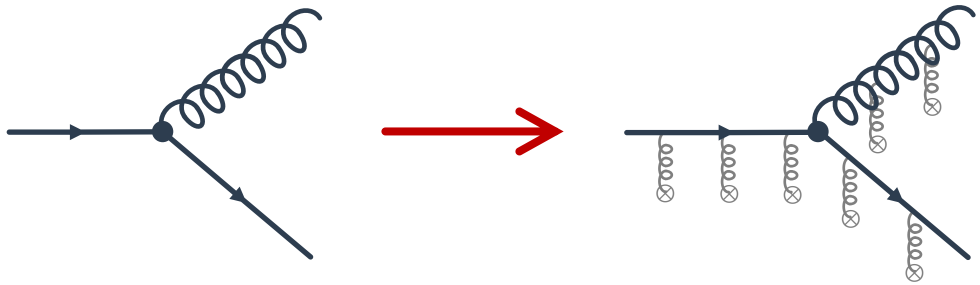

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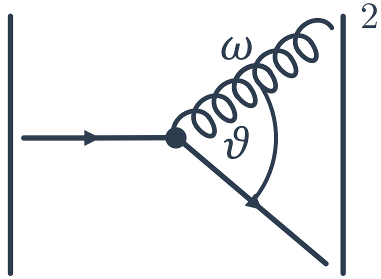
Medium-induced emission

LO radiation in vacuum:



$$\frac{dI^{vac}}{d\omega d\vartheta} \sim \frac{2\alpha_s}{\pi}\frac{C_i}{\omega}\frac{1}{\vartheta} \longrightarrow$$

soft and collinear singularity

Medium-induced emission

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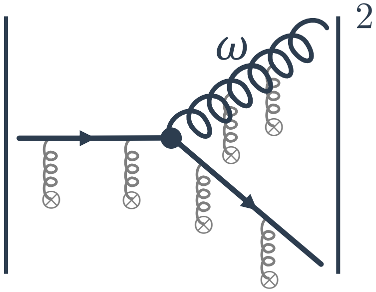

$$\frac{dI}{d\omega d\vartheta} = \frac{dI^{vac}}{d\omega d\vartheta} + \frac{dI^{med}}{d\omega d\vartheta} = \text{complicated integral}$$

[Isaksen,Tywoniuk(2023)]

- $I^{med} > 0$: induced emissions
- I^{med}: no collinear divergence: ϑ is integrated

Approximate solutions:

1. analytic:
   - harmonic oscillator [BDMPSZ(1997)]
   - opacity expansion [GLV-Wiedemann(2000)]
   - improved opacity expansion [Mehtar-Tani(2020)]

2. numeric: [Feal,Vazquez(2018)]
   [Andres,Aploinario,Dominigues(2020)]
   [Schlichting,Soudi(2021)]

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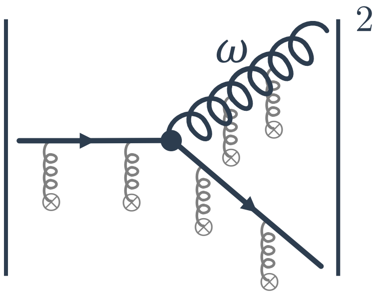

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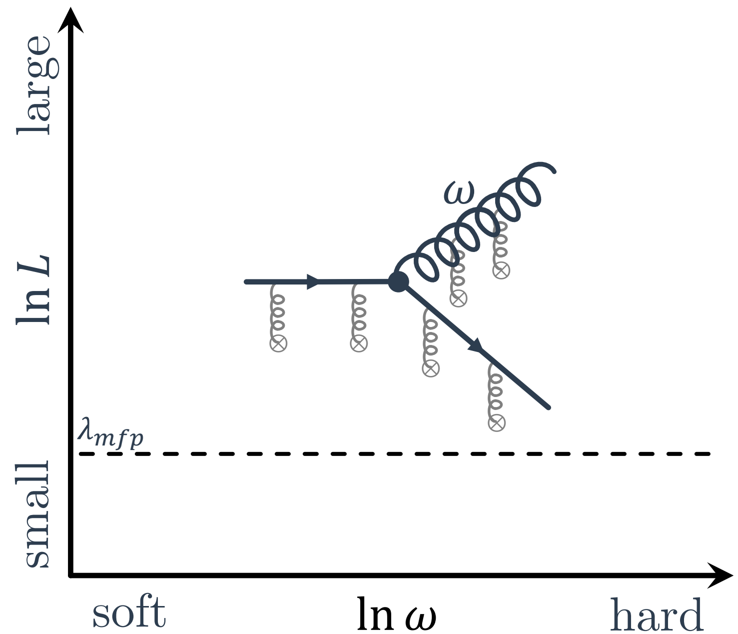
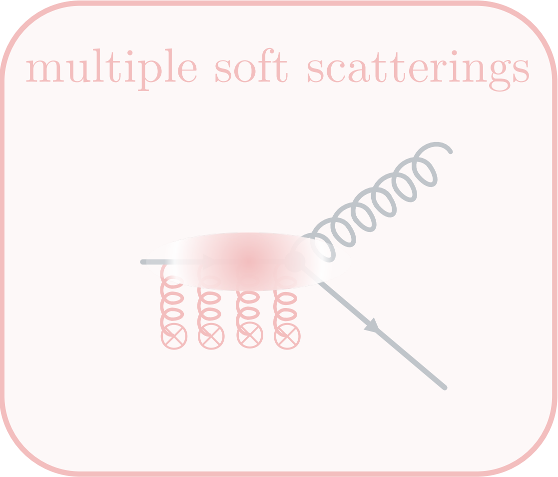
Unified picture of MIE

