Parton cascades at DLA: the role of the evolution variable

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In collaboration with:

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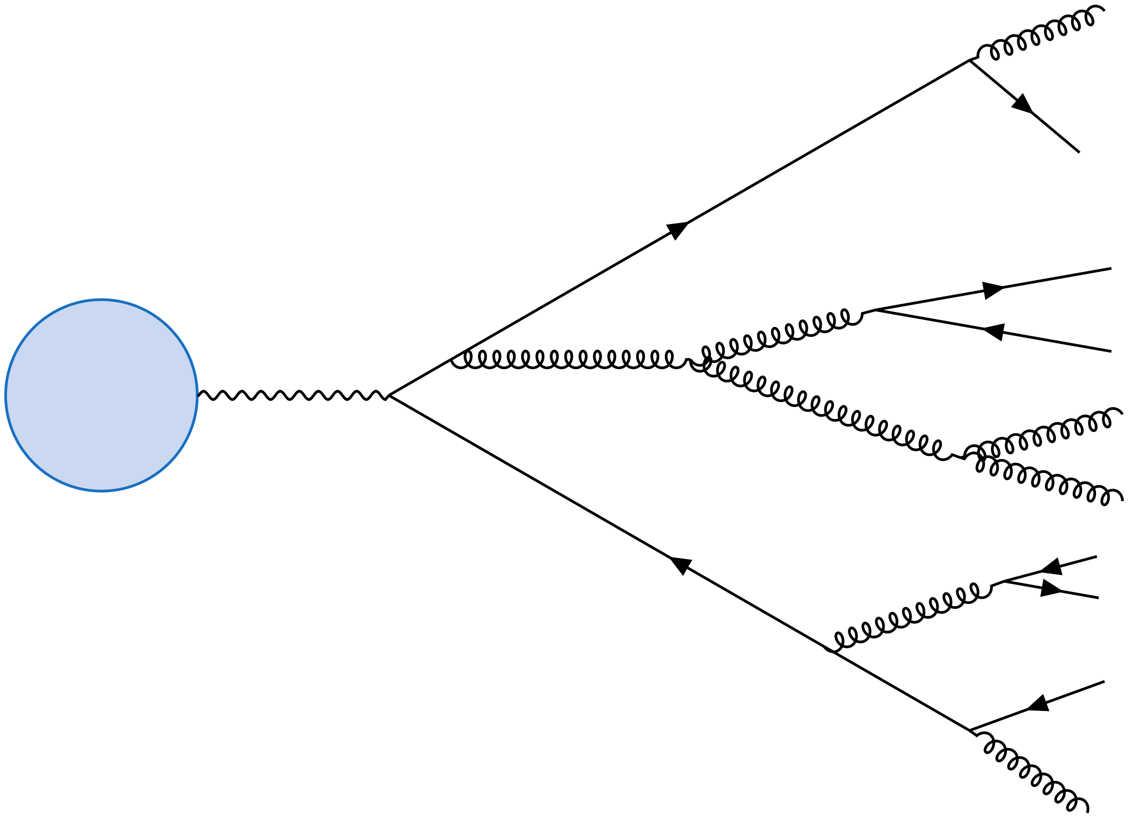
LIP TÉCNICO LISBOA

HP2023, Thursday 30th March

Why do we care about parton showers?



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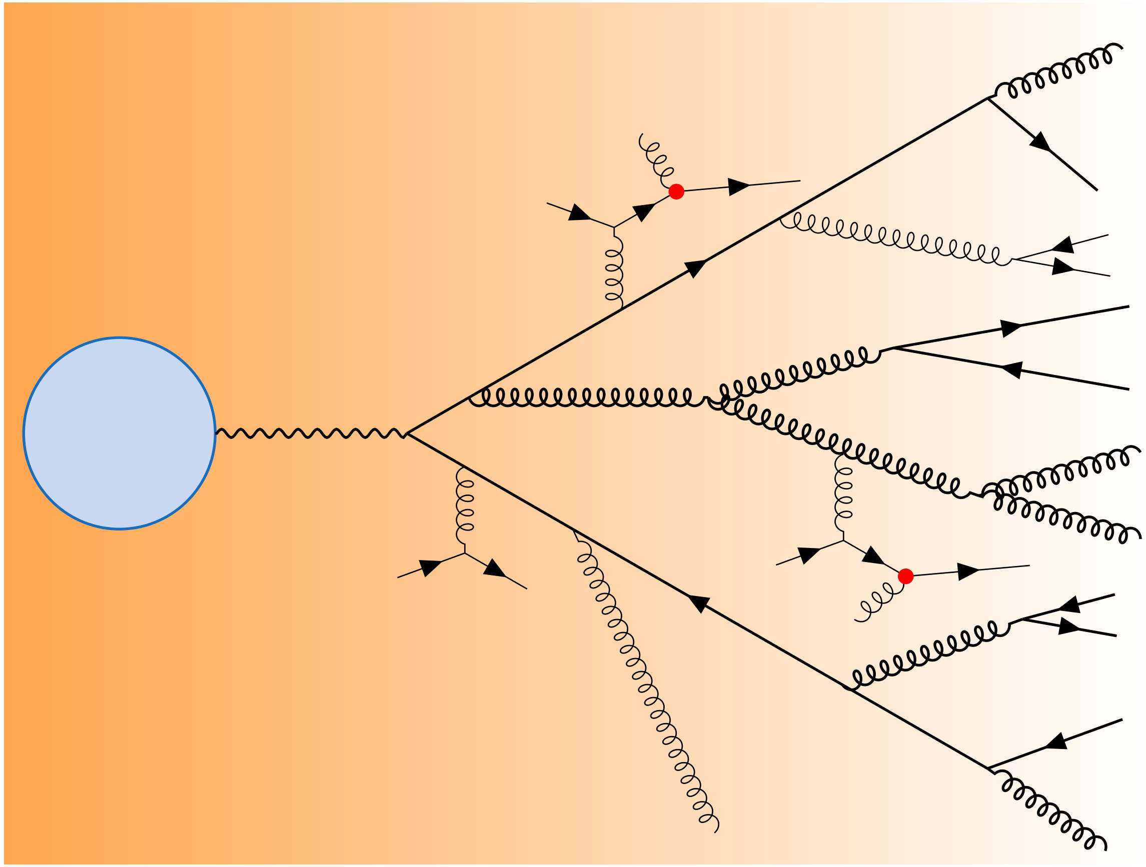

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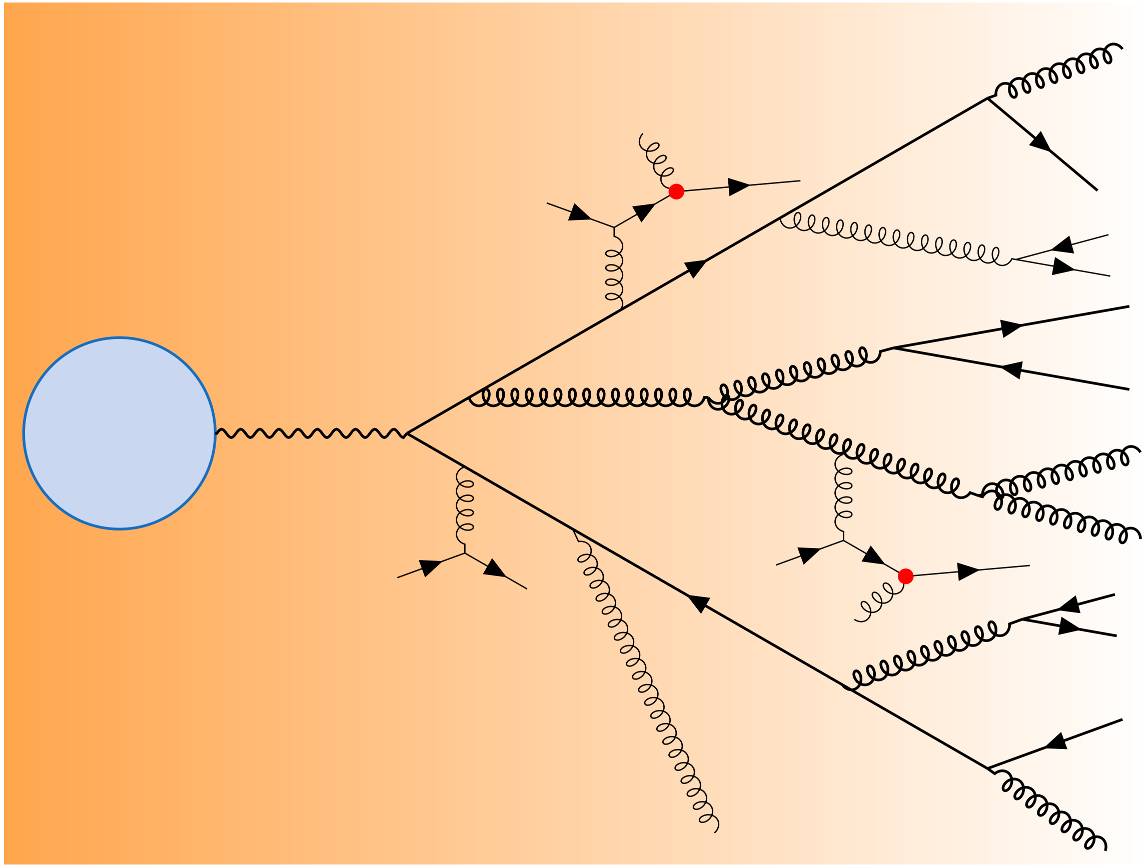

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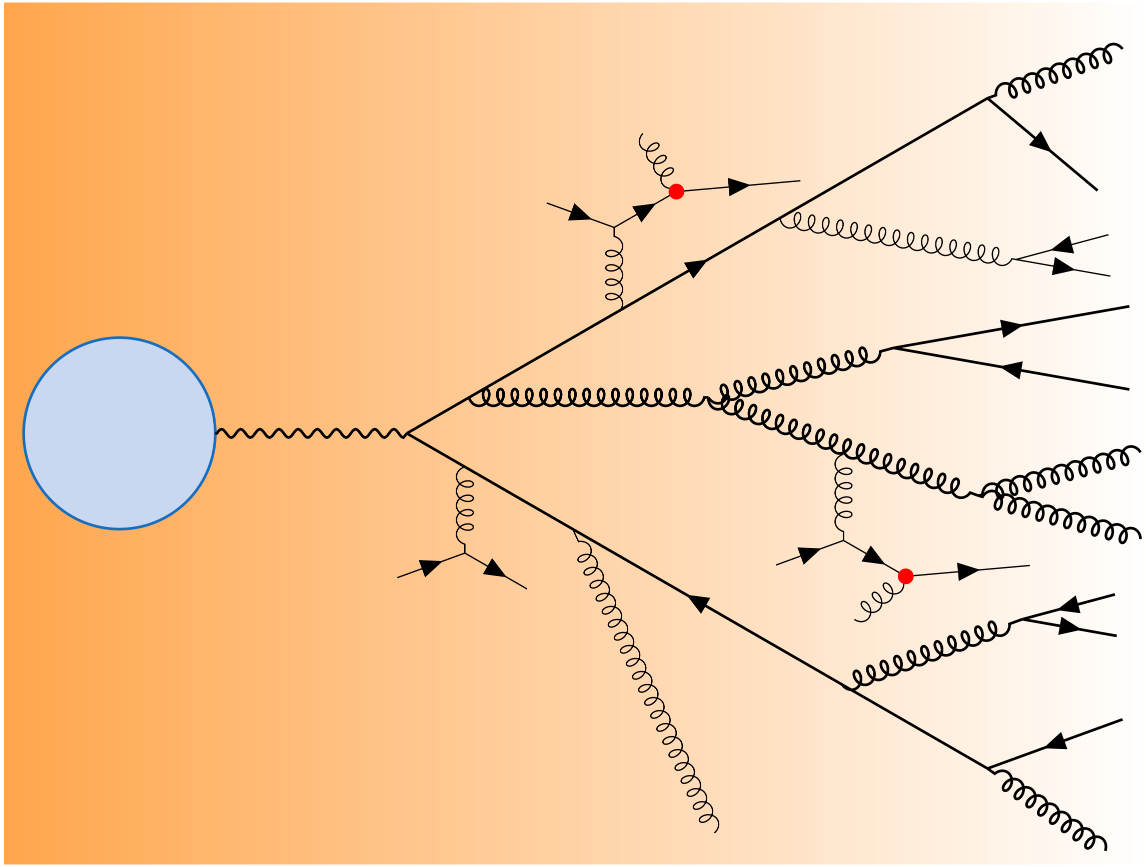

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Is jet quenching sensitive to the ordering of vacuum-like emissions?

