Hard parton dispersion in the quark-gluon plasma, non-perturbatively

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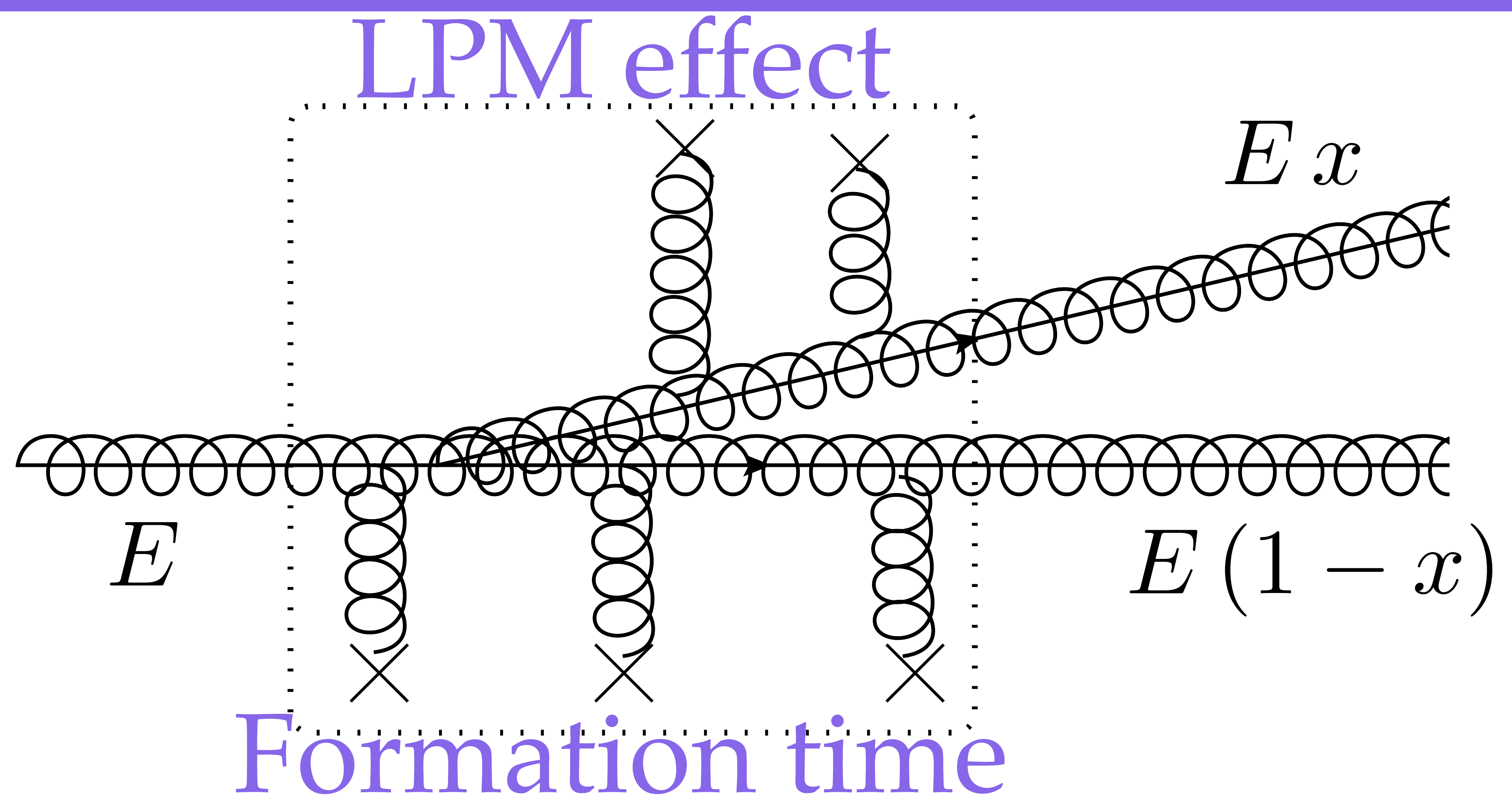
In this talk

