Classical vs. Quantum Corrections to Jet Broadening in a Weakly Coupled QGP

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Based on 2207.08842

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BDMPS-Z Splitting Probability

$$\frac{dP^{a}_{b\,c}}{dz} = \frac{\alpha P^{a}_{b\,c}(z)}{(z(1-z)P)^2} \mathrm{Re} \int_0^\infty dt_1 \int_{t_1}^\infty dt_2 \times \nabla_{\mathbf{B_1}} \cdot \nabla_{\mathbf{B_2}} \{G(\mathbf{B_2}, t_2; \mathbf{B_1}, t_1) - \text{vac.}\} \quad (1)$$

$G(\mathbf{B_2}, t_2; \mathbf{B_1}, t_1)$ is Green's function of Hamiltonian, $\mathcal{H}$ describing momentum diffusion in directions transverse to jet propagation
$\mathcal{H}$ depends on:

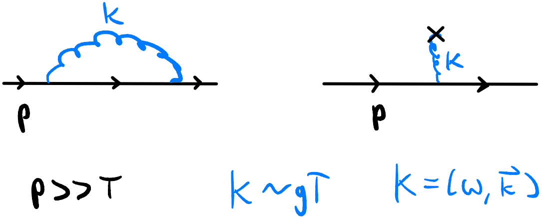
- asymptotic masses, m_∞
- transverse scattering rate, $\mathcal{C}(k_\perp)$

$m_\infty, \mathcal{C}(k_\perp)$ can be computed in **finite temperature perturbation theory**

Outline

- Both receive m_∞ and $\mathcal{C}(k_\perp)$ **classical** contributions i.e corrections coming from exchange of gluons between medium and parton that are $\lesssim gT$

$$\implies n_B(\omega) \equiv \frac{1}{\exp\left(\frac{\omega}{T}\right) - 1} \gg 1$$



Mapping onto EQCD

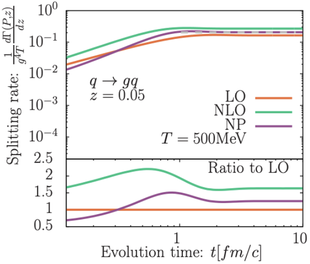
- Thanks to observation from [Caron-Huot, 2009], these classical corrections can be computed in **Electrostatic QCD** (EQCD)
- EQCD is a 3 dimensional theory of static modes

  $\Rightarrow$ Can be studied on the **lattice**!

  $\Rightarrow$ Paved way for **non-perturbative** (NP) determination of classical corrections to $\mathcal{C}(k_\perp)$!
- For m_∞ situation, see talk by Jacopo Ghiglieri, Tue 10h50

- Series of papers [Panero et al., 2014, Moore et al., 2021, Schlichting and Soudi, 2021], culminated with NP determination of in-medium splitting rate for medium of finite size

- Difference between rate from LO kernel and NP kernel can be up to 50%!

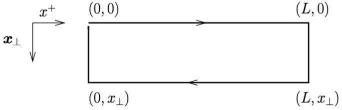


Outline

- Transverse momentum broadening coefficient, $\hat{q}(\mu)$ can be related to the transverse scattering rate, $\mathcal{C}(k_\perp)$

$$\hat{q}(\mu) = \int^{\mu} \frac{d^2 k_\perp}{(2\pi)^2} k_\perp^2 \mathcal{C}(k_\perp)$$

$$\lim_{L\to\infty} \langle W(x_\perp) \rangle = \exp(-\mathcal{C}(x_\perp) L)$$



[Ghiglieri and Teaney, 2015]

- $W(x_\perp)$ is a Wilson loop defined in the $(x^+, x_\perp)$ plane [Casalderrey-Solana and Teaney, 2007, D'Eramo et al., 2011, Benzke et al., 2013]

$\hat{q}$ Corrections – Status

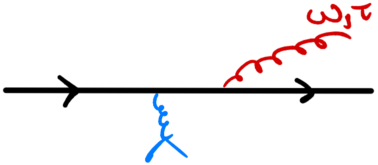
- $\mathcal{O}(g)$ classical contributions to $\hat{q}$ from soft scale $\sim gT$ calculated perturbatively in a **weakly coupled QGP** by [Caron-Huot, 2009]
- $\mathcal{O}(g^2)$ corrections found to have double logarithmic enhancement $\sim \ln^2(L_{\text{med}}/\tau_{\text{min}})$ by [Liou et al., 2013](**LMW**) and separately by [Blaizot et al., 2014](**BDIM**) for a medium with **static scattering centers**
- These are **radiative, quantum corrections**, coming from keeping track of the recoil during the medium-induced emission of a gluon
- Resummation of double logs performed recently [Caucal and Mehtar-Tani, 2022]

