Quantum simulation of jet evolution in a medium

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In collaboration with
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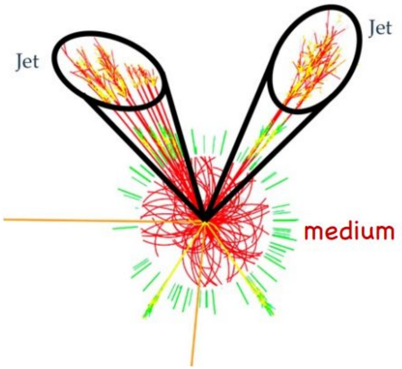
Outline

1. Methods and physical set up
2. Quantum simulation algorithm
3. Results
   - Jet evolution
   - Momentum broadening

Tools to understand jet physics

- Experiments: designing observables, ...
- Analytical calculations: improved approximation, multiple scattering, …
- Numerical methods: Monte Carlo simulations, light-front Hamiltonian, ...
- Quantum simulation?



From LF Hamiltonian formalism to quantum simulation

Classical light-front Hamiltonian formalism

Scattering in Time-Dependent Basis Light-Front Quantization, PRD 88 (2013) 065014

Ultrarelativistic quark-nucleus scattering, PRD 101 (2020) 7, 076016

Scattering and gluon emission in a color field, PRD 104 (2021) 5, 056014

Quantum simulation

Single-particle digitization strategy for quantum computation of a $\phi4$ scalar field theory, PRA 103 (2021) 4, 042410

A quantum strategy to compute the jet quenching parameter, Eur.Phys.J.C 81 (2021) 10, 862

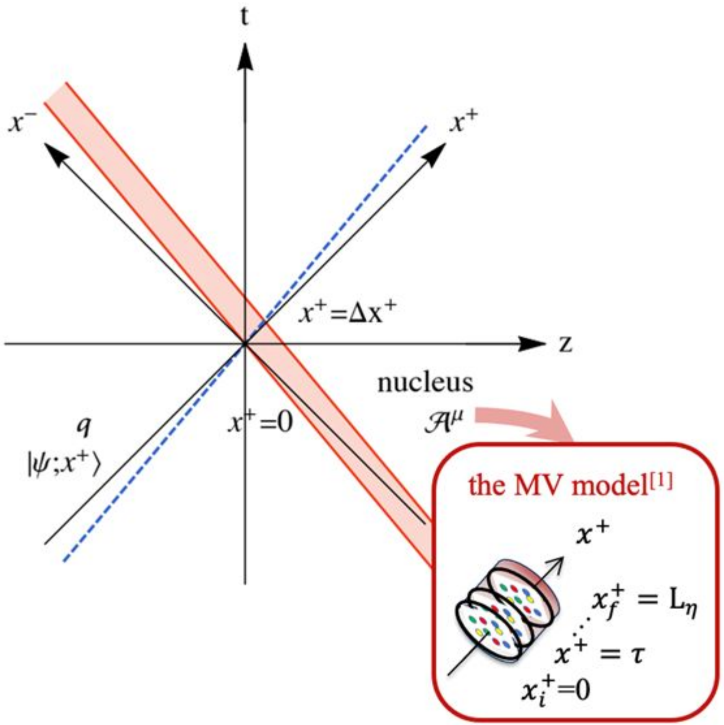
Quantum simulation of nuclear inelastic scattering, PRA 104 (2021) 1, 012611

Medium induced jet broadening in a quantum computer, PRD 106 (2022) 7, 074013

Quantum simulation of jet evolution in a medium (work in progress)

Physical set up

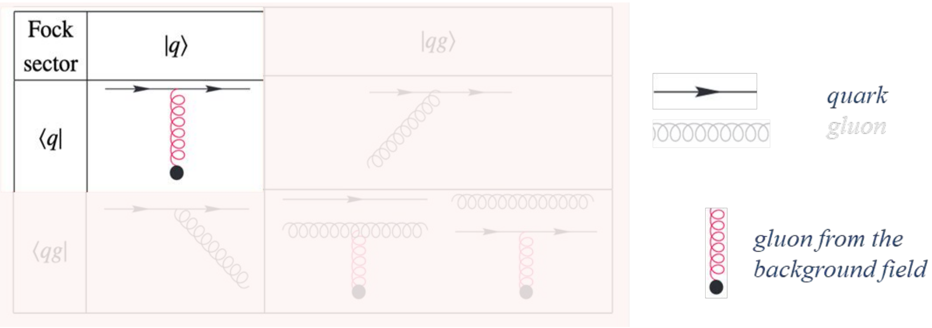
Li, Zhao, Maris, Chen, Li, Tuchin, Vary PRD101.076016 (2020)
Barata, Du, Li, WQ, Salgado, PRD106,074013 (2022)



High-energy quark moving close to the light cone scattering on a dense nucleus medium