- Medium-induced radiation and the asymptotic mass m_∞
- The asymptotic mass, classical modes and convergence
- Interplay of lattice EQCD and pQCD for m_∞
- Based on
  - Moore Schlusser **PRD102** (2020)
  - JG Moore Schicho Schlusser **JHEP02** (2021)
  - JG Schicho Schlusser Weitz *in progress*

Medium-induced radiation



- Key ingredient
  - in the description of jet modification
  - in thermalisation&transport: effective number-violating 1↔2 process, efficient chemical equilibration and energy transport, *bottom-up thermalisation* Baier Mueller Schiff Son (2001)

Medium-induced radiation

- Probability I: vacuum DGLAP x emission vertices x transverse diffusion

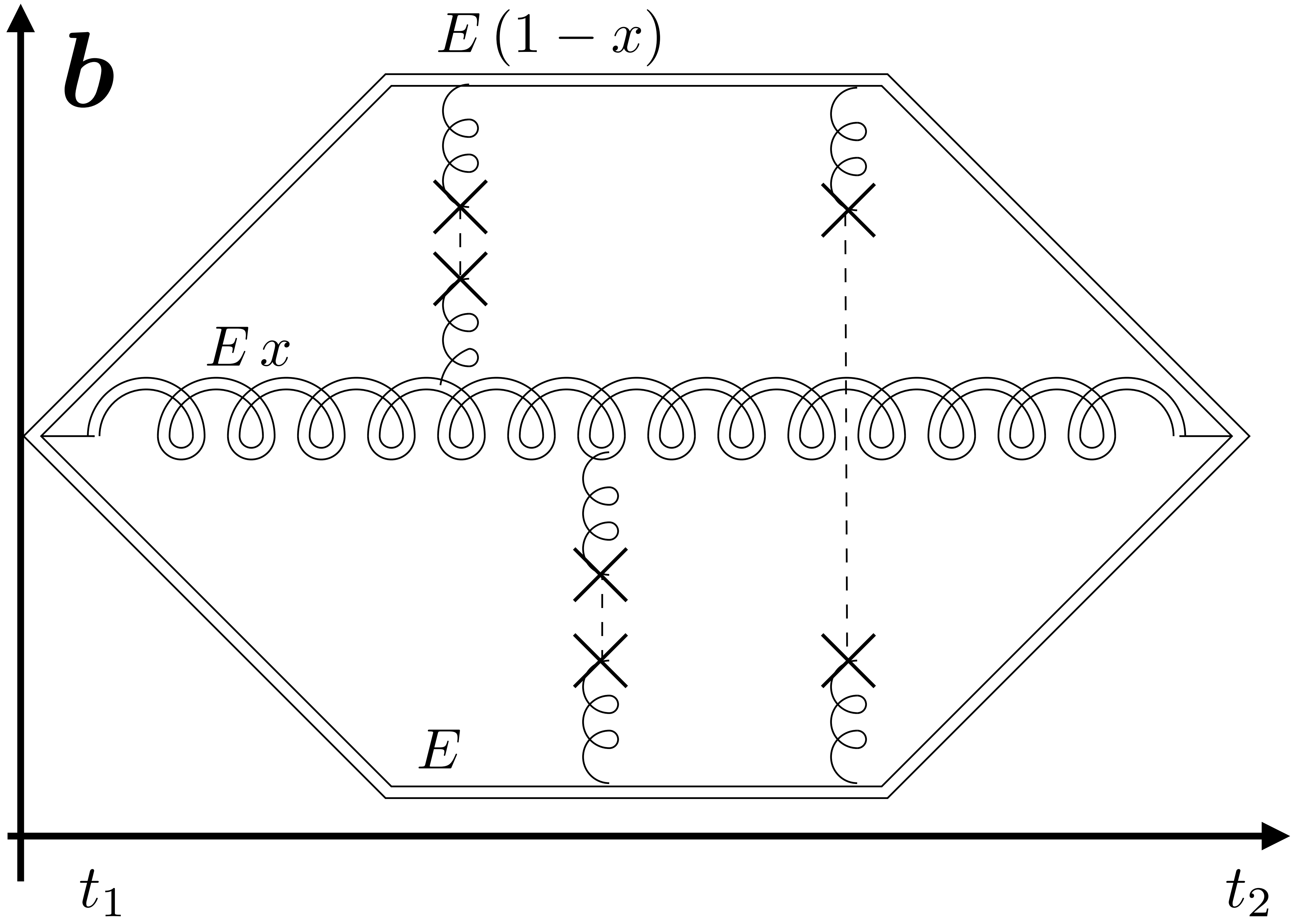
$$\frac{dI}{dx} = \frac{\alpha_s P_{1\to 2}(x)}{[x(1-x)E]^2}\mathrm{Re}\int_{t_1<t_2} dt_1 dt_2 \boldsymbol{\nabla}_{\boldsymbol{b}_2}\cdot\boldsymbol{\nabla}_{\boldsymbol{b}_1}\left[\langle \boldsymbol{b}_2, t_2|\boldsymbol{b}_1, t_1\rangle\big|_{\boldsymbol{b}_2=\boldsymbol{b}_1=0} - \text{vac.}\right]$$