Physical Picture

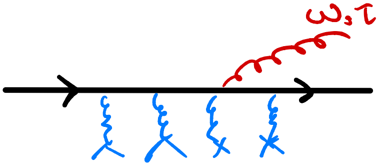
- **LMW** and **BDIM** argued that these quantum corrections come from the single-scattering regime

$$k_\perp^2 = \frac{\omega}{\tau}$$

ω = energy of radiated gluon
τ = formation time of radiated gluon



single scattering



multiple scattering

Motivation

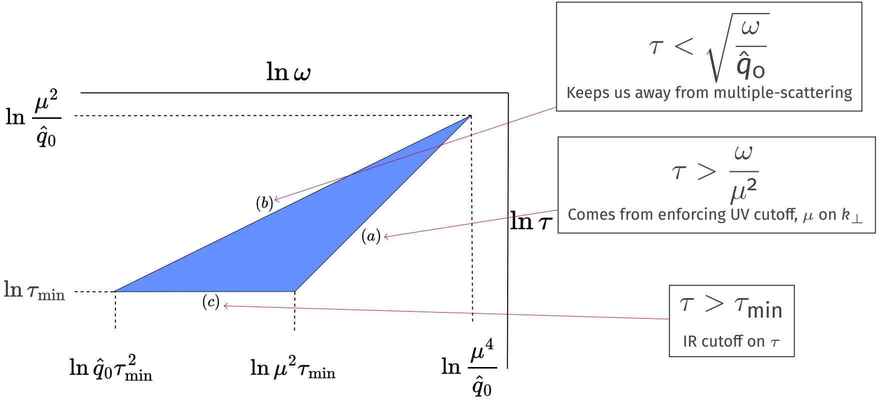
- **LMW** and **BDIM** calculations used the **Harmonic Oscillator Approximation (HOA)**, which is more well-suited to multiple scattering regime
- Both calculations also assume medium to be composed of **static scattering centers**

  $\rightarrow$ Not clear how phase spaces of classical $\mathcal{O}(g)$ and quantum $\mathcal{O}(g^2)$ corrections are connected or if there is some overlap

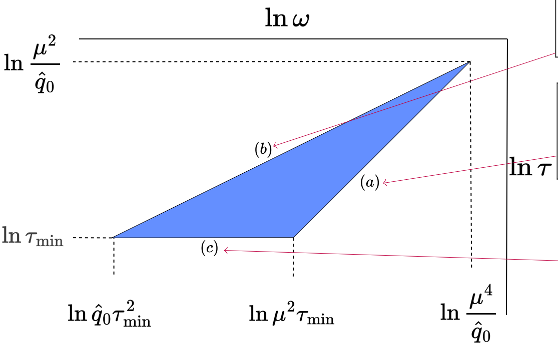
Which is larger: $K\mathcal{O}(g)$ or $\ln^2(\#)\mathcal{O}(g^2)$?

Hard to say... But can definitely make a start by revisiting computation of quantum corrections

Double Logs from the literature



$$\delta \hat{q}_{\rm LMW}(\mu) = \frac{\alpha_s C_R}{\pi} \hat{q}_0 \int_{\tau_{\rm min}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0 \tau^2}^{\mu^2 \tau} \frac{d\omega}{\omega} = \frac{\alpha_s C_R}{2\pi} \hat{q}_0 \ln^2 \frac{\mu^2}{\hat{q}_0 \tau_{\rm min}}$$



$$\tau < \sqrt{\frac{\omega}{\hat{q}_0}}$$

Keeps us away from multiple-scattering

$$\tau > \frac{\omega}{\mu^2}$$

Comes from enforcing UV cutoff, μ on $k_\perp$

$$\tau > \tau_{\min}$$

IR cutoff on τ

$$\delta \hat{q}_{\text{LMW}}(\mu) = \frac{\alpha_s C_R}{\pi} \hat{q}_0 \int_{\tau_{\min}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0 \tau^2}^{\mu^2 \tau} \frac{d\omega}{\omega} \overset{\mu^2 = \hat{q}_0 L_{\text{med}}}{=} \frac{\alpha_s C_R}{2\pi} \hat{q}_0 \ln^2 \frac{L_{\text{med}}}{\tau_{\min}}$$