First, a look at vacuum showers

Building differently ordered cascades

No-emission probability:

$$\Delta(s_{\text{prev}}, s) = \exp\left\{-\frac{\alpha C_R}{\pi}\int_{s}^{s_{\text{prev}}}\frac{\mathrm{d}\mu}{\mu}\int_{z_{\text{cut}}(\mu)}^{1}\frac{\mathrm{d}z}{z}\right\}$$

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$$s \to t_{\mathrm{form}}^{-1} = \frac{|\boldsymbol{\ell}|^2}{2p^+ z(1-z)}$$

(Formation time)

$$s \to \tilde{m}^2 = 2p^+ t_{\mathrm{f}}^{-1}$$

(Virtuality)

$$s \to \tilde{\theta}^2 = \frac{|\boldsymbol{\ell}|^2}{(p^+)^2[z(1-z)]^2}$$

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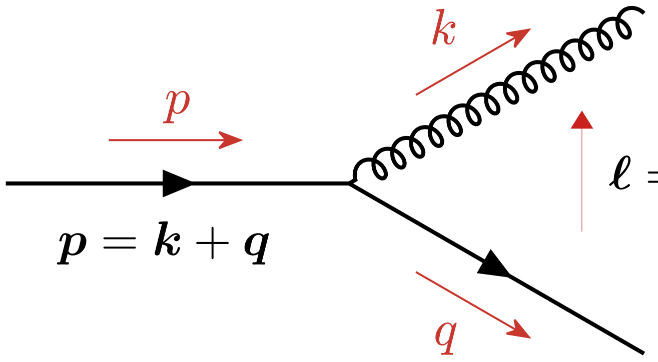

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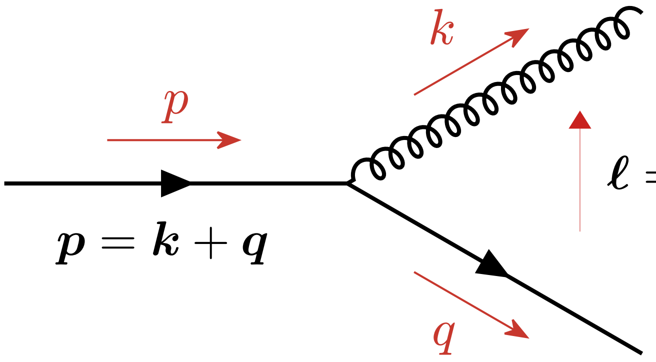

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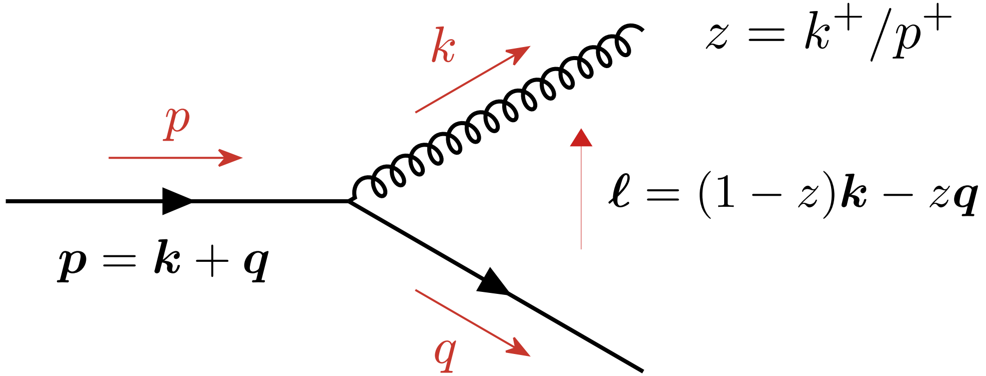

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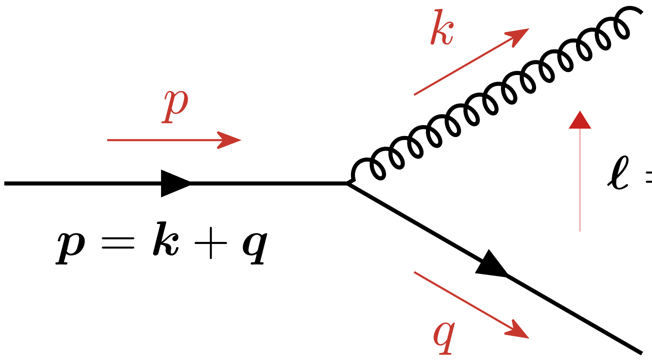

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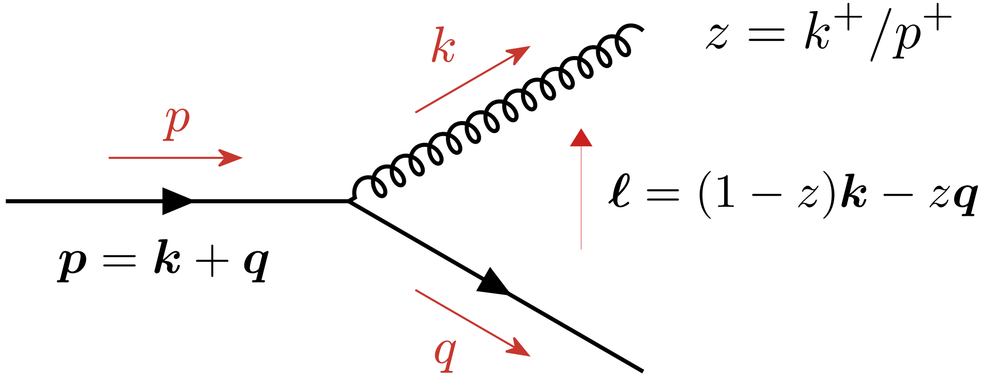

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Ensure that $|\boldsymbol{\ell}|^2 > k_{\text{had}}^2$

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- Splittings happen above some hadronization scale $|\boldsymbol{\ell}|^2 > k_{\text{had}}^2$
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- To avoid large angles: $\tilde{\theta} < 2\sqrt{2}$

Opening angle:

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Determining Splitting Kinematics

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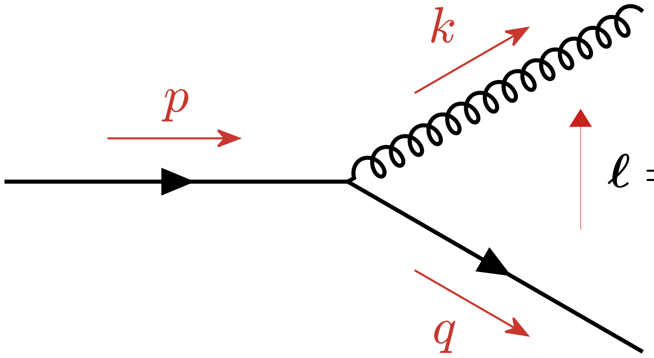

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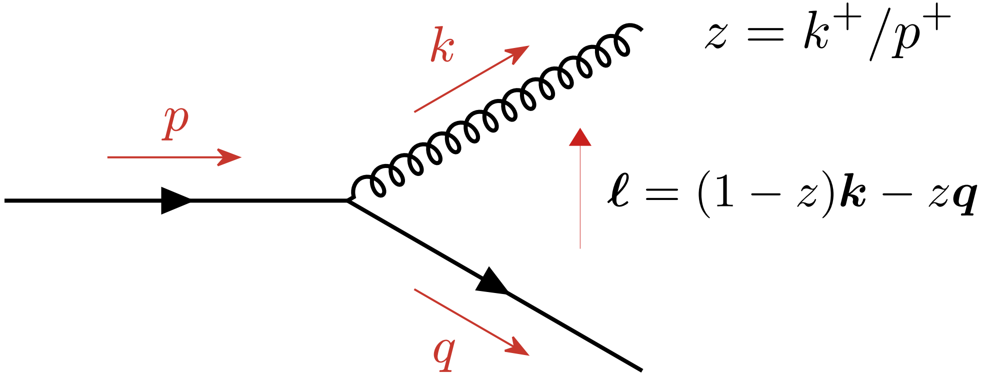

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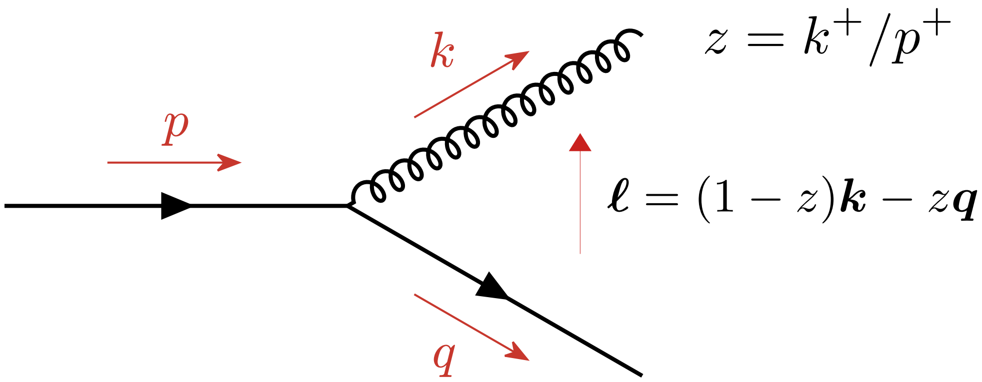

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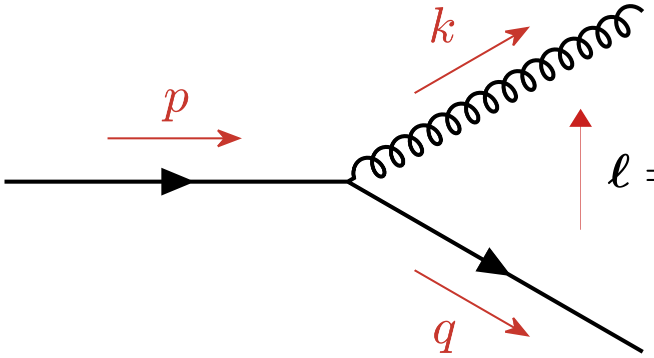