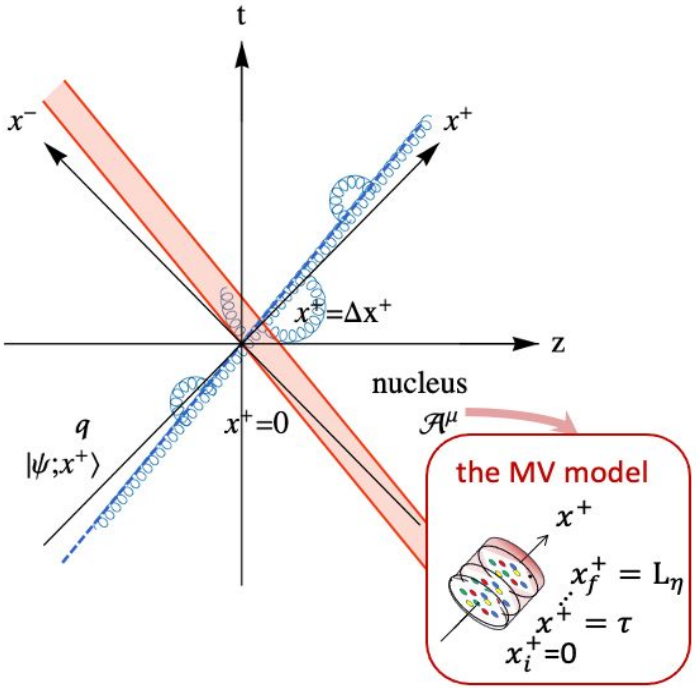
The light-front Hamiltonian in the $|q\rangle$ Fock sector:

$$P^-(x^+) = P^-_{\mathrm{KE}} + V_A(x^+)$$



Physical set up

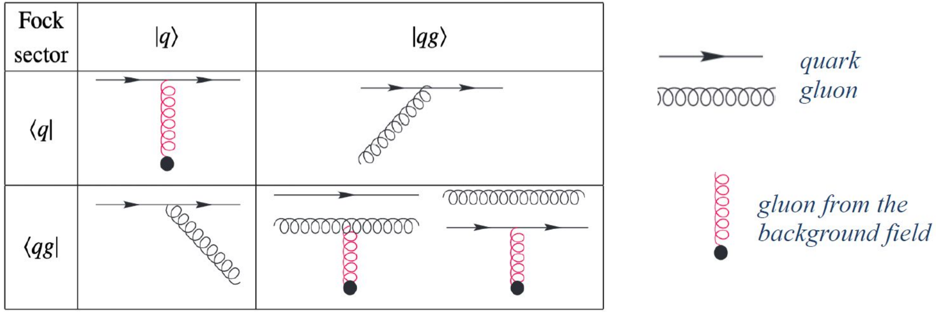
Li, Lappi, Zhao, PRD104.056014 (2021)



High-energy quark moving close to the light cone scattering on a dense nucleus medium

The light-front Hamiltonian in the $|q\rangle + |qg\rangle$ Fock sector:

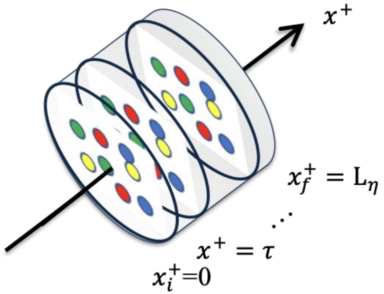
$$P^-(x^+) = P^-_{\text{KE}} + V(x^+) = P^-_{\text{KE}} + \left\{ V_{qg} + V_{\mathcal{A}}(x^+) \right\}$$



The medium and evolution

The stochastic background field uses the McLerran-Venugopalan (MV) model

McLerran, Venugopalan, PRD49, 2233; PRD49, 3352; PRD50, 2225 (1994)



$$\langle\langle \rho_a(\vec{x}_\perp, x^+)\rho_b(\vec{y}_\perp, y^+)\rangle\rangle = g^2\tilde{\mu}^2\delta_{ab}\delta^2(\vec{x}_\perp - \vec{y}_\perp)\delta(x^+ - y^+)$$

$$(m_g^2 - \nabla_\perp^2)\mathcal{A}_a^-(\vec{x}_\perp, x^+) = \rho_a(\vec{x}_\perp, x^+)$$

Light-front time evolution of the probe, decomposed as sequence of unitary operators

$$|\psi_{L_\eta}\rangle = U(L_\eta;0)\,|\psi_0\rangle \equiv \mathcal{T}_+ e^{-i\int_0^{L_\eta} \mathrm{d}x^+\, P^-(x^+)}\,|\psi_0\rangle \qquad U(L_\eta;0) = \prod_{k=1}^{N_t} U(x_k^+; x_{k-1}^+)$$