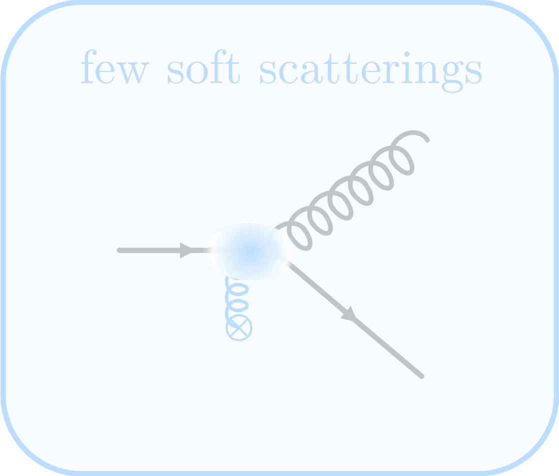
Leading physical picture of $\frac{dI^{med}}{d\omega} = \int d\vartheta \frac{dI^{med}}{d\omega d\vartheta}$:

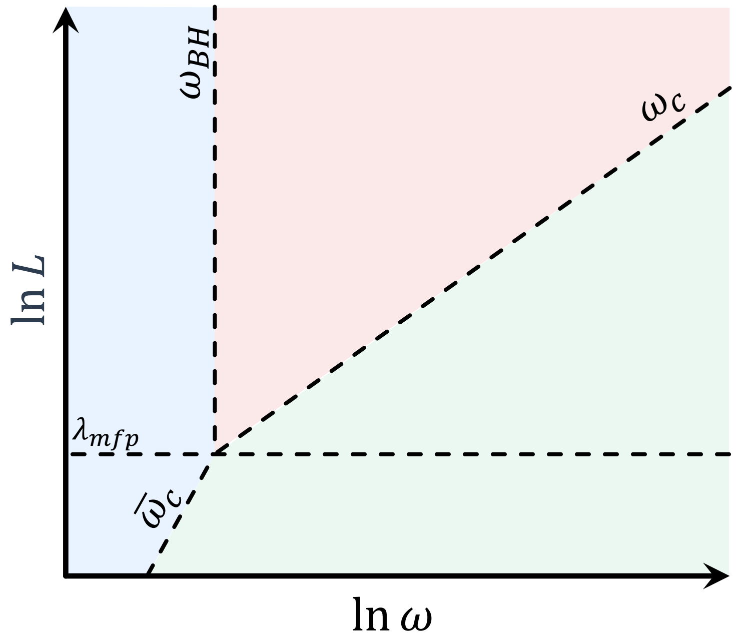


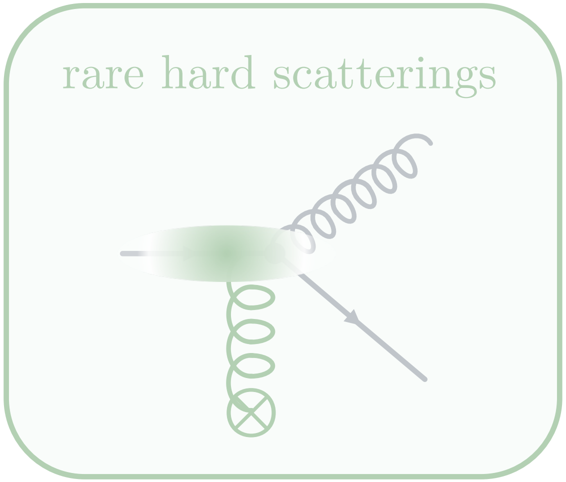
Unified picture of MIE

Physical picture of $\frac{dI^{med}}{d\omega} = \int d\vartheta \, \frac{dI^{med}}{d\omega d\vartheta}$:









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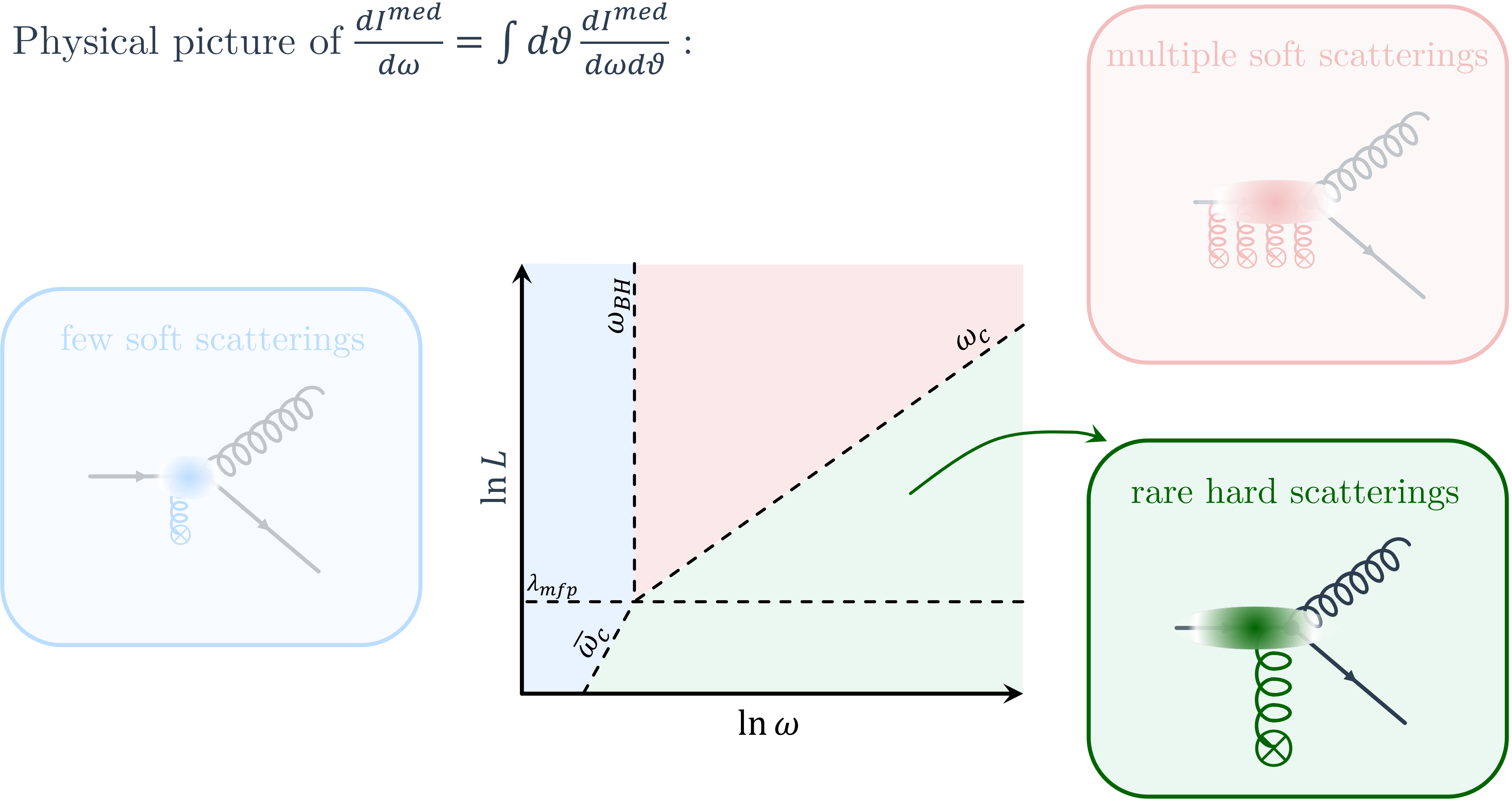

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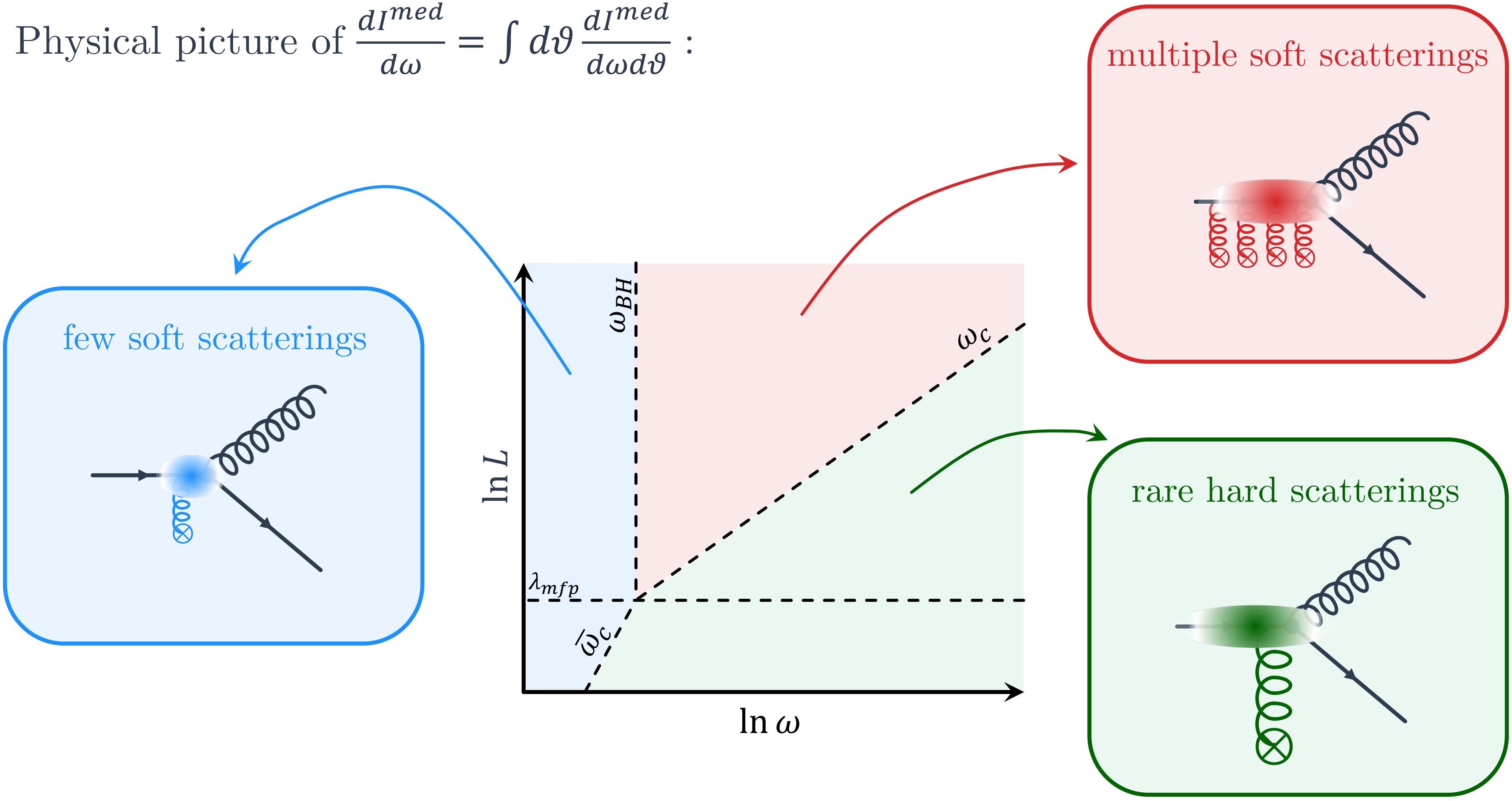
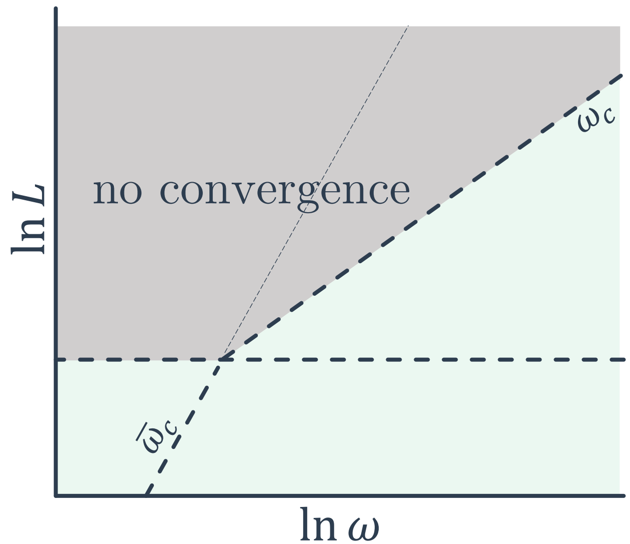