- Transverse diffusion under this Hamiltonian

$$\mathcal{H} = -\frac{\nabla^2_{\boldsymbol{b}}}{2x(1-x)E} + \sum_i \frac{m^2}{2E_i} - i\mathcal{C}(\boldsymbol{b}, x\boldsymbol{b}, (1-x)\boldsymbol{b})$$

Real part: phase accumulation (with in-medium masses)

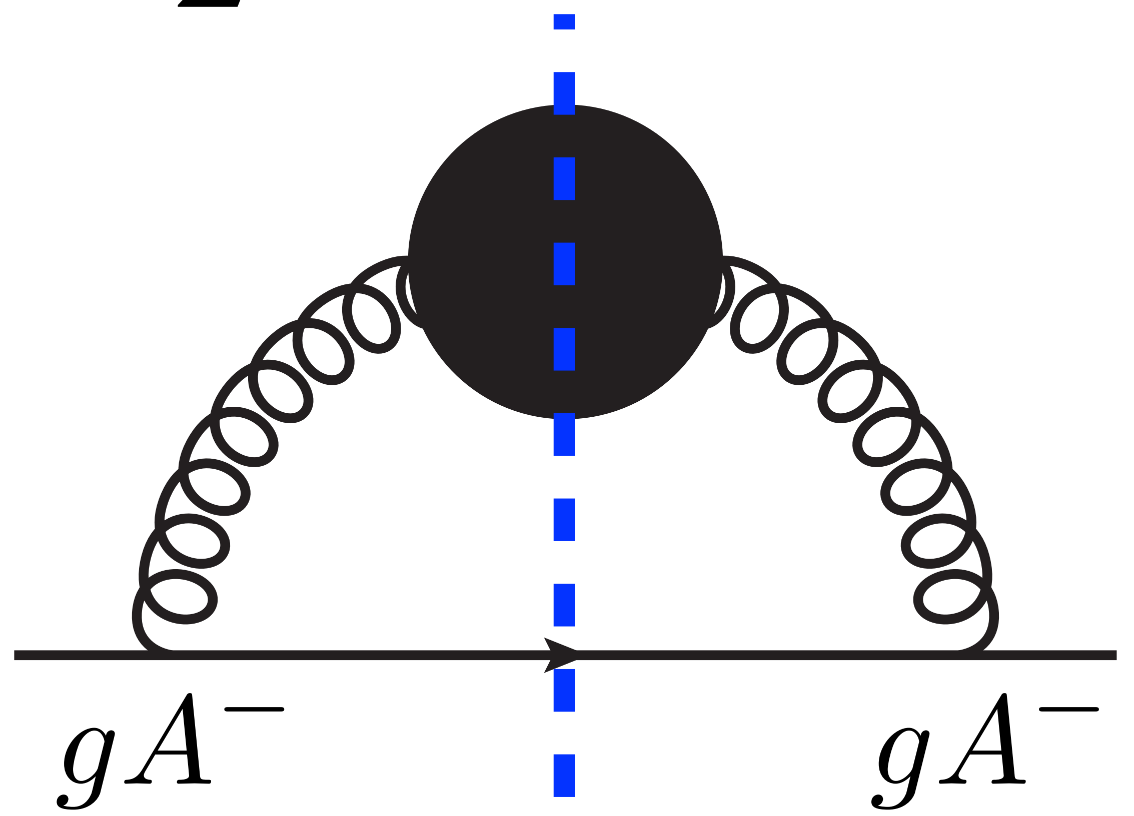
Imaginary part: Wilson lines encoding *scattering kernel* with the medium