Quantum simulation algorithm

Wiesner, 9603028 (1996); Zalka, 9603026 (1996)

1. **Define problem Hamiltonian**
2. **<u>Encode Hamiltonian onto basis</u>**
3. Prepare initial states
4. Evolution
5. Measurement protocol

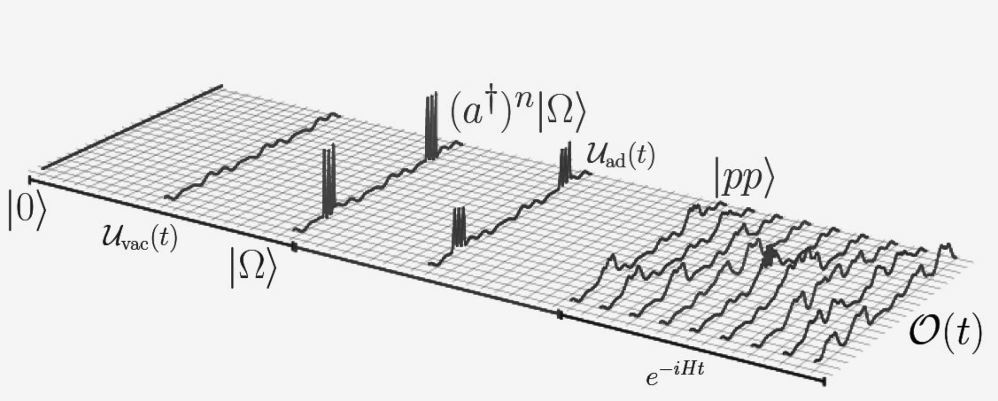


Image from Lamm's talk at Fermilab (2021)

Basis encoding

Barata, Mueller, Tarasov, Venugopalan, PRA103, 4, 042410(2021)

To encode the quantum state, we use single-particle representation. The general quantum state

$$|\psi\rangle = \underbrace{|q\rangle \cdots |q\rangle}_{n} \otimes \underbrace{|g\rangle \cdots |g\rangle}_{m} \longrightarrow |\tilde{\psi}\rangle = \prod_{i=1}^{m} \left(|e_{g_i}\rangle \otimes |g_i\rangle \right) \otimes \prod_{i=1}^{n} \left(|e_{q_i}\rangle \otimes |q_i\rangle \right)$$

$|e_{q_i}\rangle, |e_{g_i}\rangle$

existence registers

Within $|q\rangle + |qg\rangle$ Fock sectors, the occupancy registers can be omitted

$$|q_{\rm dressed}\rangle = |z\rangle \otimes \underbrace{\left(|g_x\rangle |g_y\rangle |c_g\rangle \right)}_{|g\rangle} \otimes \underbrace{\left(|q_x\rangle |q_y\rangle |c_q\rangle \right)}_{|q\rangle}$$

$$|z\rangle = |0\rangle \Leftrightarrow \text{Fock } |q\rangle, k_g^+ = 0, k_q^+ = K$$
$$|z\rangle = |1\rangle \Leftrightarrow \text{Fock } |qg\rangle, k_g^+ = 1, k_q^+ = K - 1$$
$$\cdots$$

take care of unphysical states

Resource cost at $N_C = 2$

$$N_{\rm tot} = 2^7 \lceil K \rceil N_\perp^4 \rightarrow n_Q = (7 + 4 \log N_\perp + \log \lceil K \rceil)$$

Lattice

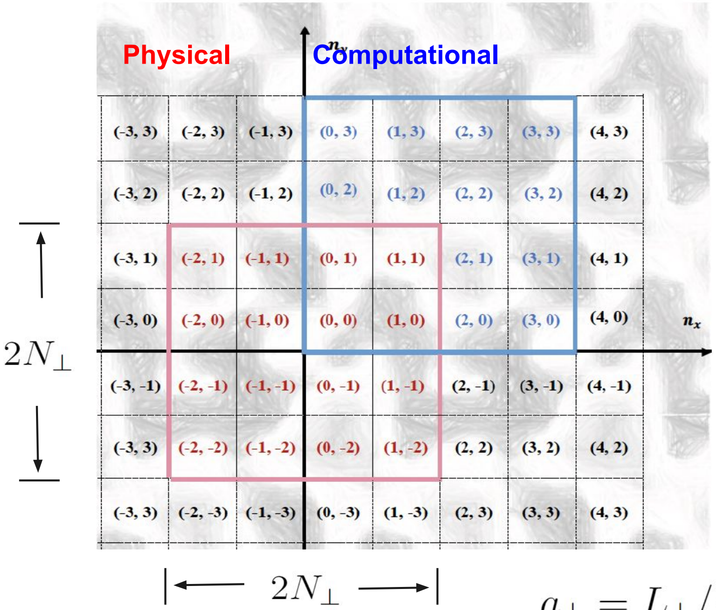
- Transverse lattice is periodical, for both position and momentum space via quantum fourier transform (FT)

$$(n_x, n_y) \Longleftrightarrow (n_x + i2N_\perp, n_y + j2N_\perp)$$

$$\vec{r}_\perp = (n_x, n_y)a_\perp \qquad \vec{p}_\perp = (k_x, k_y)b_\perp$$

- Longitudinal direction is periodical (anti-periodical) for bosons (fermions)

$$p_l^+ = \frac{2\pi}{L}k_l^+, \quad k_q^+ = \frac{1}{2}, \frac{3}{2}, \cdots, \quad k_g^+ = 1, 2, 3, \cdots$$