Unified picture of MIE

Expansion in scatterings:

$$\bar{\omega}_c = \frac{\mu^2 L}{2}$$

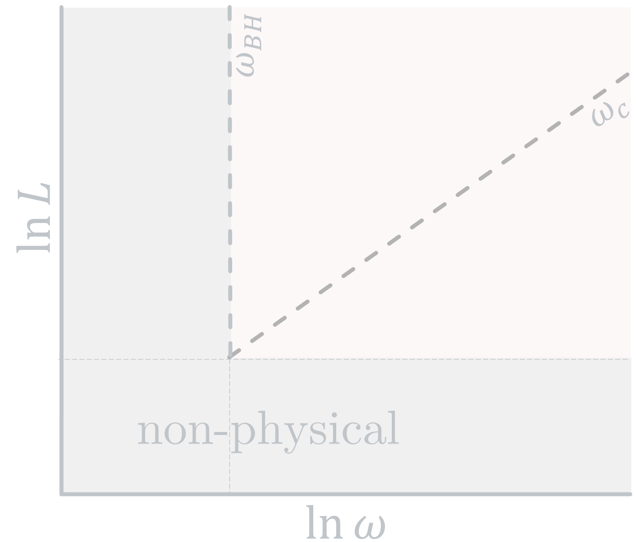
$$\omega \frac{dI}{d\omega} = \begin{cases} \bar{\alpha} \sum_{n=1}^{\infty} \left(\frac{L}{\lambda}\right)^n f_n\left(\frac{\omega}{\bar{\omega}_c}\right), & \omega \ll \bar{\omega}_c \\ \bar{\alpha} \sum_{n=1}^{\infty} \left(\frac{L}{\lambda}\frac{\bar{\omega}_c}{\omega}\right)^n f_n'\left(\frac{\bar{\omega}_c}{\omega}\right), & \bar{\omega}_c \ll \omega \end{cases}$$



Multiple soft scatterings:

$$\omega_c = \frac{L}{\lambda}\bar{\omega}_c$$

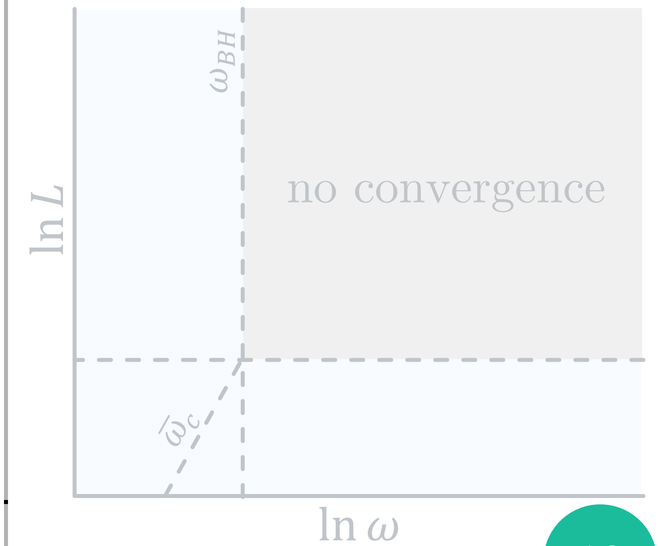
$$\omega \frac{dI^{HO}}{d\omega} = \text{Closed form}, \quad \omega_{BH} \ll \omega \ll \omega_c$$