Outline

How can we adapt BDIM/LMW result to weakly coupled QGP?

$$\delta\hat{q}_{\mathrm{LMW}}(\mu) = \frac{\alpha_s C_R}{\pi}\hat{q}_0 \int_{\tau_{\min}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0\tau^2}^{\mu^2\tau} \frac{d\omega}{\omega}$$

↓

$$\delta\hat{q}_{\mathrm{LMW}}(\mu) = \frac{\alpha_s C_R}{\pi}\hat{q}_0 \int_{\tau_{\min}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0\tau^2}^{\mu^2\tau} \frac{d\omega}{\omega}(1 + 2n_B(\omega))$$

$\Longrightarrow$ $n_B \equiv \frac{1}{e^{\frac{\omega}{T}} - 1}$ can be ignored iff $\omega_{\min} = \hat{q}_0\tau_{\min}^2 \gg T$

Is this consistent with single scattering?

$$\delta\hat{q}_{\mathrm{LMW}}(\mu) = \frac{\alpha_s C_R}{\pi}\hat{q}_0 \int_{\tau_{\min}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0\tau^2}^{\mu^2\tau} \frac{d\omega}{\omega}$$

$\hat{q}_0 \sim g^4 T^3$

Need to demand $g^4 T^3 \tau_{\min}^2 \gg T$

$\Rightarrow \tau_{\min}$ should be $\gg \dfrac{1}{g^2 T}$

But $\dfrac{1}{g^2 T}$ is the mean free time between multiple scatterings!

$\Rightarrow$ Would lead us away from single scattering regime!

$\Longrightarrow$ In order to stay away from multiple scattering regime, must account for thermal effects

$$\delta\hat{q}_{\mathrm{LMW}}(\mu) = \frac{\alpha_s C_R}{\pi}\hat{q}_0 \int_{\tau_{\min}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0\tau^2}^{\mu^2\tau} \frac{d\omega}{\omega}$$

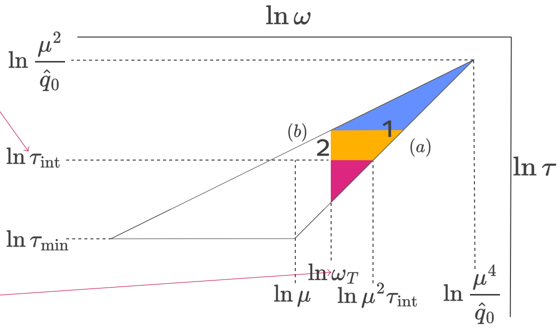
Introduce intermediate regulator
$\tau_{\mathrm{int}} \ll 1/g^2T$

$\omega_T \sim T$

$$\delta\hat{q}_{1+2}(\mu) = \frac{\alpha_s C_R}{\pi}\hat{q}_0 \int_{\tau_{\mathrm{int}}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0\tau^2}^{\mu^2\tau} \frac{d\omega}{\omega}(1 + 2n_B(\omega))$$
$$= \frac{\alpha_s C_R}{2\pi}\hat{q}_0\left\{ \ln^2 \frac{\mu^2}{\hat{q}_0\tau_{\mathrm{int}}} - \frac{1}{2}\ln^2 \frac{\omega_T}{\hat{q}_0\tau_{\mathrm{int}}^2}\right\}$$

Double Logs in a Weakly Coupled QGP

Intermediate regulator τ_{int}



$\omega_T \equiv 2\pi T e^{-\gamma_E}$

$$\delta \hat{q}_{1+2}(\mu) = \frac{\alpha_s C_R}{2\pi} \hat{q}_0 \left\{ \ln^2 \frac{\mu^2}{\hat{q}_0 \tau_{\mathrm{int}}} - \frac{1}{2} \ln^2 \frac{\omega_T}{\hat{q}_0 \tau_{\mathrm{int}}^2} \right\}$$