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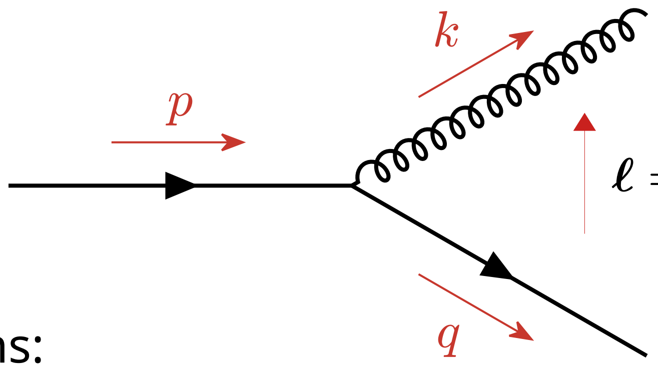

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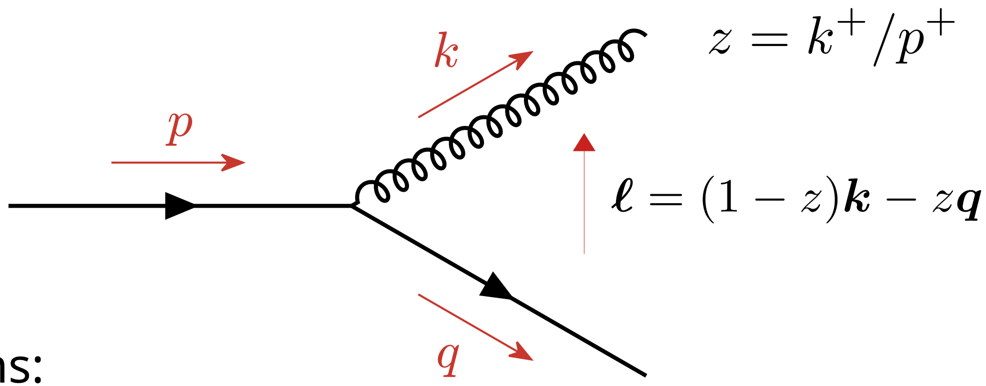

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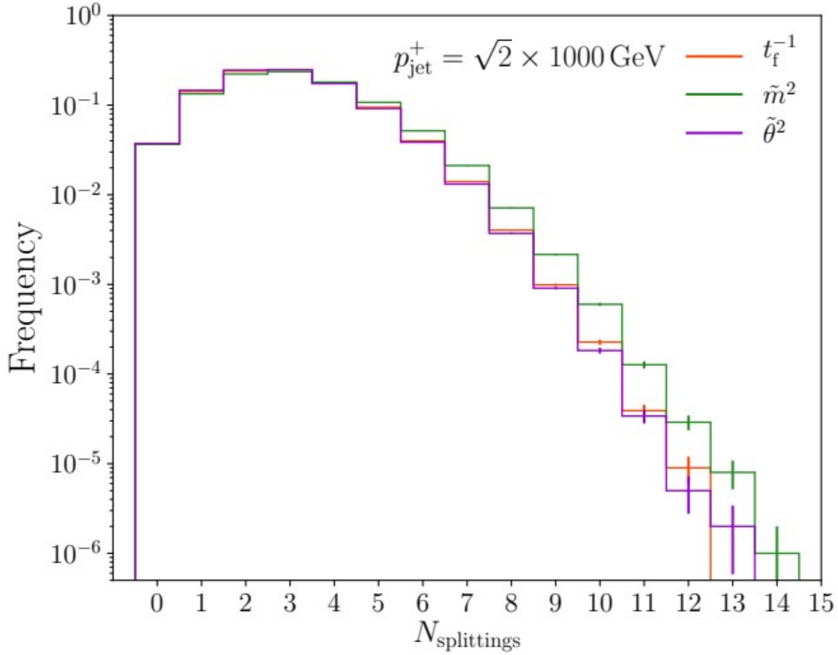
$$\boldsymbol{p} = \boldsymbol{k} + \boldsymbol{q} \qquad \boldsymbol{q} = -\boldsymbol{\ell} + (1-z)\boldsymbol{p}$$

Differences in ordering choices

Splittings along the quark branch



Different orderings → Different phase-space for allowed splittings

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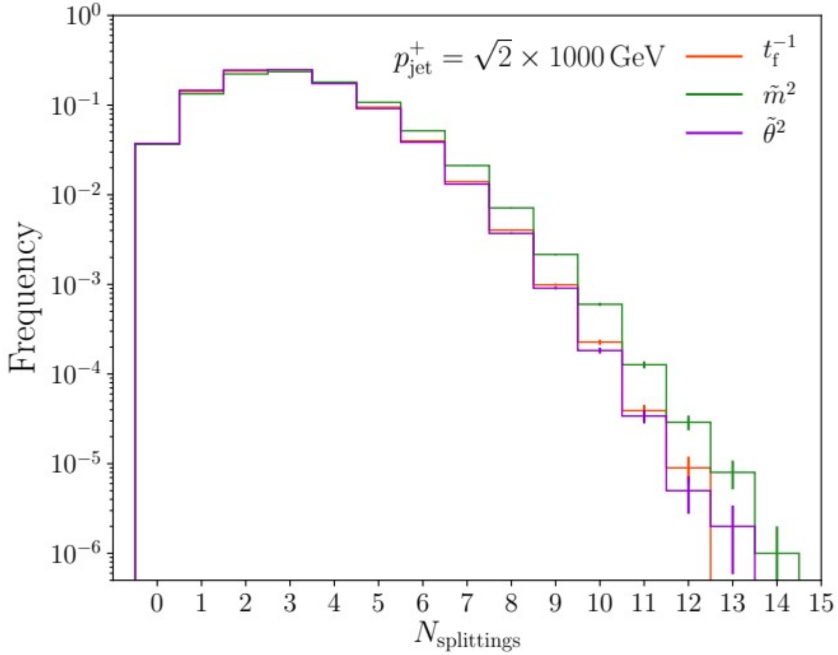

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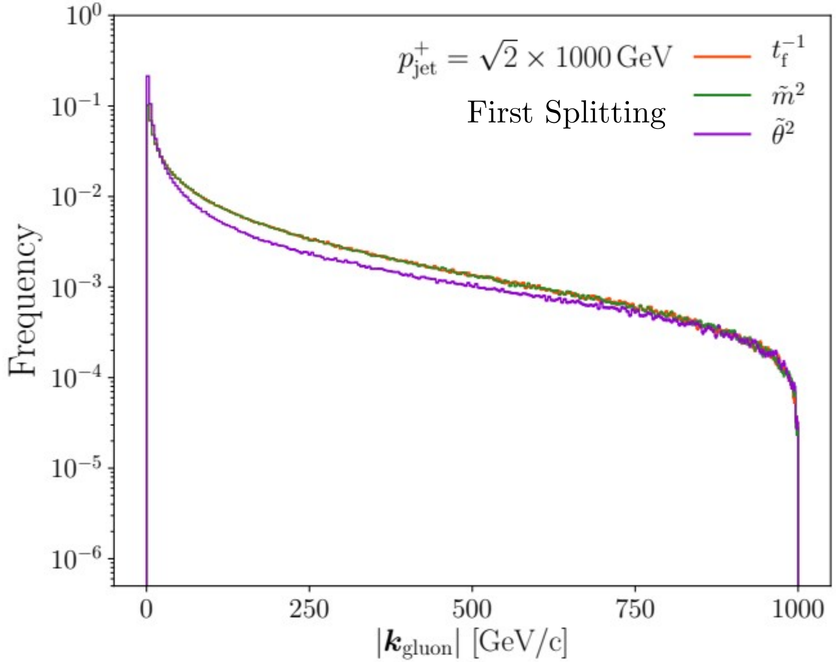
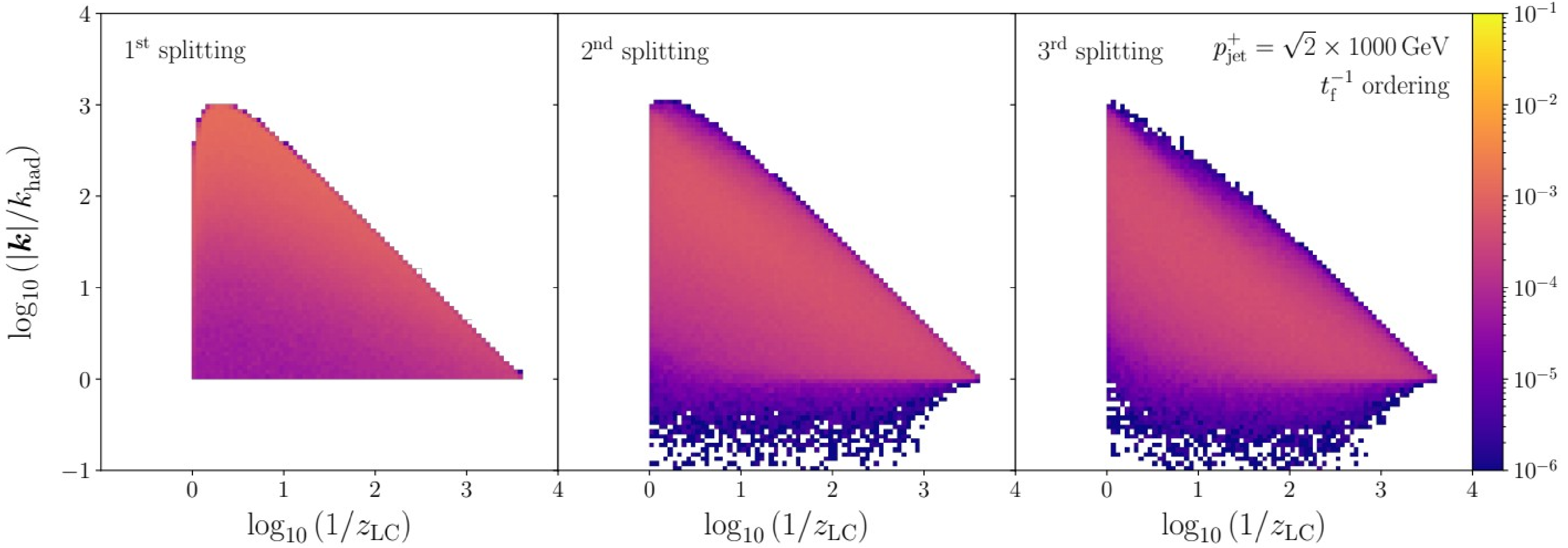
Transverse momentum of 1st splitting