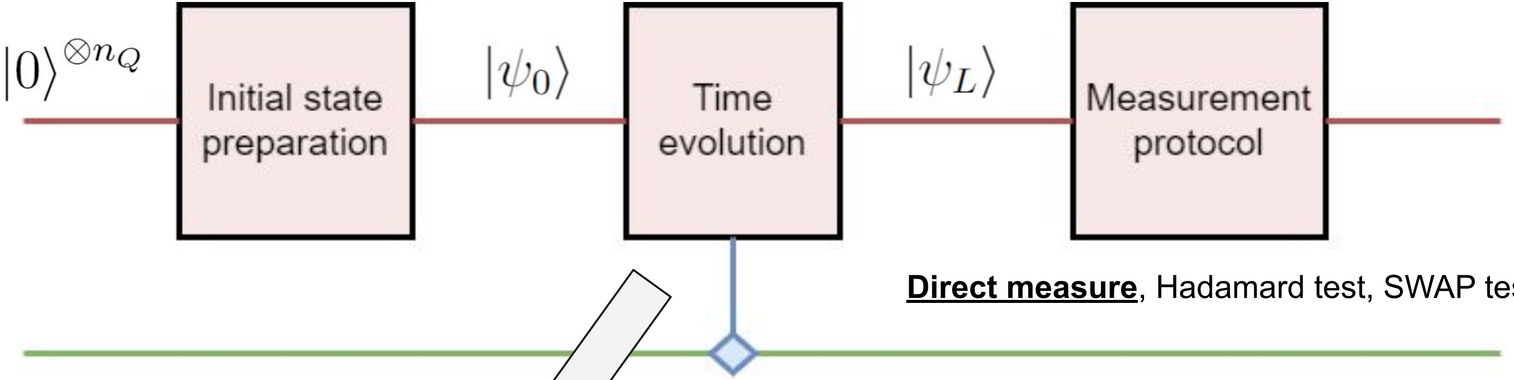


$$a_\perp = L_\perp / N_\perp$$

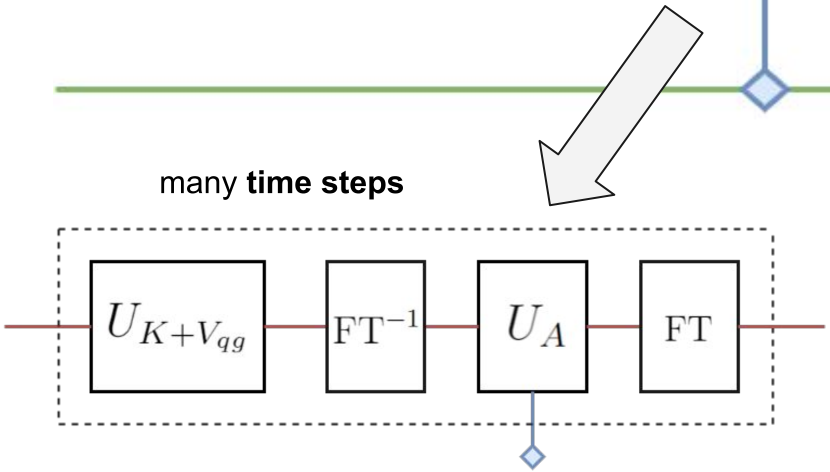
$$b_\perp = \pi / L_\perp$$

Workflow of quantum evolution

Amplitude level



Direct measure, Hadamard test, SWAP test, etc



FT allows convenient simulation in the respective basis, esp when Hamiltonians are extremely sparse

$$U_A(x^+, \delta x^+)\,|\boldsymbol{r}\rangle = \exp(-iV_A\delta x^+)\,|\boldsymbol{r}\rangle$$
$$U_K(\delta x^+)\,|\boldsymbol{p}\rangle = \exp(-iP^-_{\mathrm{KE}}\delta x^+)\,|\boldsymbol{p}\rangle$$
$$U_{V_{qg}}(\delta x^+)\,|\boldsymbol{p}\rangle = \exp(-iV_{qg}\delta x^+)\,|\boldsymbol{p}\rangle$$