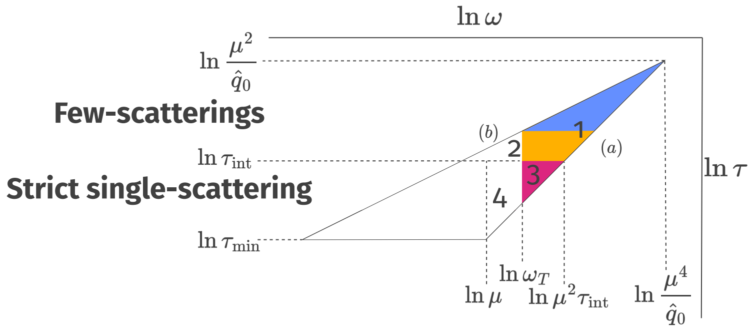
Strict Single-Scattering

- Compute $\mathcal{C}(k_\perp)$ using HTL resummation instead of Random Colour Approximation
- Investigate which logs are produced by *soft, collinear* modes through a *semi-collinear* [Ghiglieri et al., 2013, Ghiglieri et al., 2016] process associated with formation time $\tau_{\text{semi}} \sim 1/gT$

Only spacelike interactions with medium

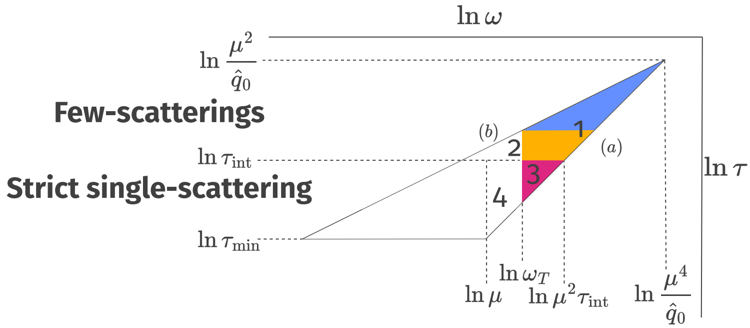
Now timelike interactions are allowed too

$\Rightarrow$ Going beyond instantaneous approximation!

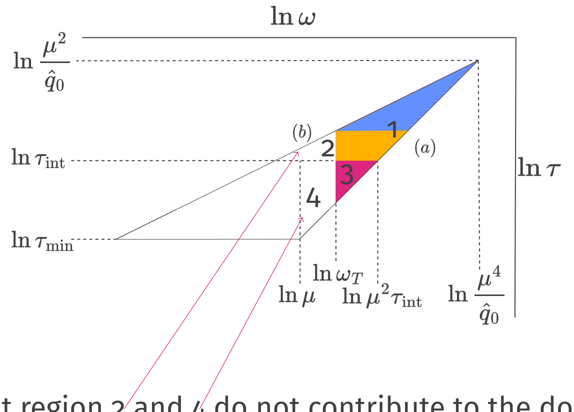


$$\delta\hat{q}_{3+4}(\mu) = \frac{\alpha_s C_R}{2\pi}\hat{q}_0 \ln^2 \frac{\mu^2\tau_{\rm int}}{\omega_T} + \text{subleading logs}$$

Total Double Log Contribution



$$\delta\hat{q}_{\mathrm{GW}}(\mu) = \delta\hat{q}_{1+2}(\mu) + \delta\hat{q}_{3+4}(\mu) = \frac{\alpha_s C_R}{4\pi}\hat{q}_0 \ln^2 \frac{\mu^4}{\hat{q}_0\omega_T}$$



Why is it that region 2 and 4 do not contribute to the double Logs?

Vacuum and Quantum Correction Cancellation

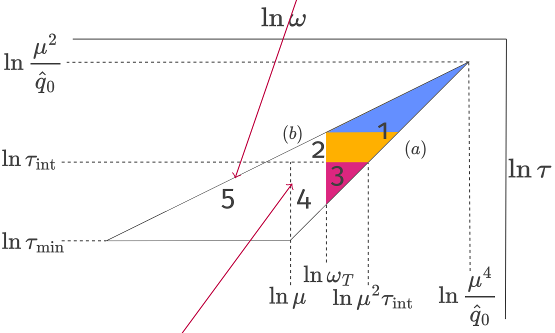
First, note that

$$\lim_{\frac{\omega}{T}->0}\Big(1+2n_B(\omega)\Big)=1+\frac{2T}{\omega}-1 \tag{2}$$