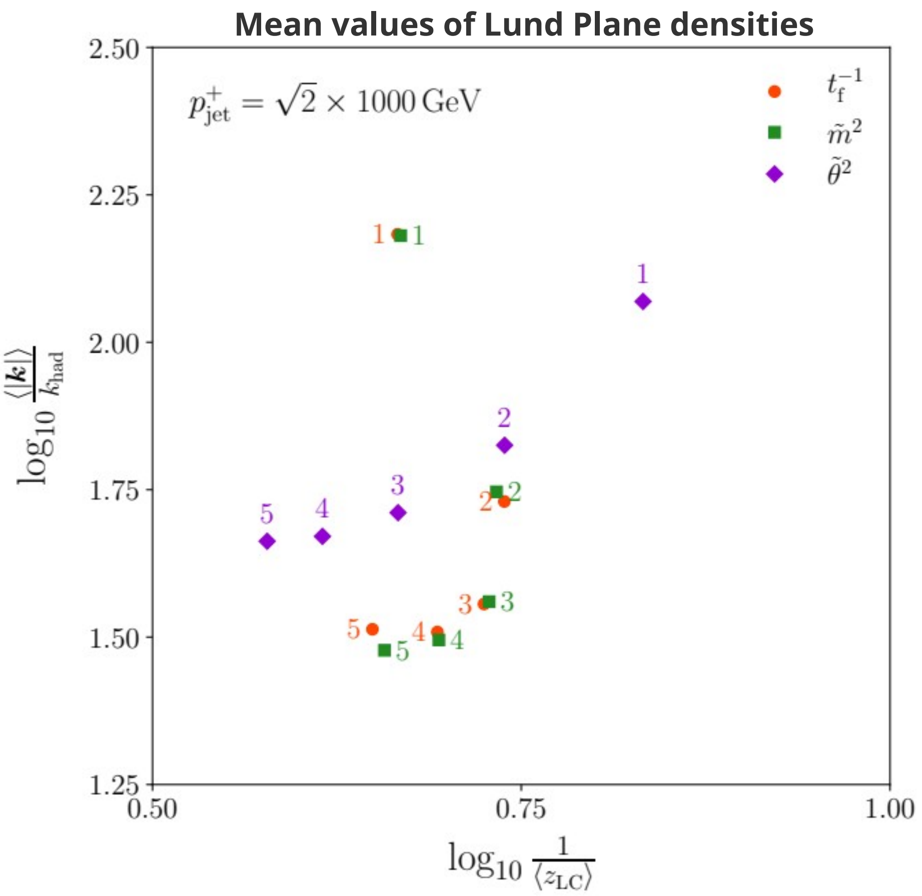
Transverse momentum distribution follows $\frac{\mathrm{d}|\boldsymbol{k}|^2}{|\boldsymbol{k}|^2}$

Lund Plane Densities



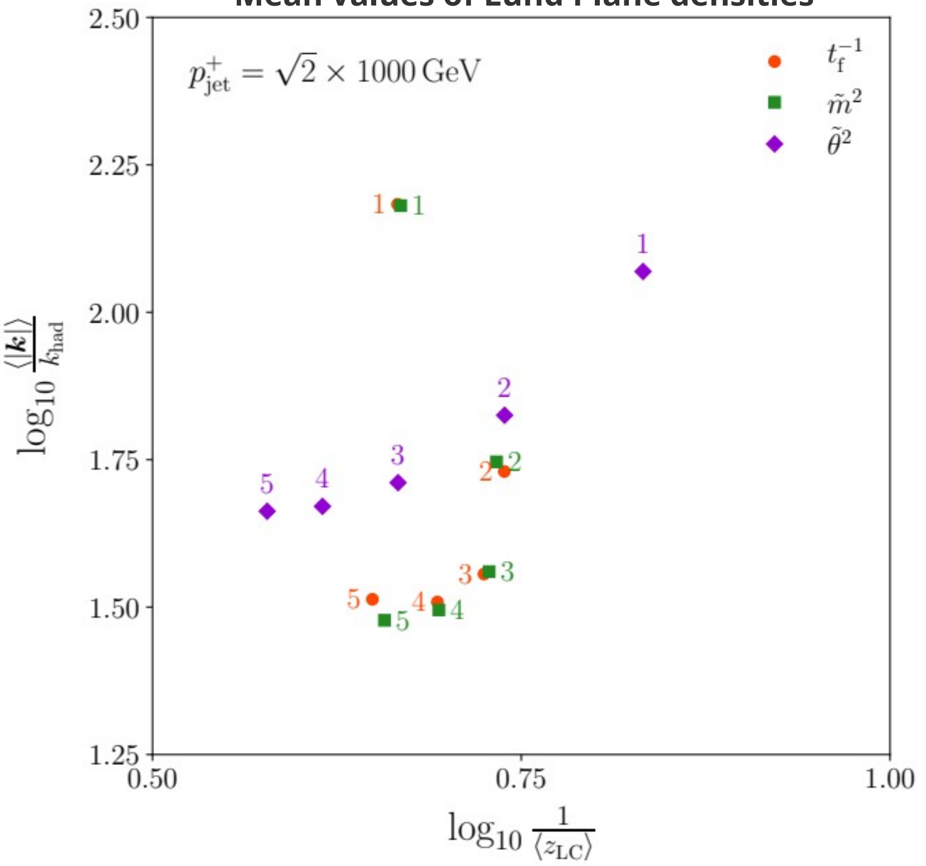
Shower evolution: Transverse momentum decreases, momentum fraction increases.

Lund Plane Trajectories



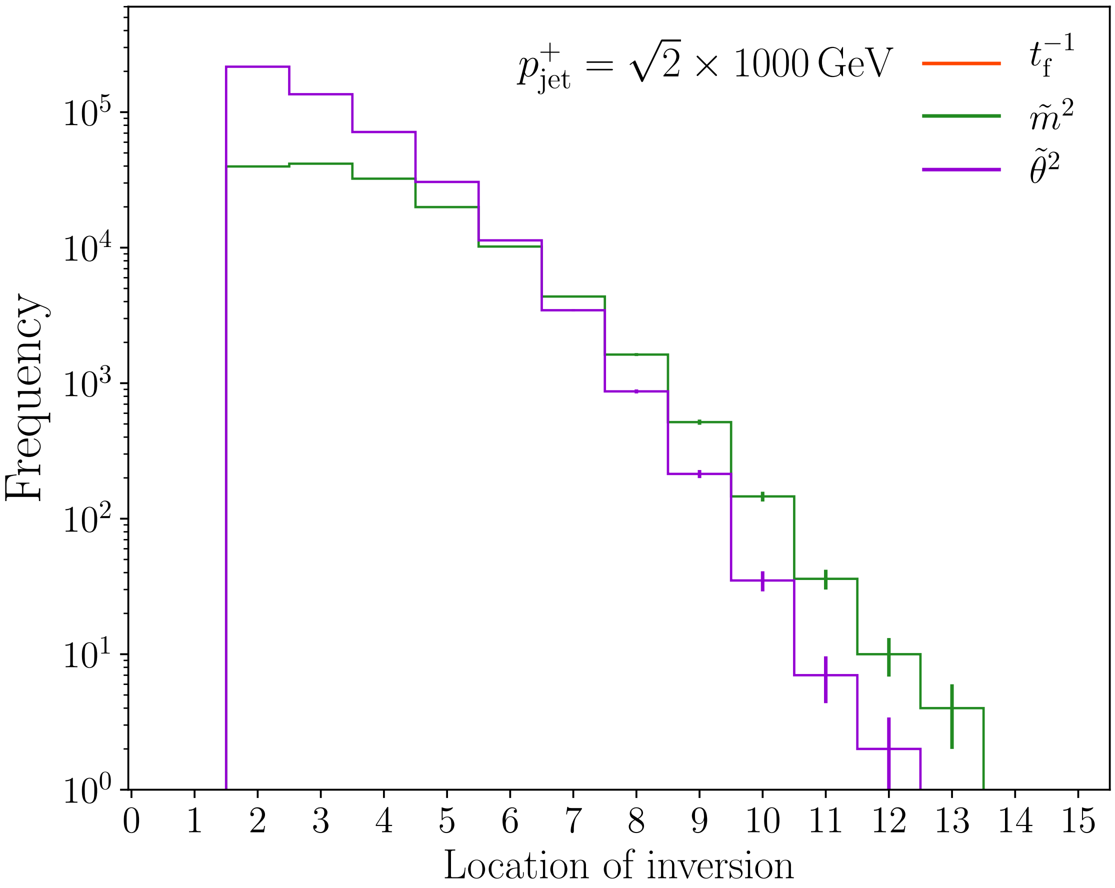
Lund Plane Trajectories

Mean values of Lund Plane densities



Differences between phase-space trajectories

Formation Time Inversions



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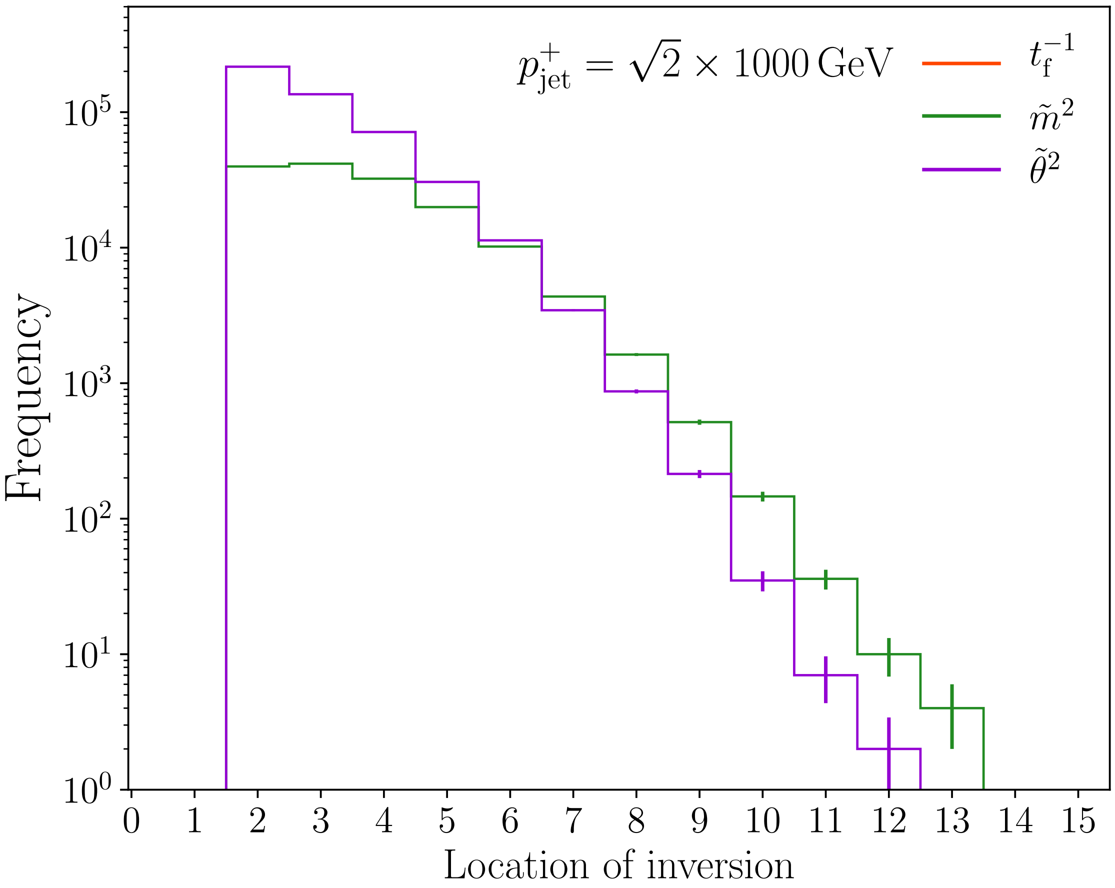