Ham => pauli strings => quantum gates

Simulation parameters

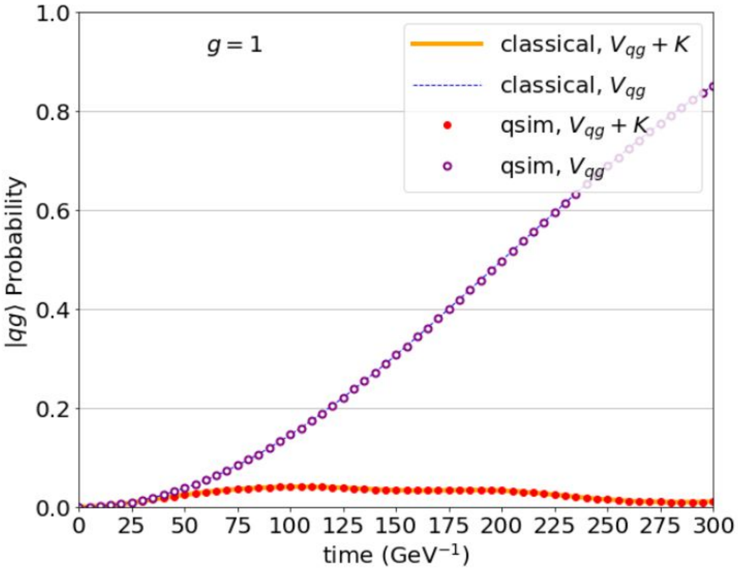
- Our initial state for the jet is a superposition color state with zero transverse momenta: $(p_x, p_y) = (0, 0)$
- Duration of static medium: $L_\eta = 50\,\mathrm{GeV}^{-1} \approx 10\,\mathrm{fm}$
- 5 stochastic fields are used for configuration average
- Computational lattice:

  Transverse direction: $N_\perp = 1$ Longitudinal direction: $K = 3.5$
- SU(2) color $N_C = 2$
- Spin non-flip case
- Selected values of saturation scales $Q_s^2 = C_F \frac{(g^2\tilde{\mu})^2 L_\eta}{2\pi}$ $C_F = (N_c^2 - 1)/(2N_c)$
- Sufficient time steps in trotterization and layers in background medium

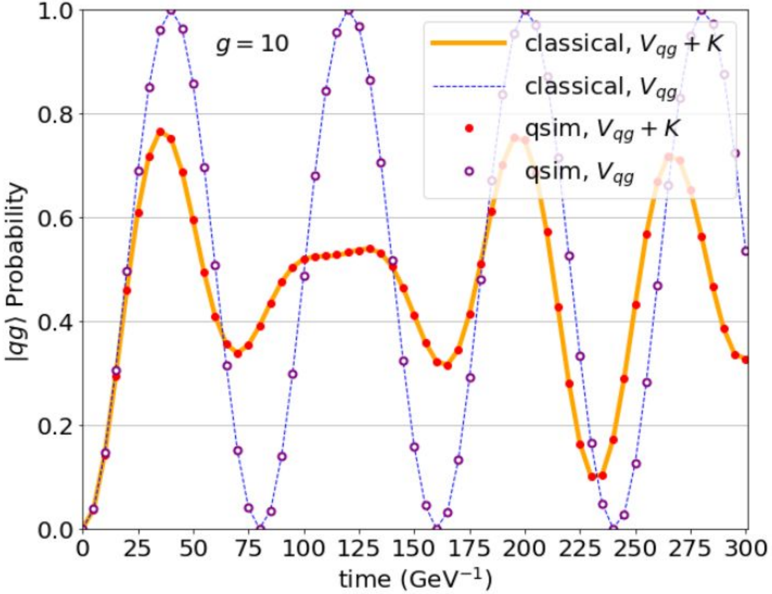
We present our preliminary results of jet evolution in vacuum and in medium, using IBM Qiskit quantum simulators