The absence of the IR scale in any logarithms can then be seen by looking at the following integral, with $\nu_{IR} \ll T \ll \nu_{UV}$

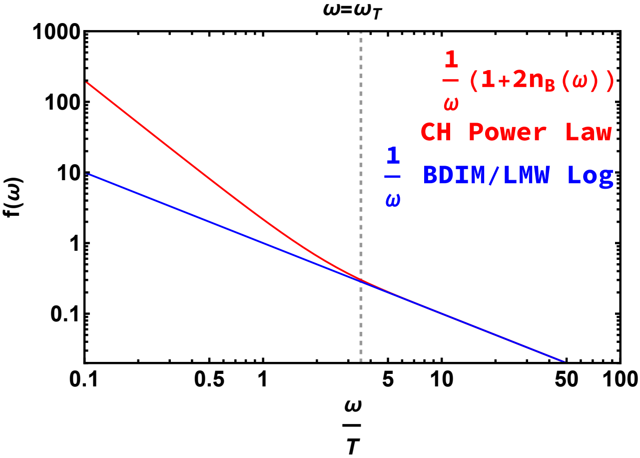
$$\begin{aligned}\int_{\nu_{IR}}^{\nu_{UV}}\frac{d\omega}{\omega}\Big(\underbrace{1}_{\text{vacuum}}+\underbrace{2n_B(\omega)}_{\text{thermal}}\Big) &= \underbrace{\ln\frac{\nu_{uv}}{\nu_{IR}}}_{\text{vacuum}}+\underbrace{\frac{2T}{\nu_{IR}}-\ln\frac{2\pi T}{\nu_{IR}e^{\gamma_E}}+\ldots}_{\text{thermal}} \\ &= \frac{2T}{\nu_{IR}}+\ln\frac{\nu_{UV}e^{\gamma_E}}{2\pi T}+\ldots\end{aligned} \tag{3}$$

Region of phase space from which classical $\mathcal{O}(g)$ corrections emerge [Caron-Huot, 2009]



How can we understand the transition to power law enhancement in regions 2 and 4?

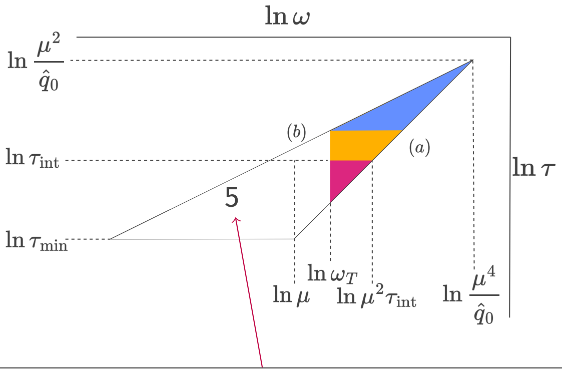
Can understand transition by looking at ω integrand, $f(\omega)$



$$\omega_T \equiv 2\pi T e^{-\gamma_E}$$

$$n_B(\omega) \equiv \frac{1}{e^{\frac{\omega}{T}} - 1}$$

Our results include power law corrections depending on our IR cutoff



They cancel against cutoff-dependent corrections computed from [Caron-Huot, 2009]$\Rightarrow$ Non-trivial check!

Outline

Summary of Results/Conclusions

- Double logarithmic corrections in strict single scattering regime computed within the setting of a weakly coupled QGP
- Showed that original BDIM/LMW phase space overlaps with phase space for classical corrections
- Can show how our result fits with respect to these emended BDIM/LMW corrections as well as the classical corrections
- Computed corrections allowing for timelike processes and showed that they are subleading

Going Beyond Harmonic Oscillator Approximation

HOA not well-suited to single-scattering

$\Rightarrow$So how can we go beyond it?

$$\delta\hat{q}_{\mathrm{GW}}(\mu) = \frac{\alpha_s C_R}{4\pi}\hat{q}_0 \ln^2 \frac{\mu^4}{\hat{q}_0\omega_T} \longrightarrow \frac{\alpha_s C_R}{4\pi}\hat{q}(\rho)\ln^2 \frac{\#}{\omega_T}$$

where $\hat{q}(\rho) \propto \ln \frac{\rho^2}{m_D^2}$

ρ separates us from neighbouring region
with simultaneously single-scattering and multiple scatterings