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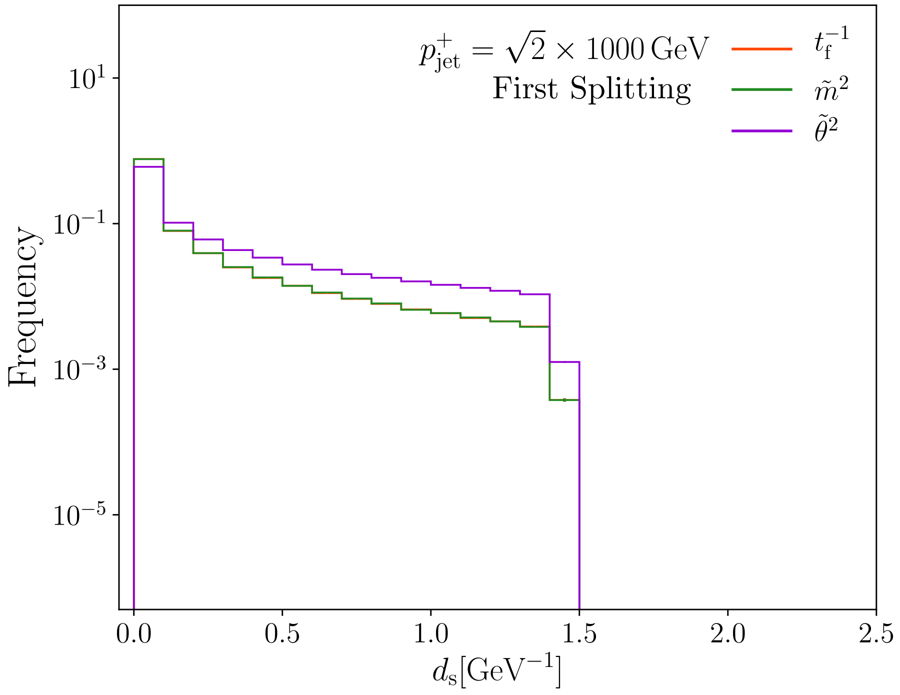
Does this discrepancy translate into differences in quenching magnitude?

Let’s look at jet quenching!

Simple (Pseudo-)Quenching Models



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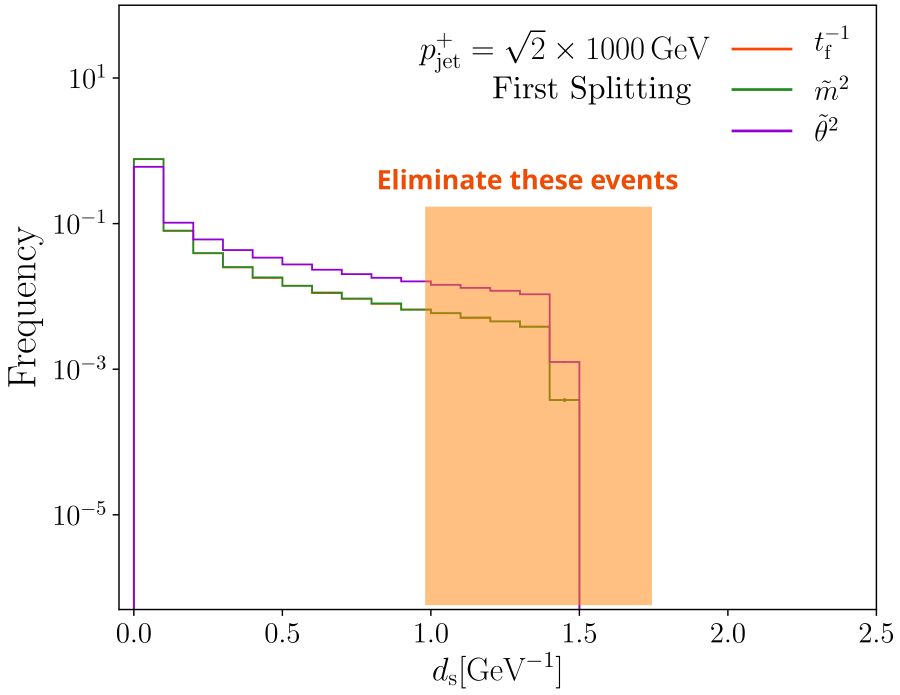

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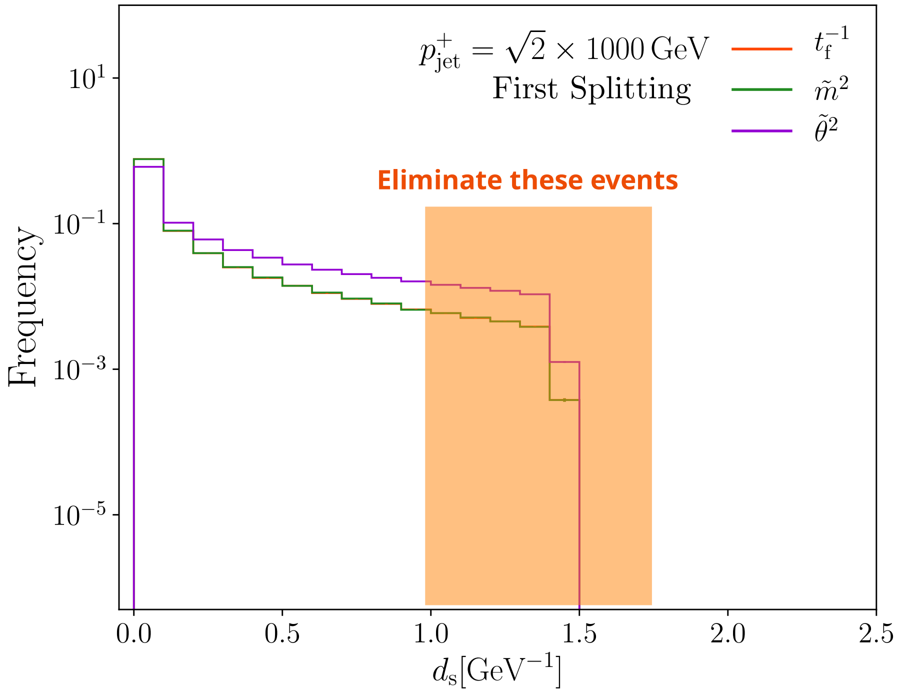

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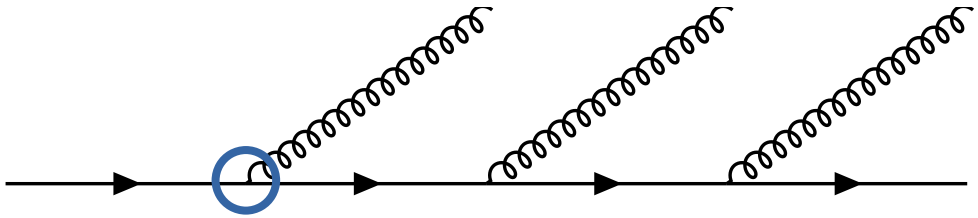

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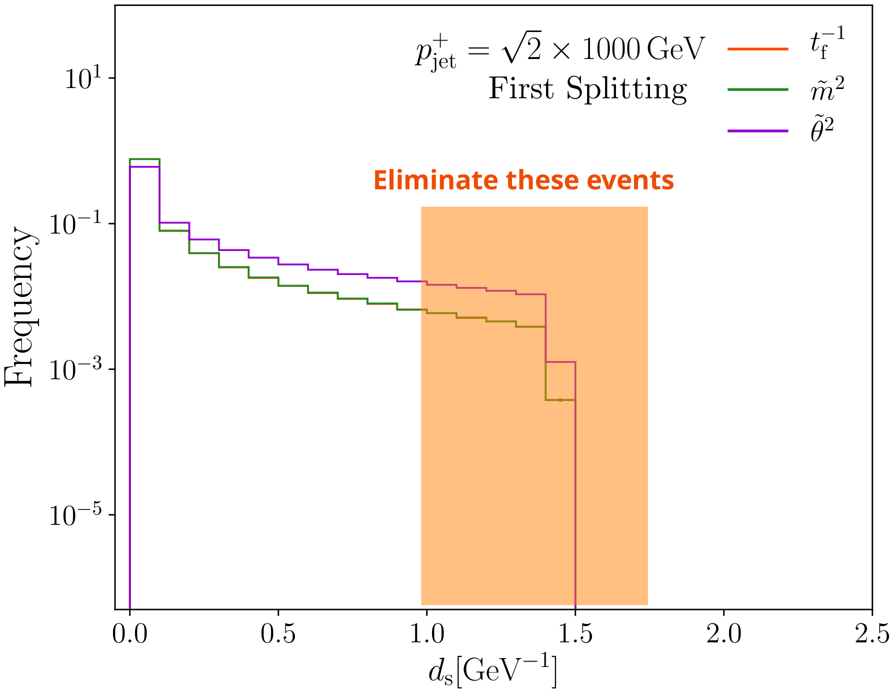

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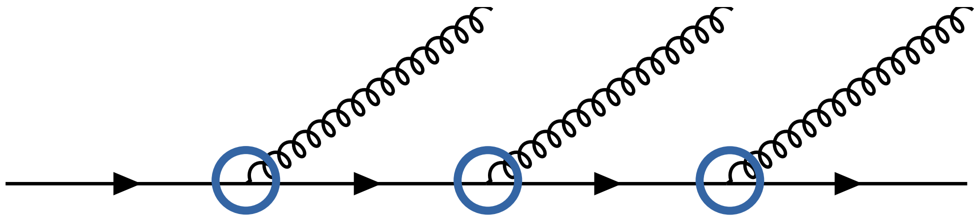
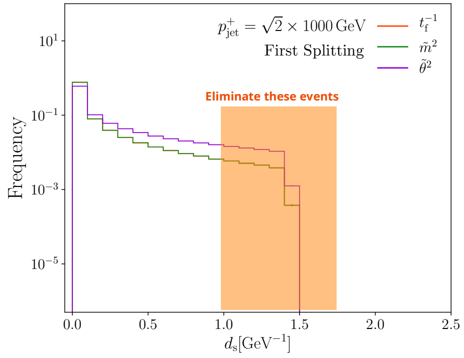
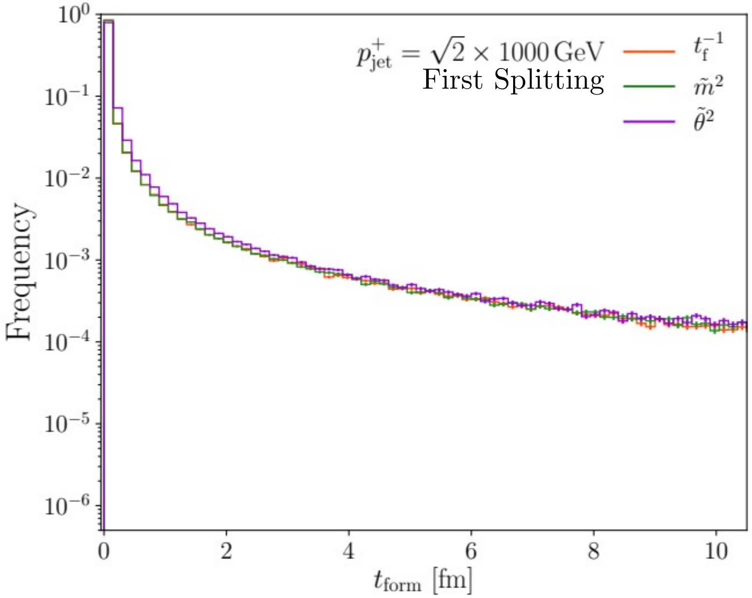