Qiskit

Results: jet evolution in vacuum



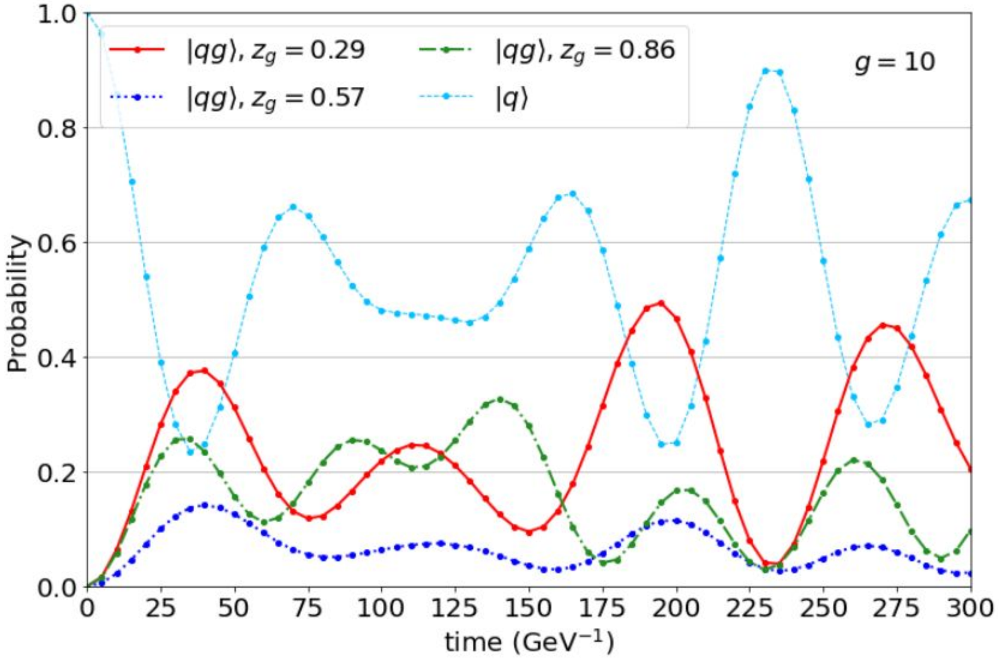
g = 1

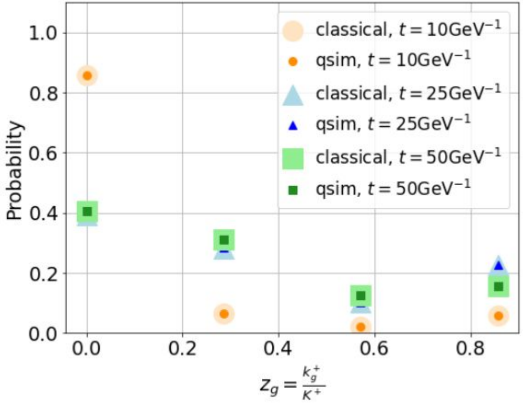


g = 10

Quantum & classical in agreement:
(1) Vqg oscillation decohered by kinetic terms (2) more qg component at larger coupling