Expansion in real scatterings:

$$\omega_{BH} = \frac{\bar{\omega}_c}{L/\lambda}$$

$$\omega \frac{dI}{d\omega} = \begin{cases} \text{Opacity expansion}, L \ll \lambda \\ \bar{\alpha} \frac{L}{\lambda} \sum_{n=1}^{\infty} g_n\left(\frac{\omega}{\omega_{BH}}\right), \omega \ll \omega_{BH} \end{cases}$$



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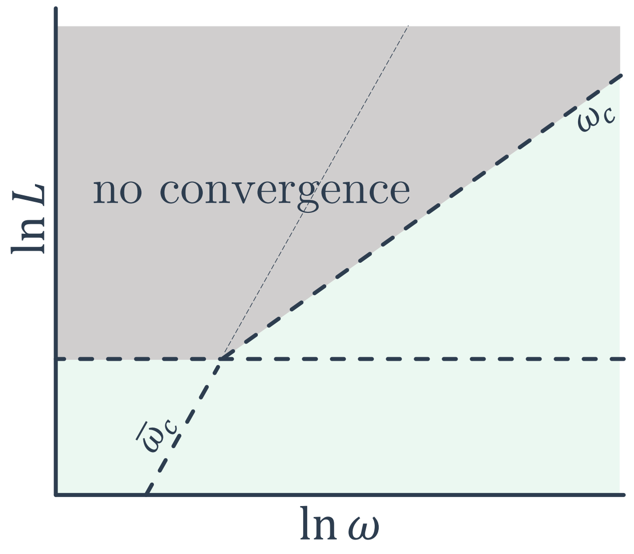

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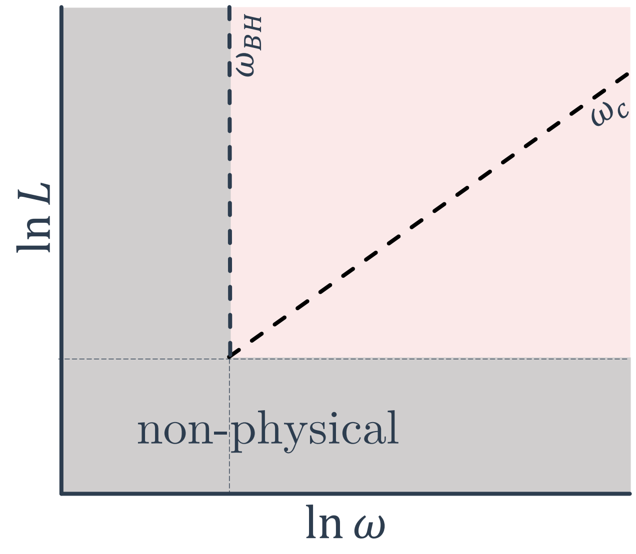