- Option 1: Apply only to first splitting
- Option 2: Apply to whole quark branch

Simple (Pseudo-)Quenching Models





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  - And if $t_{\rm form} < L$

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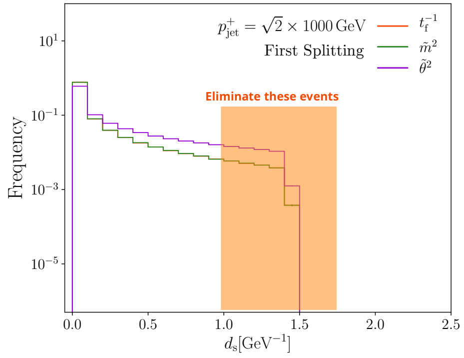

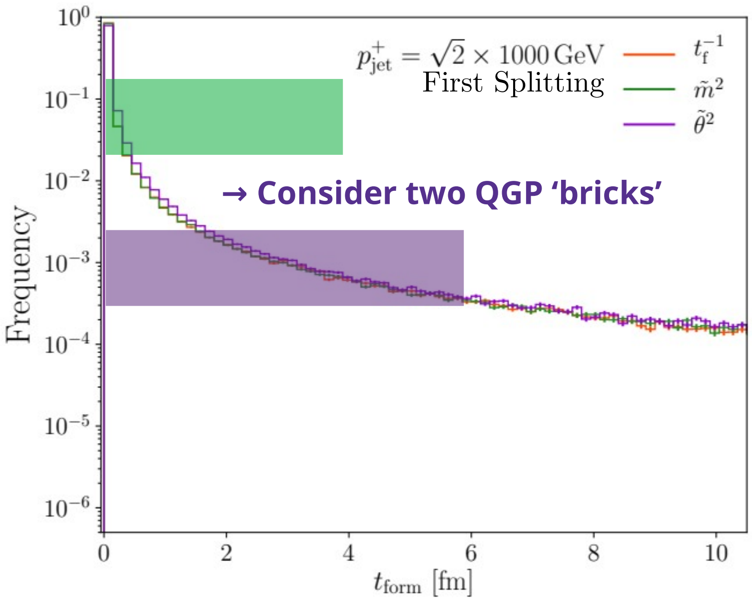

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L [fm]	4		6
$\hat{q}$ [GeV^2/fm]	2	5	5
t_{f}^{-1}	1.093 %	3.066 %	5.861 %
$\tilde{m}^2$	1.095 %	3.087 %	5.875 %
$\tilde{\theta}^2$	3.951 %	9.054 %	15.607 %

Apply quenching condition to the <u>first splitting</u>

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t_{f}^{-1}	1.093 %	3.066 %	5.861 %
$\tilde{m}^2$	1.095 %	3.087 %	5.875 %
$\tilde{\theta}^2$	3.951 %	9.054 %	15.607 %

Apply quenching condition to the first splitting

L [fm]	4		6
$\hat{q}$ [GeV^2/fm]	2	5	5
t_{f}^{-1}	4.578 %	11.519 %	22.030 %
$\tilde{m}^2$	4.939 %	12.371 %	23.512 %
$\tilde{\theta}^2$	4.569 %	11.452 %	22.005 %

Apply quenching condition to the entire quark branch

Influence of the first splitting on quenching

- Apply this pseudo-quenching model to all orderings
  - Compute the percentage of 'quenched' events

L [fm]	4		6
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Apply quenching condition to the <u>first splitting</u>

L [fm]	4		6
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Apply quenching condition to the <u>entire quark branch</u>

Quantifies importance of ordering scale

Summary

- We have created a toy Parton Shower Monte Carlo:
  - To explore differences between ordering variables
  - Aiming at a framework for time-ordered, medium-induced emissions
- Choice of vacuum ordering → Sensitivity to quenching at differential timescales
  - Model does not account for medium dilution, differential energy loss
  - Only implements vacuum emissions **[Medium-induced emissions needed]**
- Is jet quenching sensitive to the ordering of vacuum-like emissions?
  - Suggested by this simple model. **[Work in Progress]**

Thank you!

Acknowledgements

LIP

fct Fundação para a Ciência e a Tecnologia

TÉCNICO LISBOA

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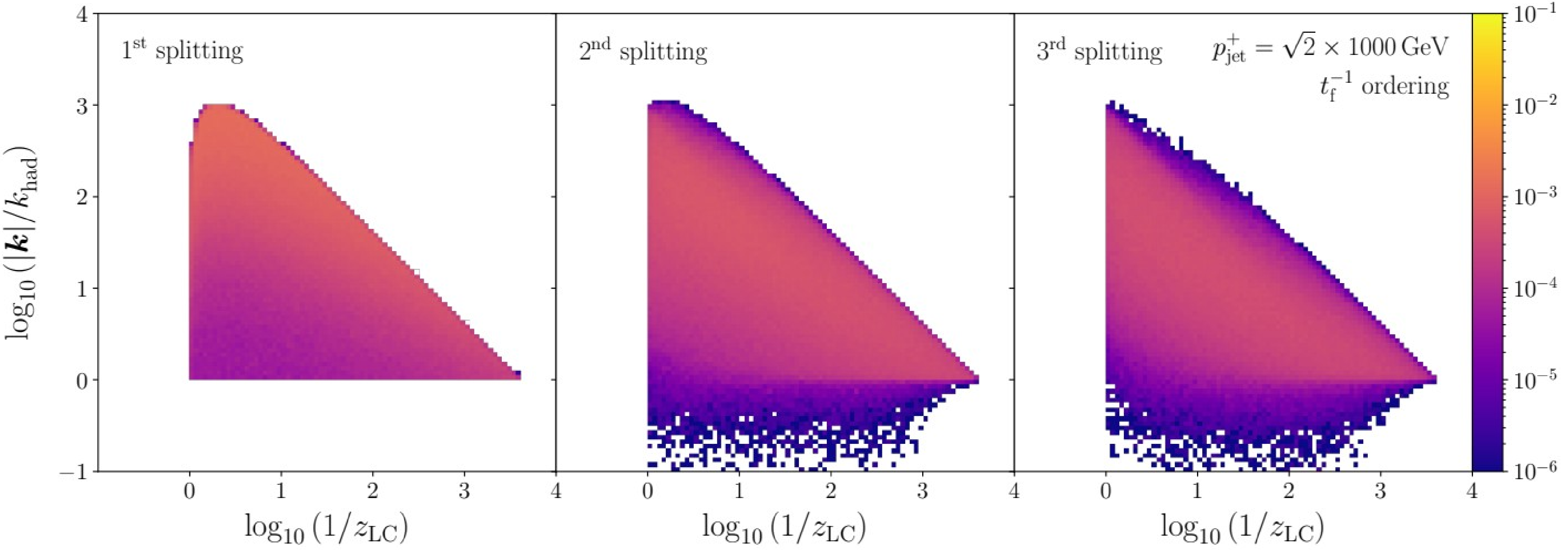
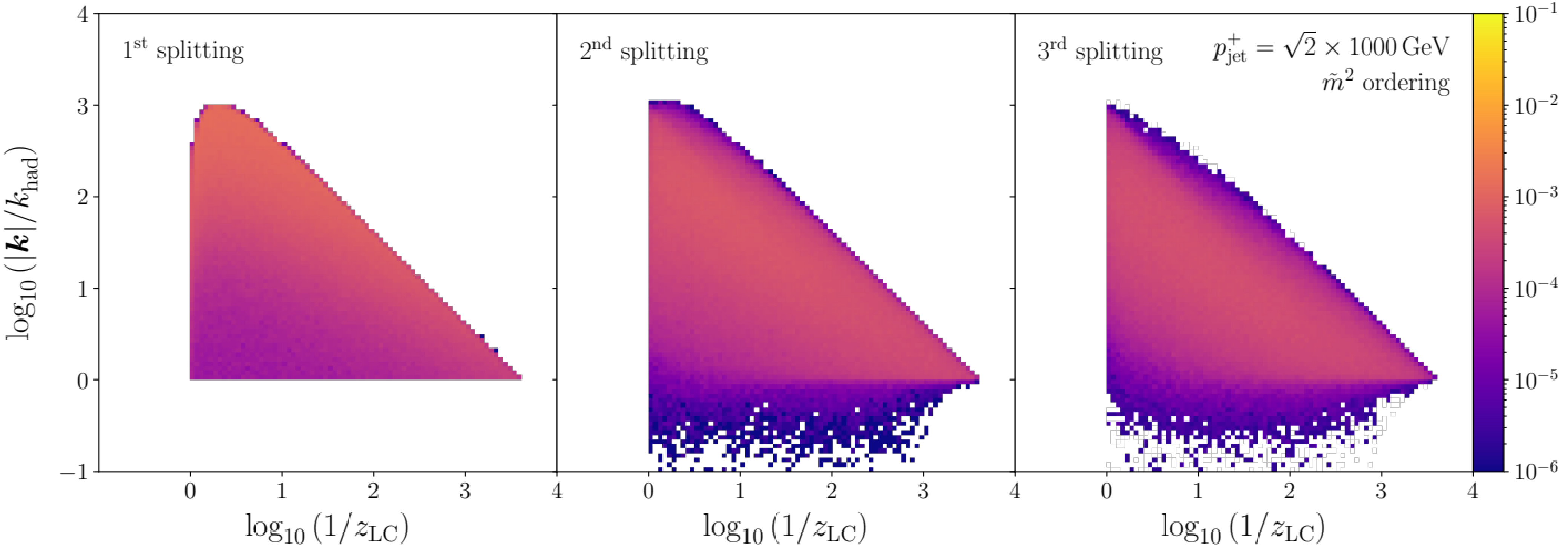
Backup Slides