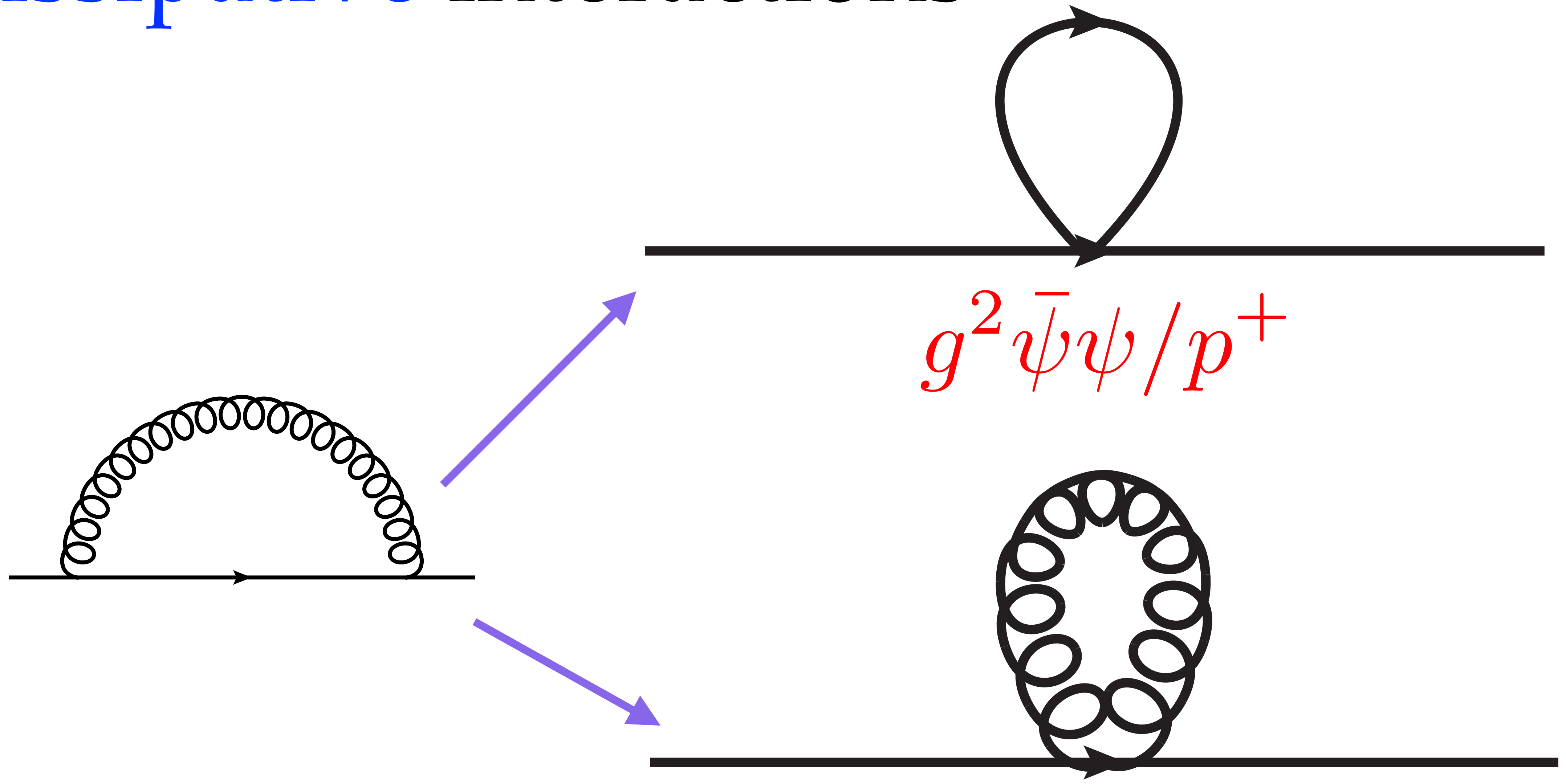
Baier Dokshitzer Mueller Peigne Schiff, Zakharov (1995-97) AMY (2001)