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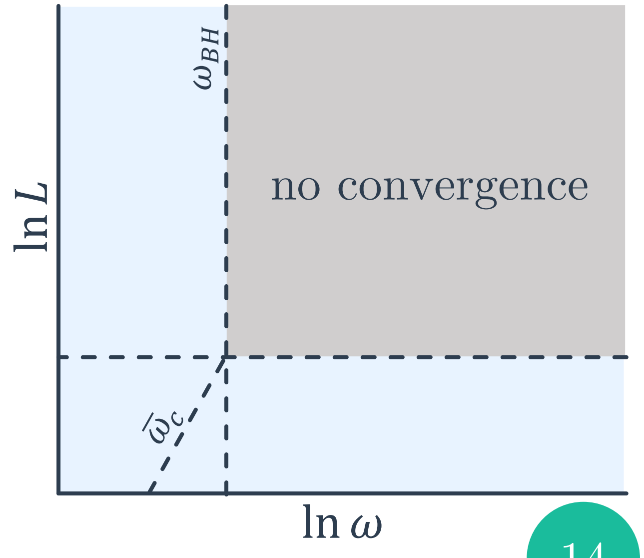
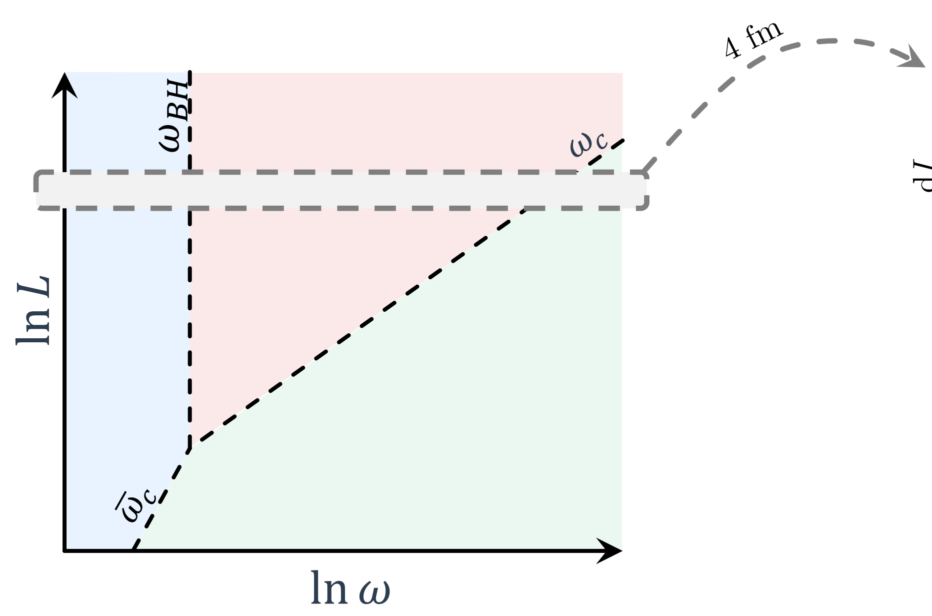
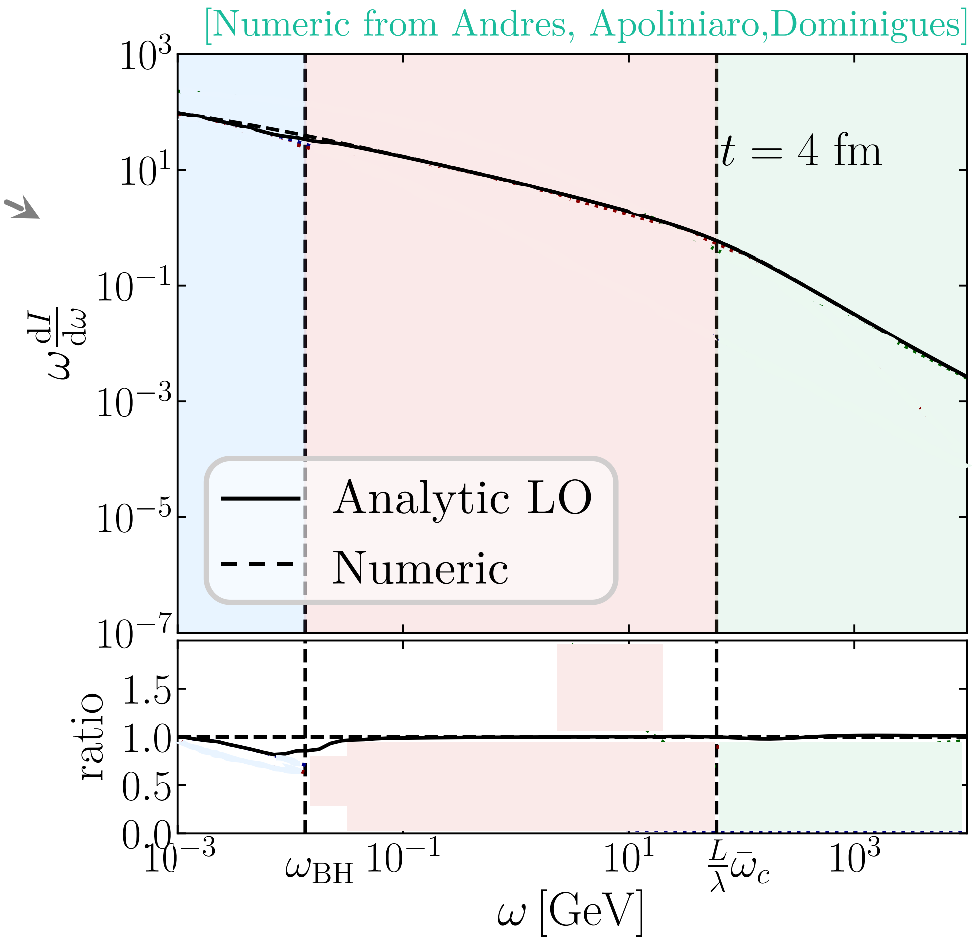


Testing the unified picture

Comparing to numerical solution:





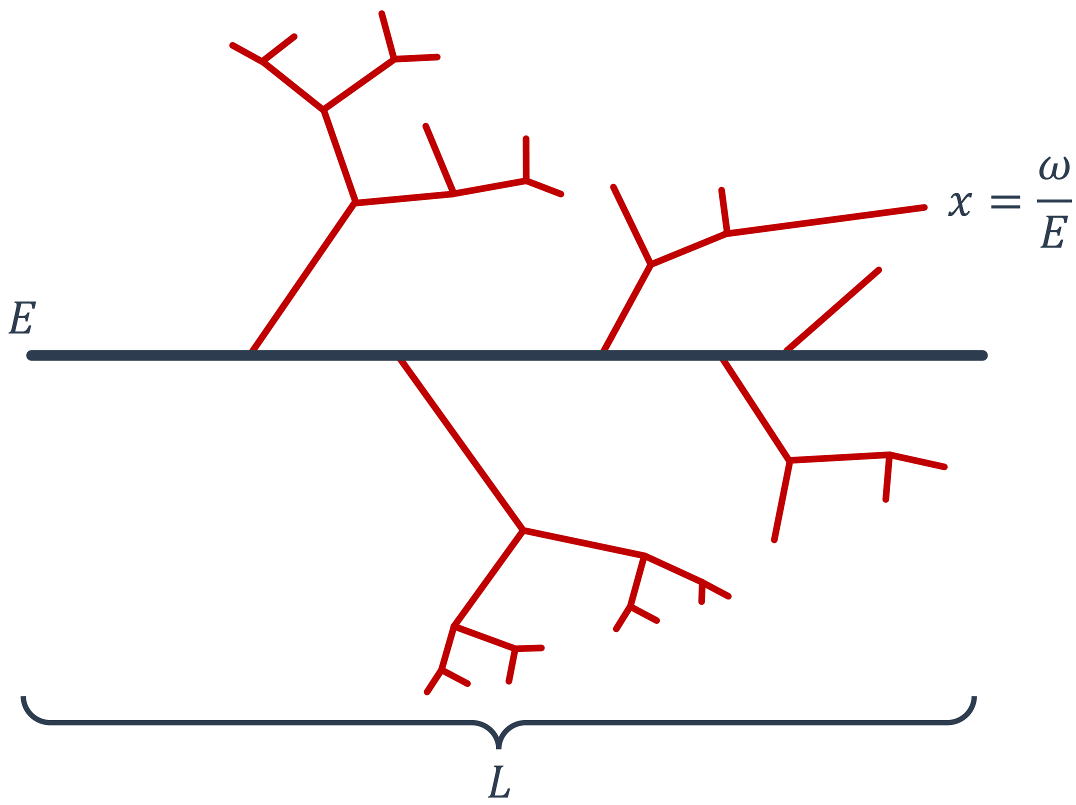
- Very good agreement.
- Computationally very effective.
- https://github.com/adam-takacs/kernels