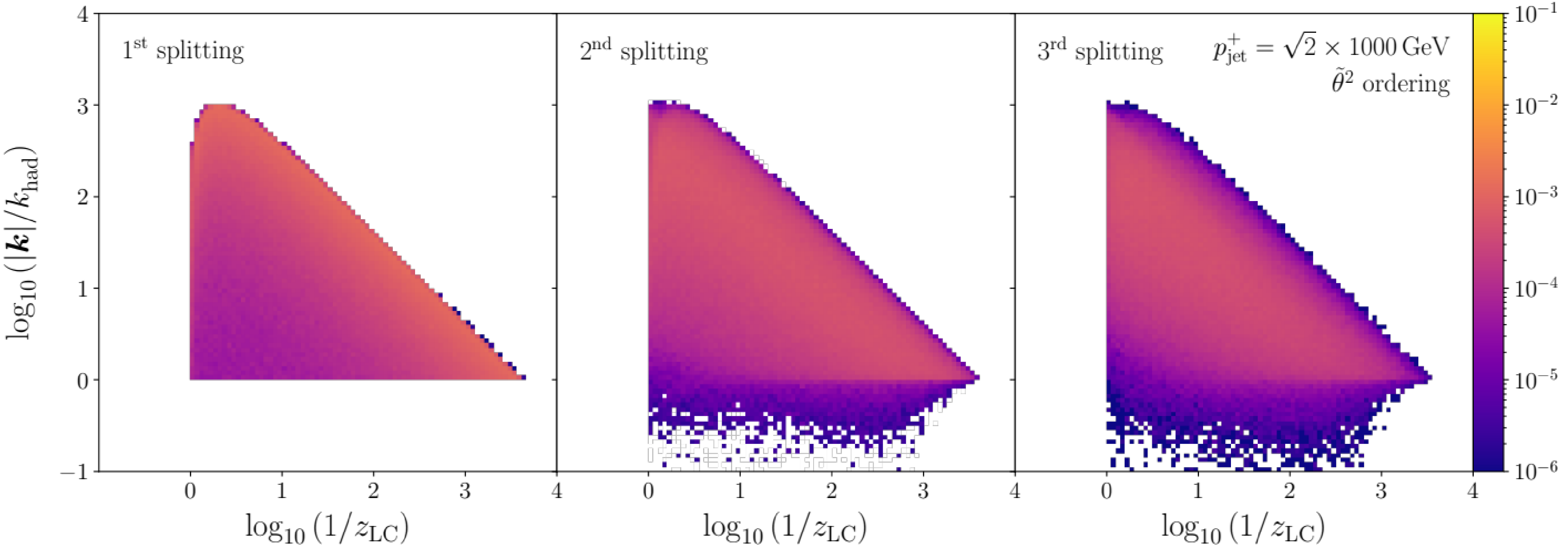
Lund Plane Densities – Time ordering



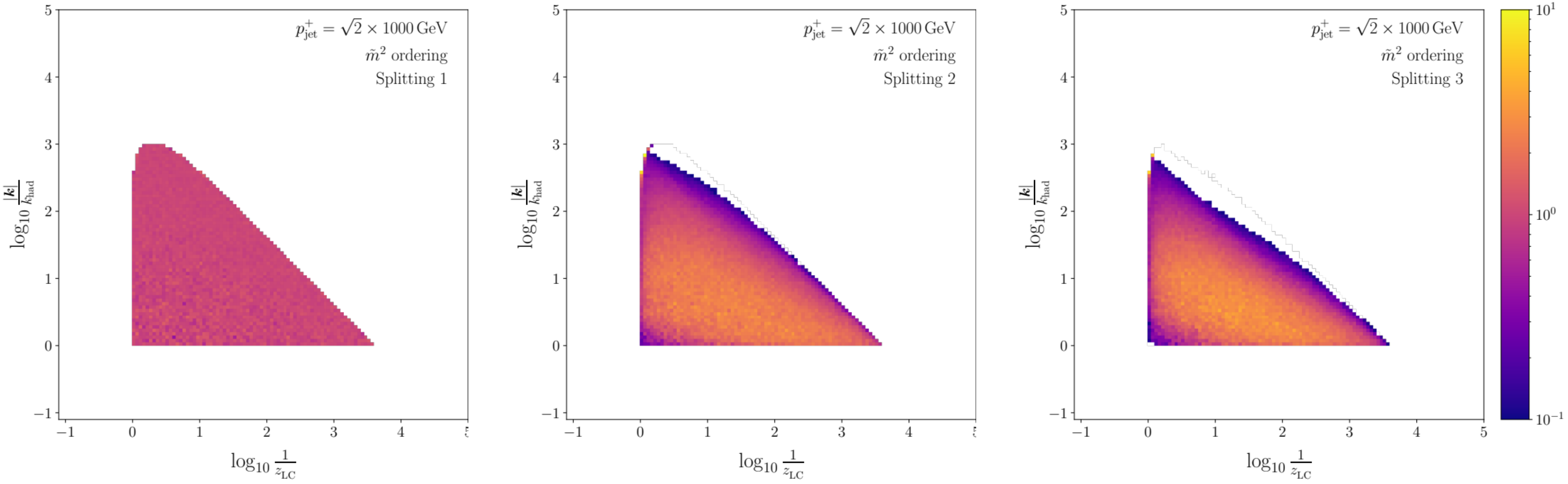
Lund Plane Densities – Virtuality ordering



Lund Plane Densities – Angular ordering

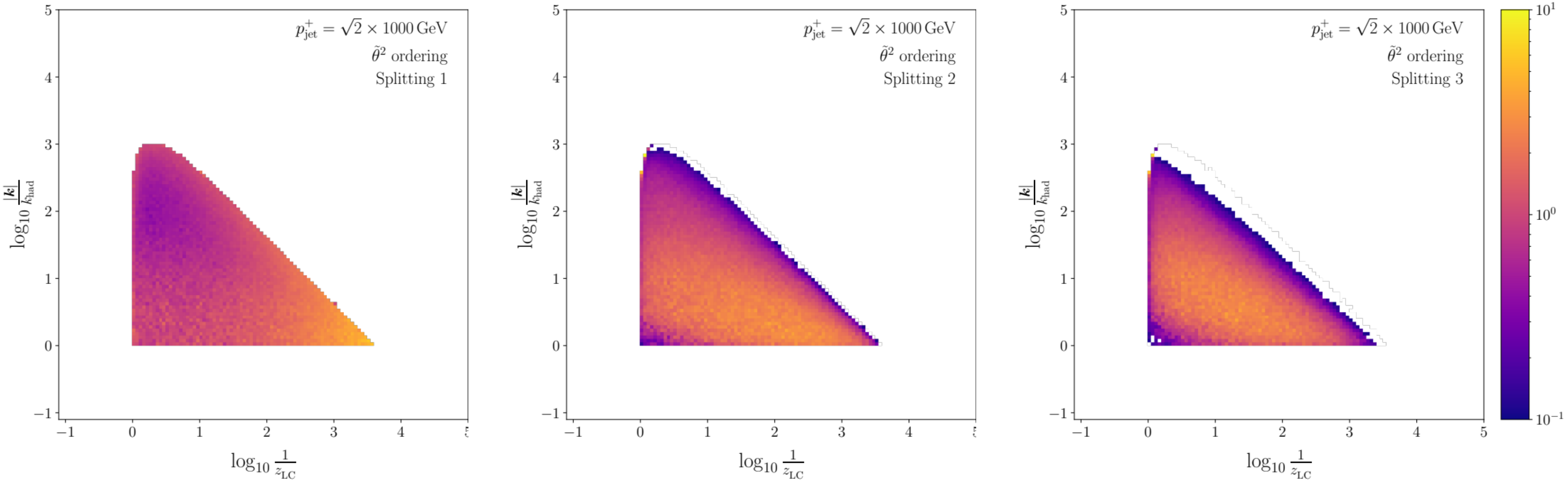


Lund Density Ratio – Mass / Formation Time



All Events

Lund Density Ratio – Angle / Formation Time



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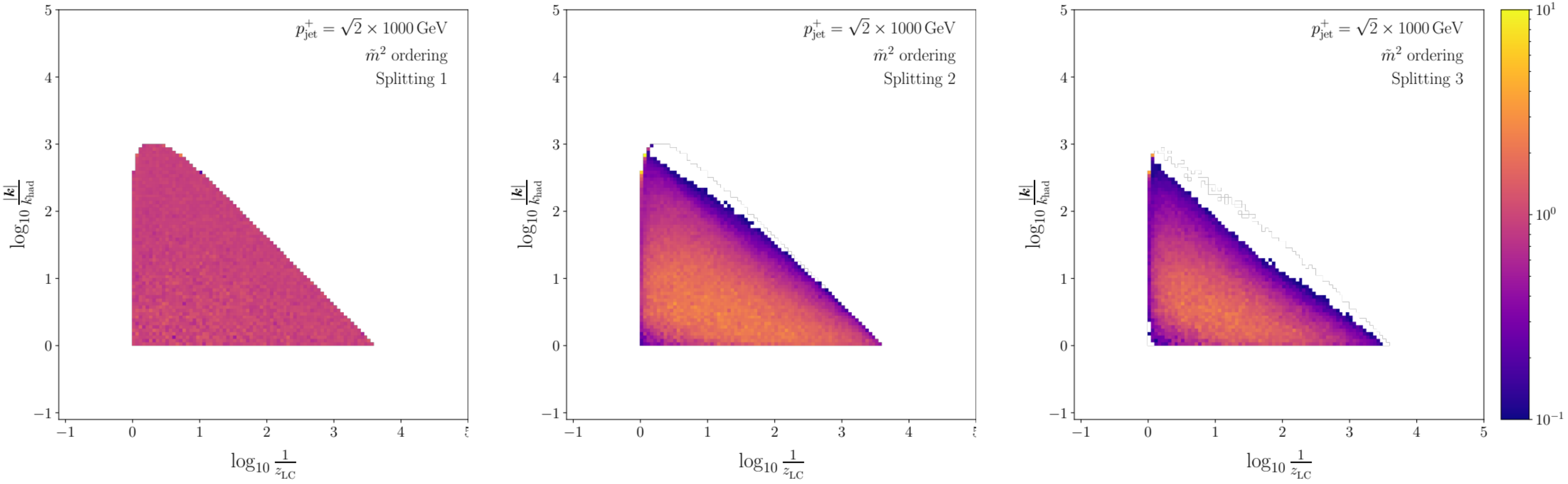
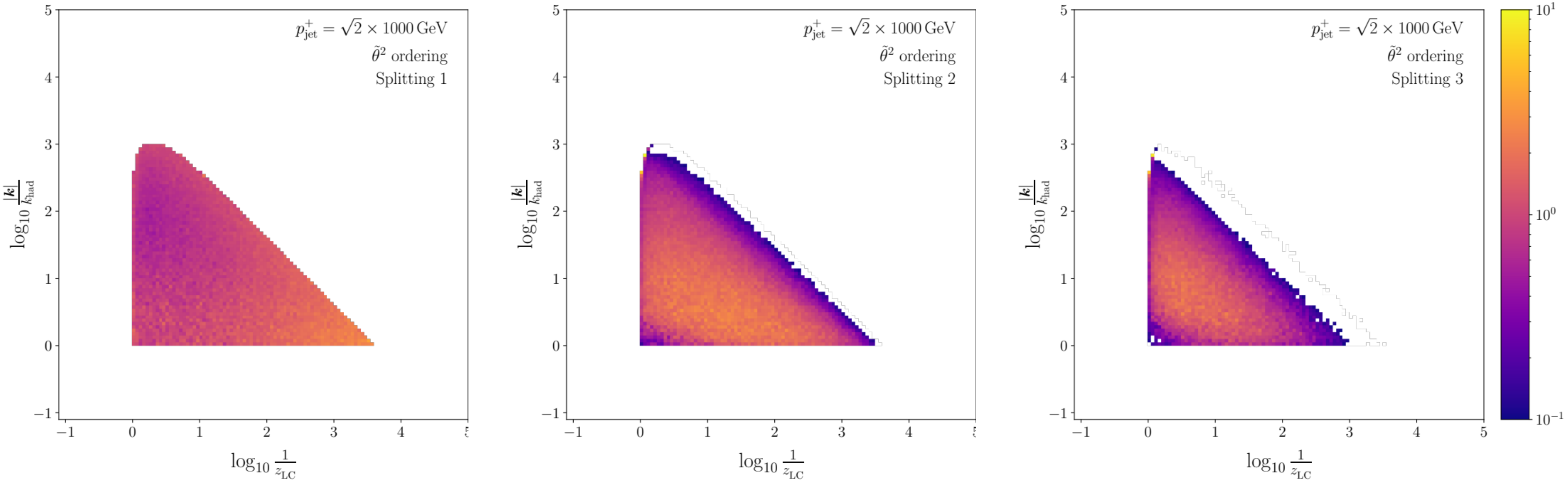