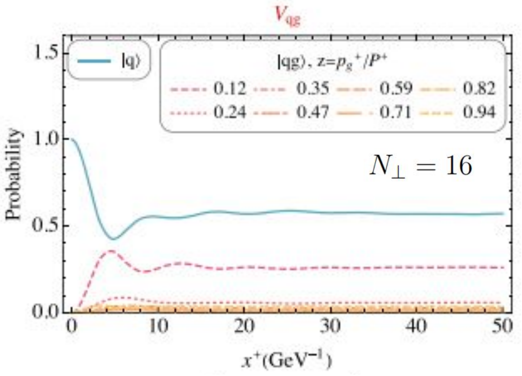
Results: jet evolution in vacuum





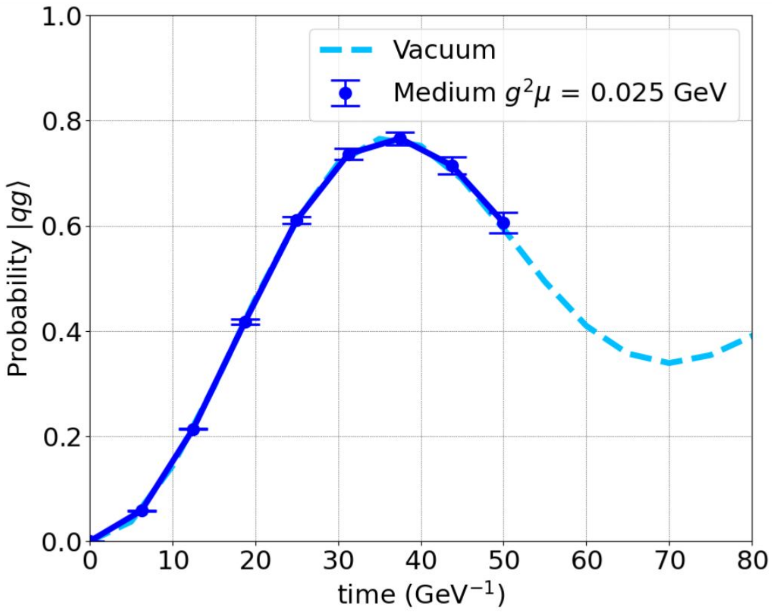
larger lattice expectation

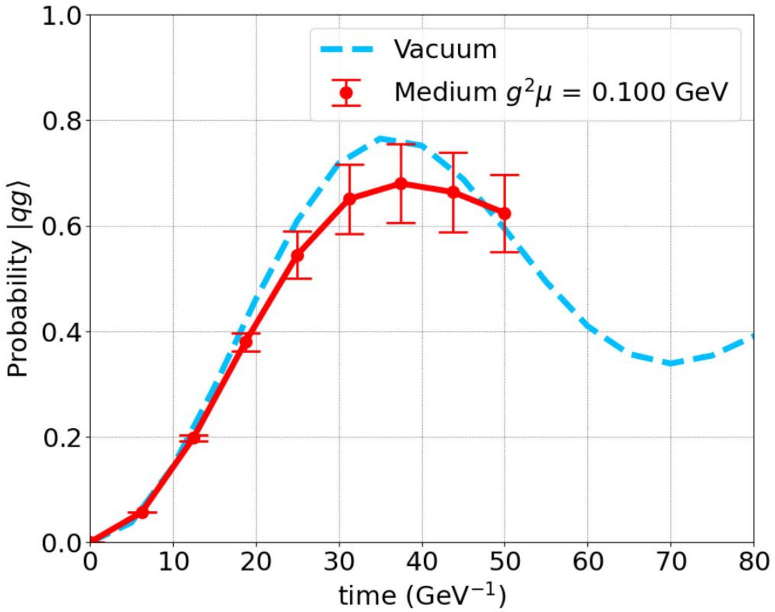
Li, Lappi, Zhao, PRD104.056014 (2021)



Results: jet evolution in medium

(Preliminary)



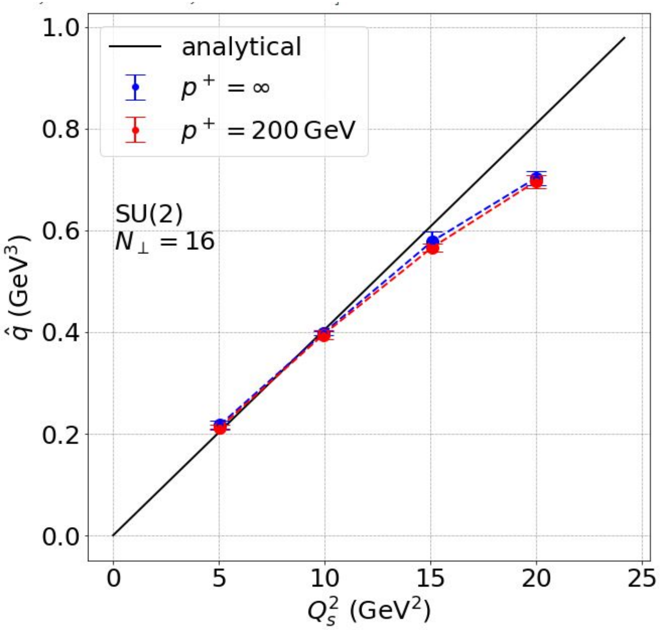


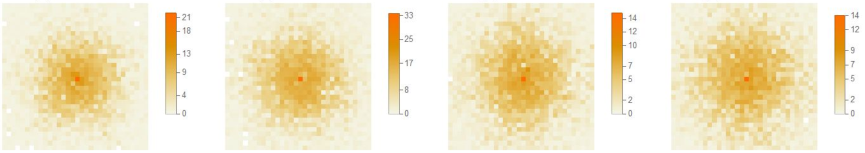
Medium-induced jet modification on a limited lattice: jet evolution through increasing medium strength, gluon splitting probability decreases

Results: momentum broadening

Fock |q> only

Barata, Du, Li, WQ, Salgado, PRD106,074013 (2022)





Analytical $$\hat{q} = \frac{g^4}{4\pi} C_F \tilde{\mu}^2 \left\{ \log\left(1 + \frac{\frac{\pi^2}{a_\perp^2}}{m_g^2}\right) - \frac{1}{1 + \frac{a_\perp^2 m_g^2}{\pi^2}} \right\}$$

(eikonal limit linear with Q_s^2)