Application of the unified picture

arXiv:2206.02811

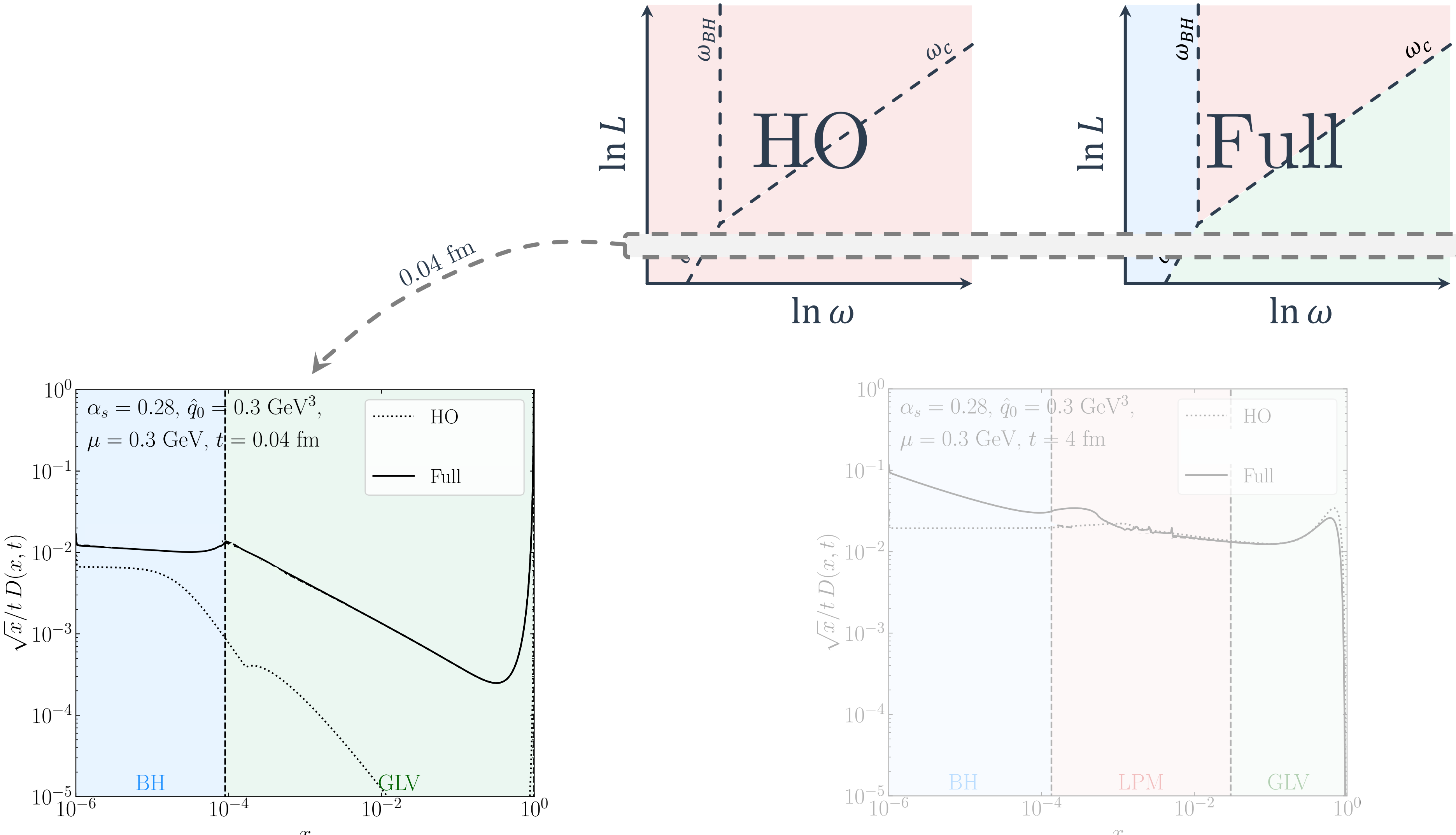
Application: MIE cascade

Medium-induced fragmentation function:



$$D(x,t) = x\frac{dN}{dx}$$

Application: MIE cascade



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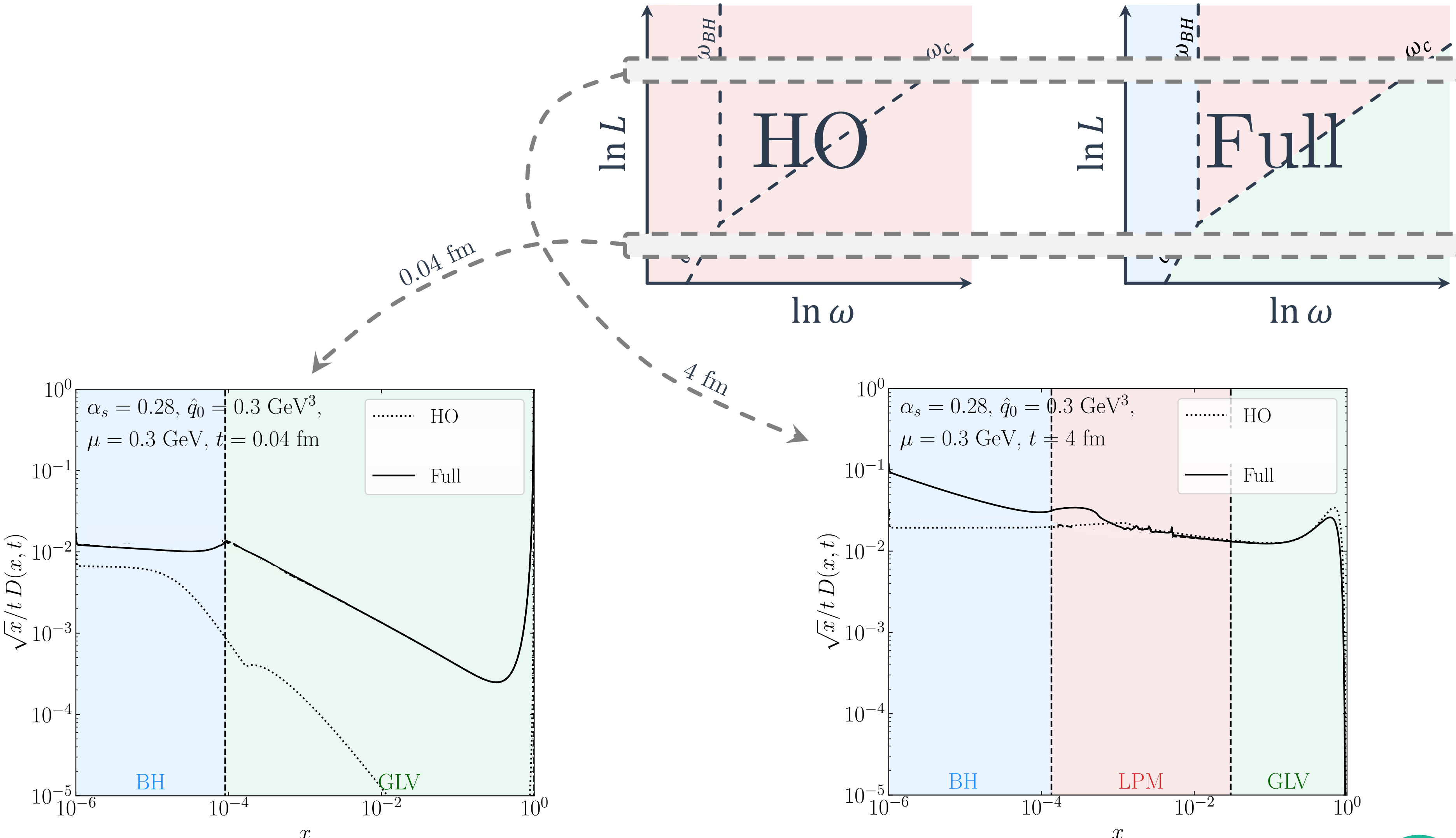

Application: accuracy

Single emission in vacuum

$$\int dz \int d\vartheta \; P^{LO}(z,\vartheta) = \mathcal{O}(\alpha_s L^2) + \mathcal{O}(\alpha_s L) + \mathcal{O}(\alpha_s)$$

logarithmic enhancement: soft & collinear emission

Power counting in vacuum

$$\Sigma(v) = \int dv' \frac{1}{\sigma_0}\frac{d\sigma}{dv'}$$

$$= 1 + \alpha_s(\Sigma_{12}L^2 + \Sigma_{11}L + \Sigma_{10})$$
$$+ \alpha_s^2(\Sigma_{24}L^4 + \Sigma_{23}L^3 + \Sigma_{22}L^2 + \cdots)$$
$$+ \cdots$$
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DL NDL

Single emission in medium

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power & logarithmic enhancement: soft emission & large medium

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P_m NP_m

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$\mathrm{P_m}$ $\mathrm{NP_m}$

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P_m NP_m

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P_m NP_m

Summary

- Understanding jet modification in medium
- Medium-induced emissions:
  - Medium-induced emission $\frac{dI}{d\omega}$
  - Including in to MC, extending to finite angles [See Iancu's talk on JetMed2.0]
- Multiple induced emissions
  - Understanding pQCD in the plasma to all orders

Thank you for the attention!