Appearance of $\hat{q}_0$ in double log is an artefact of lack of understanding of transition between single scattering and mutiple scattering regimes

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$$\delta\hat{q}_{\mathrm{GW}}(\mu) = \frac{\alpha_s C_R}{4\pi}\hat{q}_0 \ln^2 \frac{\mu^4}{\hat{q}_0\omega_T} \longrightarrow \frac{\alpha_s C_R}{4\pi}\hat{q}(\rho)\ln^2 \frac{\#}{\omega_T}$$

$$\text{where } \hat{q}(\rho) \propto \ln \frac{\rho^2}{m_D^2}$$

Need to solve transverse momentum-dependent LPM equation without **HOA** [Ghiglieri and Weitz, 2022] in order to shed light on how these issues could be addressed

Thanks for listening!

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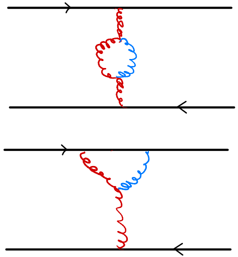
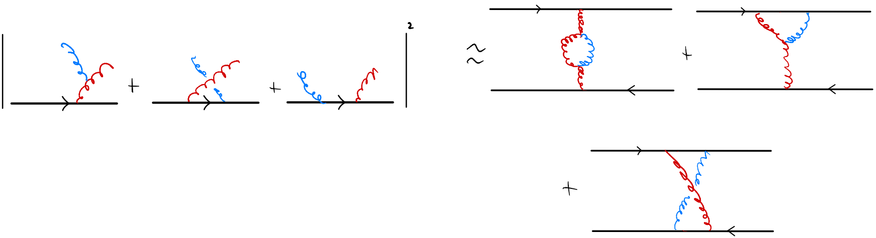
Splitting rates in QCD plasmas from a non-perturbative determination of the momentum broadening kernel $C(q_\perp)$.

Contributing diagrams

- Can think of sticking together amplitude and conjugate amplitude to get diagrams on the right
- **Black lines** represent hard parton in the amplitude and conjugate amplitude
- Red gluons are bremsstrahlung, represented by thermal propagators
- Blue gluons are those that are exchanged with the medium and are represented by HTL propagators

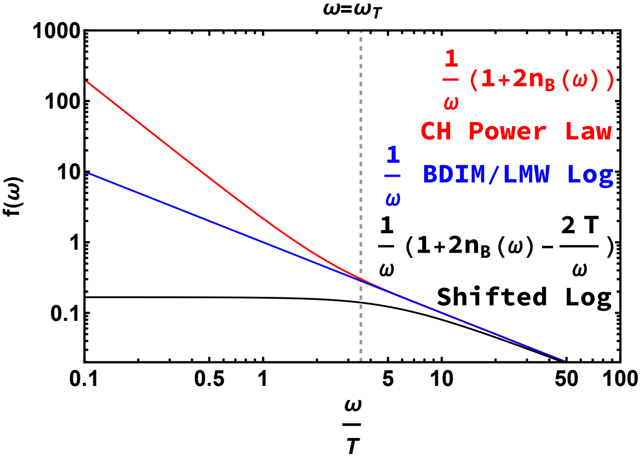


Where do these diagrams come from?



Relation to Soft Corrections

Can understand transition by looking at ω integrand, $f(\omega)$



$$\omega_T \equiv 2\pi T e^{-\gamma_E}$$

$$n_B(\omega) \equiv \frac{1}{e^{\frac{\omega}{T}} - 1}$$

Going Beyond Harmonic Oscillator Approximation

HOA not well-suited to single-scattering

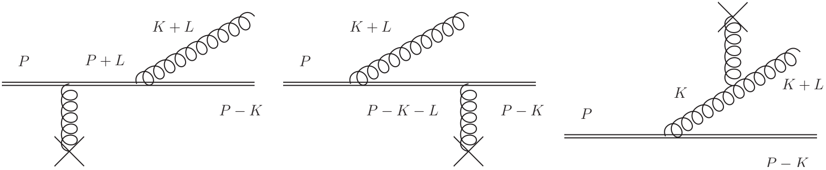
$\Rightarrow$So how can we go beyond it?