Simulation $$\hat{q} = \langle \boldsymbol{p}_\perp^2 \rangle / L_\eta$$

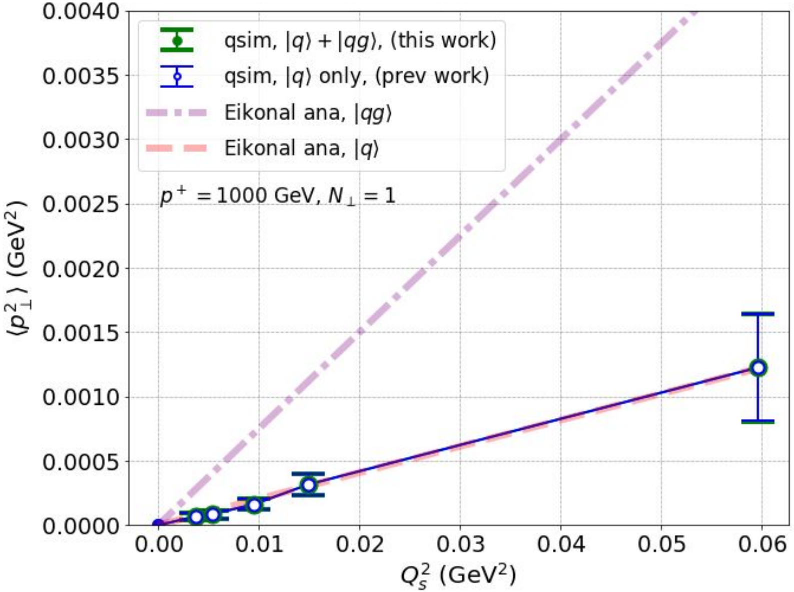
819200 shots, 11 qubits

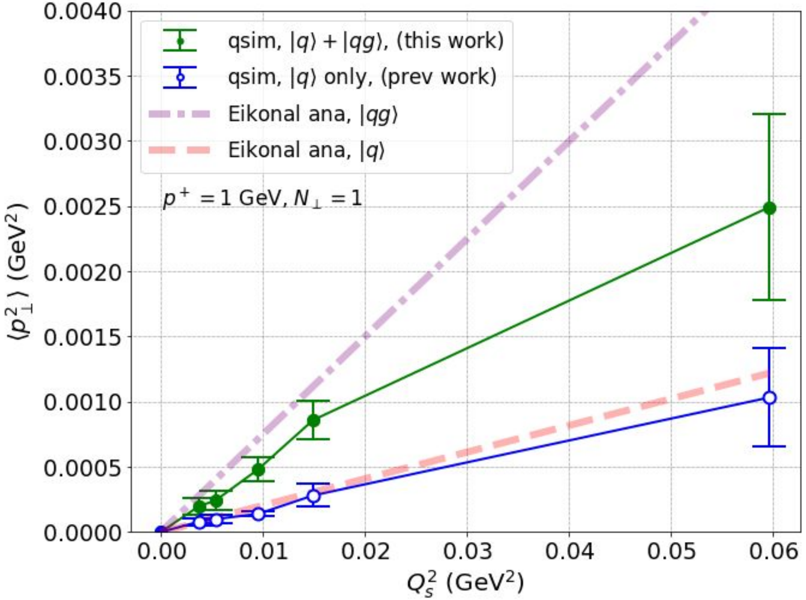
Results: momentum broadening

Fock |q> + |qg>

Barata, Du, Li, WQ, Salgado,
(work in progress)

$P_{|q\rangle}\langle p_q^2\rangle + P_{|qg\rangle}\langle p_{qg}^2\rangle$





Simulation with different longitudinal momentum agrees with analytical asymptotics

819200 shots, 9 qubits

Summary and outlook

- We extend previous study on medium-induced momentum broadening by incorporating higher Fock sector.
- Despite of having a small model space, we can study jet evolution using quantum simulators. Our results agree with classical simulations and prev simulation. More numerical analysis is underway.
- Quantum simulation can be effective in reducing the problem complexity faced in classical simulation; we expect to work on simulation with larger lattice and even higher Fock sectors in the next step.

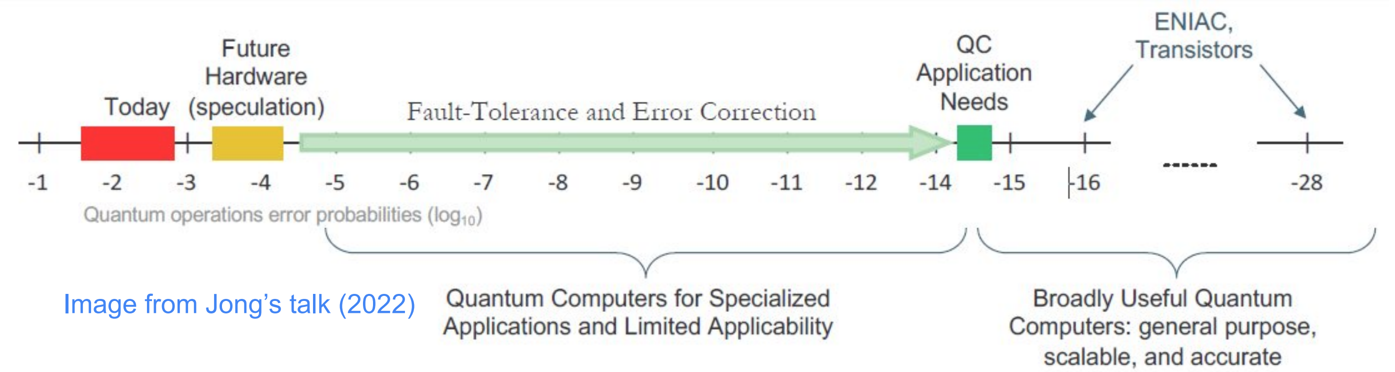


Image from Jong's talk (2022)