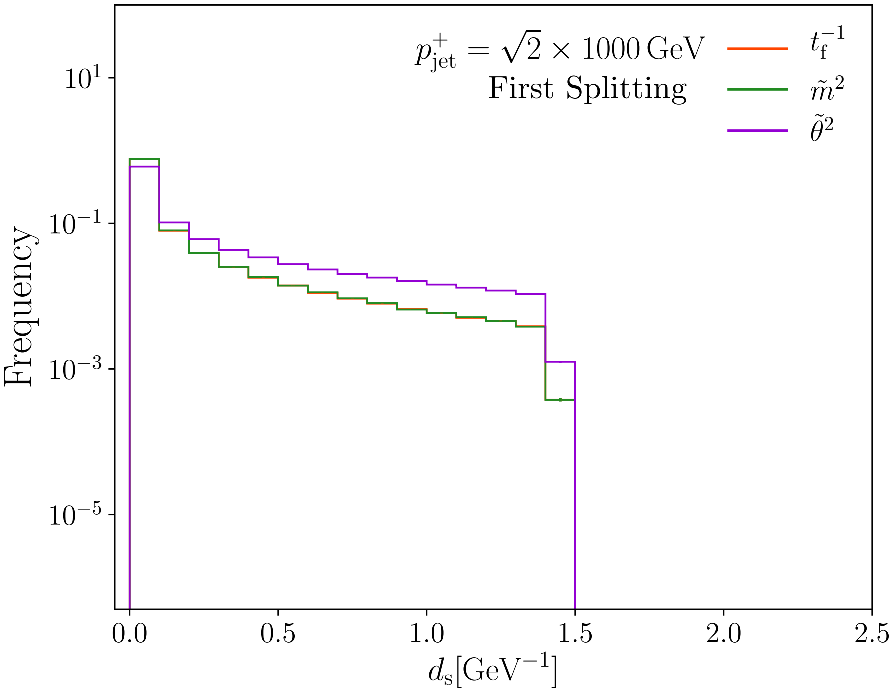
Events with at least 3 quark splittings

Lund Density Ratio – Angle / Formation Time



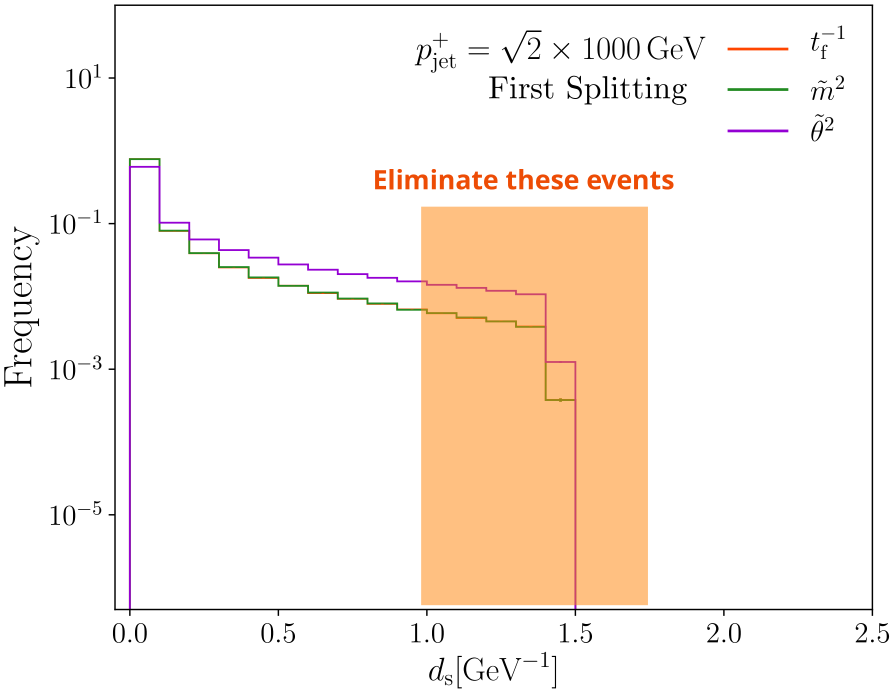
Events with at least 3 quark splittings

Simple (Pseudo-)Quenching Models



- Consider distance between daughters: $\mathrm{d}_s = \sqrt{\frac{t_{\rm form}}{k^+}}$
- **A <u>simplistic</u> model:**
  - Eliminate event if $d_s > d_{\rm coh} = \frac{1}{\sqrt{\hat{q}L}}$ **(Decoherence)**

Simple (Pseudo-)Quenching Models



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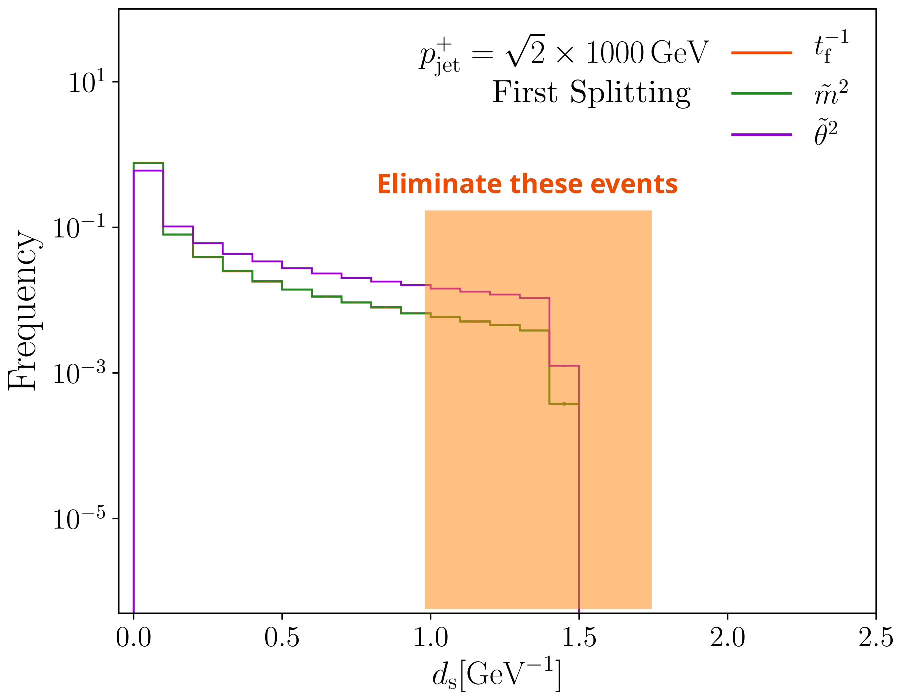
Simple (Pseudo-)Quenching Models



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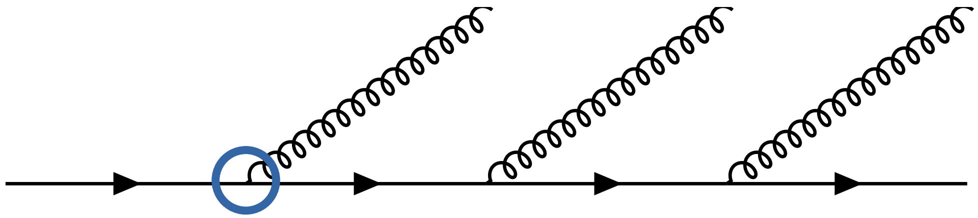
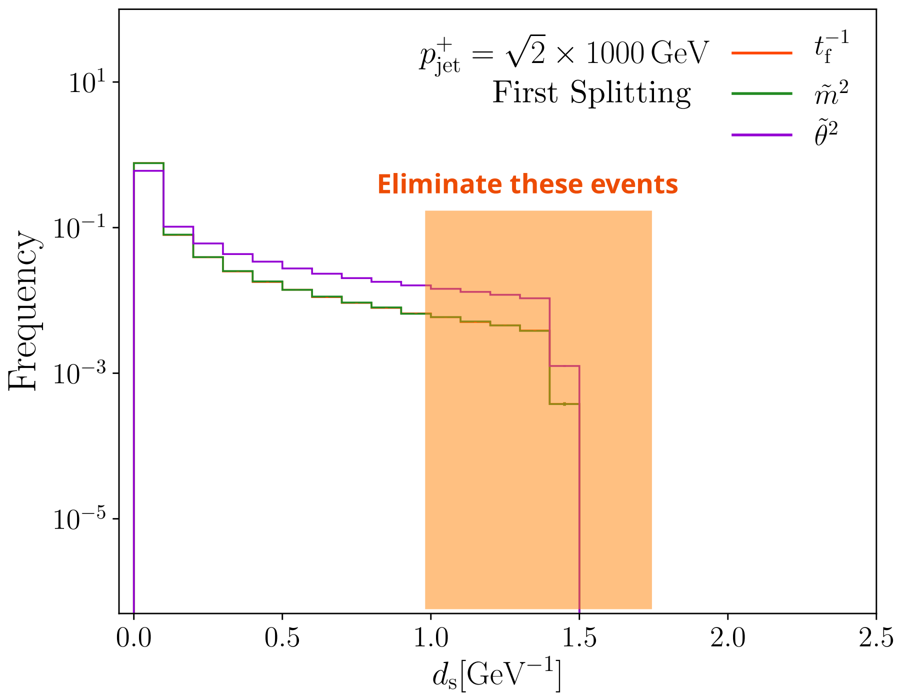


- Option 1: Apply only to first splitting

Simple (Pseudo-)Quenching Models



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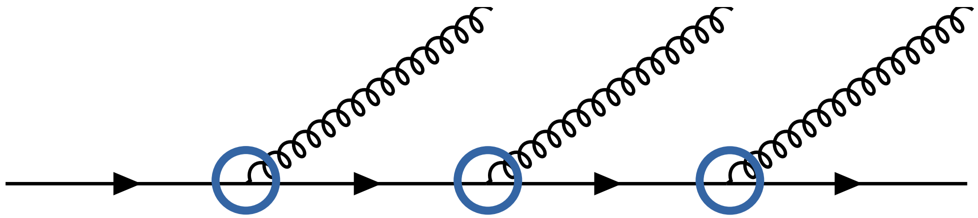

- Option 1: Apply only to first splitting
- Option 2: Apply to whole quark branch

Quenched events in simple model

- Apply this pseudo-quenching model to all orderings
  - Compute the percentage of 'quenched' events

L [fm]	4		6
$\hat{q}$ [GeV^2/fm]	2	5	5
t_{f}^{-1}	2.624 %	4.811 %	7.478 %
$\tilde{m}^2$	2.663 %	4.824 %	7.510 %
$\tilde{\theta}^2$	8.044 %	12.547 %	18.212 %

Apply quenching condition to the <u>first splitting</u>

L [fm]	4		6
$\hat{q}$ [GeV^2/fm]	2	5	5
t_{f}^{-1}	10.972 %	18.033 %	28.097 %
$\tilde{m}^2$	11.777 %	19.260 %	29.853 %
$\tilde{\theta}^2$	10.933 %	17.974 %	28.070 %

Apply quenching condition to the <u>entire quark branch</u>