$$\delta\hat{q}_{\text{GW}}(\mu) = \frac{\alpha_s C_R}{4\pi}\hat{q}_0 \ln^2 \frac{\mu^4}{\hat{q}_0\omega_T} \longrightarrow \frac{\alpha_s C_R}{4\pi}\hat{q}(\rho) \ln^2 \frac{\#}{\omega_T}$$

where $\hat{q}(\rho) \propto \ln \frac{\rho^2}{m_D^2}$

Improved Opacity Expansion [Barata et al., 2021]
could be used to solve resummation equation in order to better understand transition from single scattering to multiple scattering

Double Logs from the literature



$N = 1$ term in opacity expansion emerges from dipole picture

$$\delta\mathcal{C}(k_\perp)_{\mathrm{LMW}} = 4\alpha_s C_R \int \frac{d\omega}{\omega} \int \frac{d^2 l_\perp}{(2\pi)^2} \mathcal{C}_0(l_\perp) \frac{l_\perp^2}{k_\perp^2 (\mathbf{k}_\perp + \mathbf{l}_\perp)^2} \qquad (4)$$

Double Logs from the literature

$$\delta\mathcal{C}(k_\perp,\rho)_{\rm LMW} = 4\alpha_s C_R \int \frac{d\omega}{\omega} \int^{\rho} \frac{d^2 l_\perp}{(2\pi)^2} \mathcal{C}_0(l_\perp) \frac{l_\perp^2}{k_\perp^2 (\mathbf{k}_\perp + \mathbf{l}_\perp)^2}$$

$|\mathbf{k}_\perp + \mathbf{l}_\perp| \gg l_\perp \Rightarrow$ Single Scattering

$$\delta\mathcal{C}(k_\perp,\rho)_{\rm LMW} = 4\alpha_s C_R \int \frac{d\omega}{\omega} \int^{\rho} \frac{d^2 l_\perp}{(2\pi)^2} \mathcal{C}_0(l_\perp) \frac{l_\perp^2}{k_\perp^4}$$

$\hat{q}_0(\rho) \to \hat{q}_0 \Rightarrow$ HOA

$$\delta\mathcal{C}(k_\perp)_{\rm LMW} = 4\alpha_s C_R \hat{q}_0 \frac{1}{k_\perp^4} \int \frac{d\omega}{\omega}$$

Reminder: $\hat{q}(\mu) = \int^{\mu} \frac{d^2 k_\perp}{(2\pi)^2} k_\perp^2 \mathcal{C}(k_\perp)$

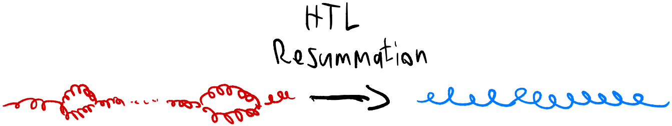
Thermal Scales in a Weakly Coupled QGP

- T, hard scale associated with energy of individual particles
  $\Rightarrow$ hard-hard interactions can be described perturbatively
- gT, soft scale associated with energy of collective excitations
  $\Rightarrow$ soft-soft interactions can also be described perturbatively
- g^2T, ultrasoft scale is associated with nonperturbative physics
  $\Rightarrow$ loops can be added at no extra cost (Linde problem)
  $\Rightarrow$ cannot use perturbation theory

HTL Effective Theory

- For hard-soft interactions, we are not so lucky either... Turns out that one can add loops for free
  $\implies$ perturbative expansion breaks down

- Hard Thermal Loop (HTL) effective theory comes to the rescue, allowing us to resum these loops



EQCD

- Rewrite frequencing integral $\int d\omega/2\pi$ as sum over Matsubara modes and integrate out all but zero Matsubara mode
  $\Rightarrow$ Dimensional Reduction

- EQCD Lagrangian

$$\mathcal{L}_{\mathrm{EQCD}} = \frac{1}{2g_{3\mathrm{d}}^2}\,\mathrm{Tr}\,F_{ij}F_{ij} + \mathrm{Tr}\,D_i\Phi D_i\Phi + m_D^2\,\mathrm{Tr}\,\Phi^2 + \lambda\left(\mathrm{Tr}\,\Phi^2\right)^2$$