Hard partons through the medium

- Imagine a hard quark propagating through a medium with $p^+ \equiv \frac{p^0 + p^z}{2} \gg T$. Dispersive and dissipative interactions



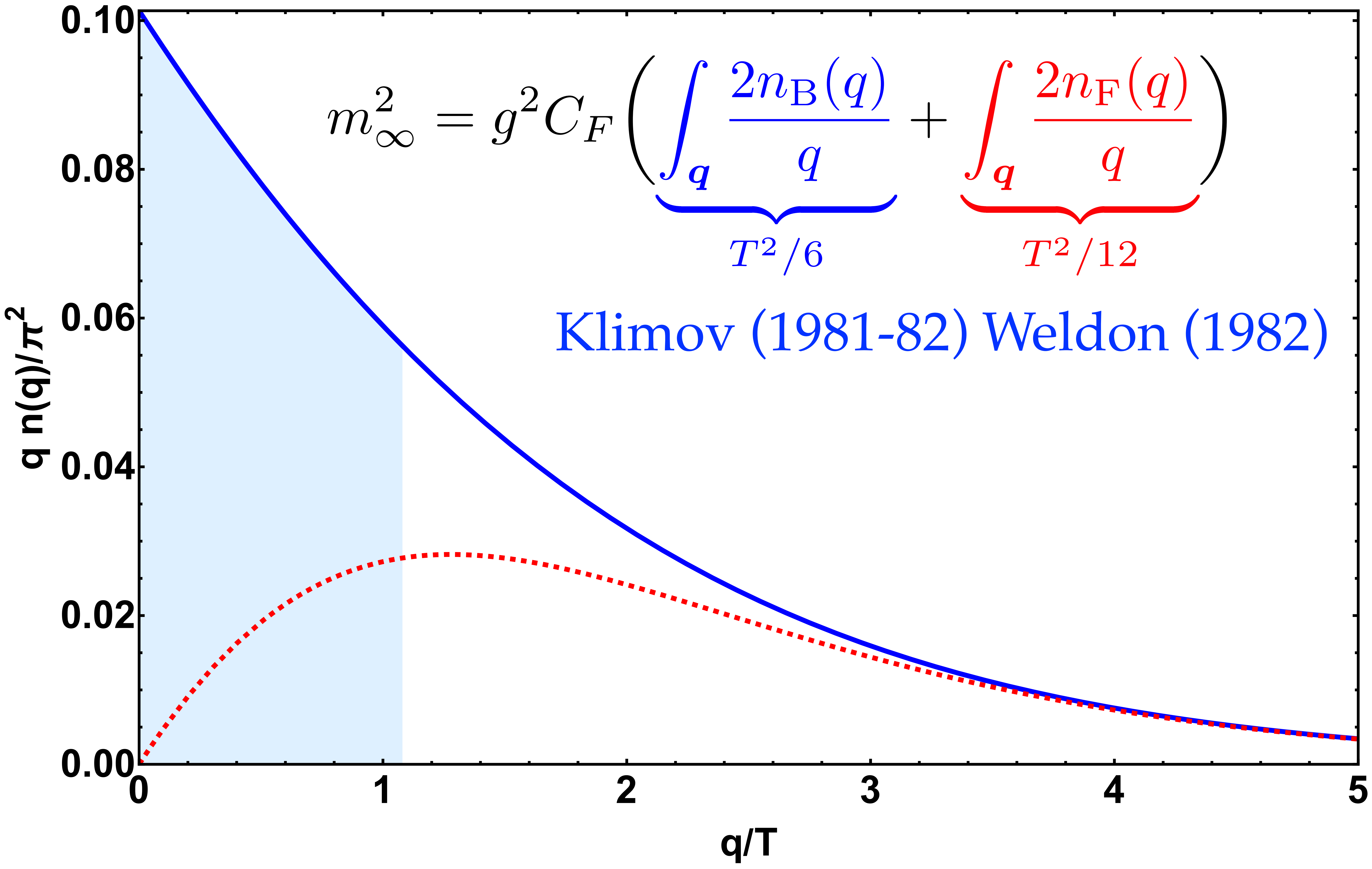
$\mathcal{C}(k_\perp) \sim g^2 \int_Q G^{--}(Q)\delta(q^-)\delta^{(2)}(\boldsymbol{q}_\perp - \boldsymbol{k}_\perp)$

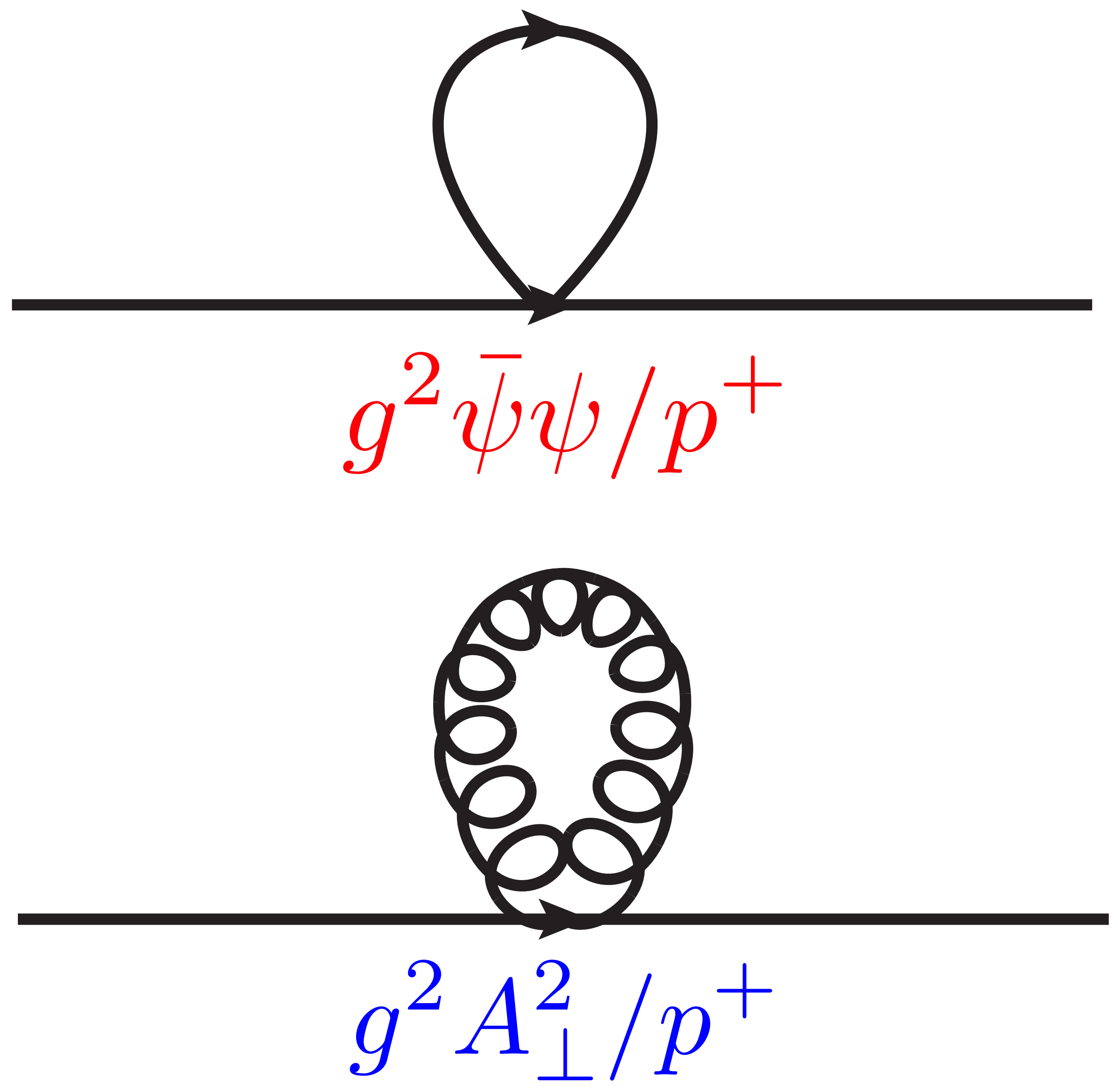


- The mass shift is then $m_\infty^2 = g^2T^2/3$ for a hard quark close to the mass shell

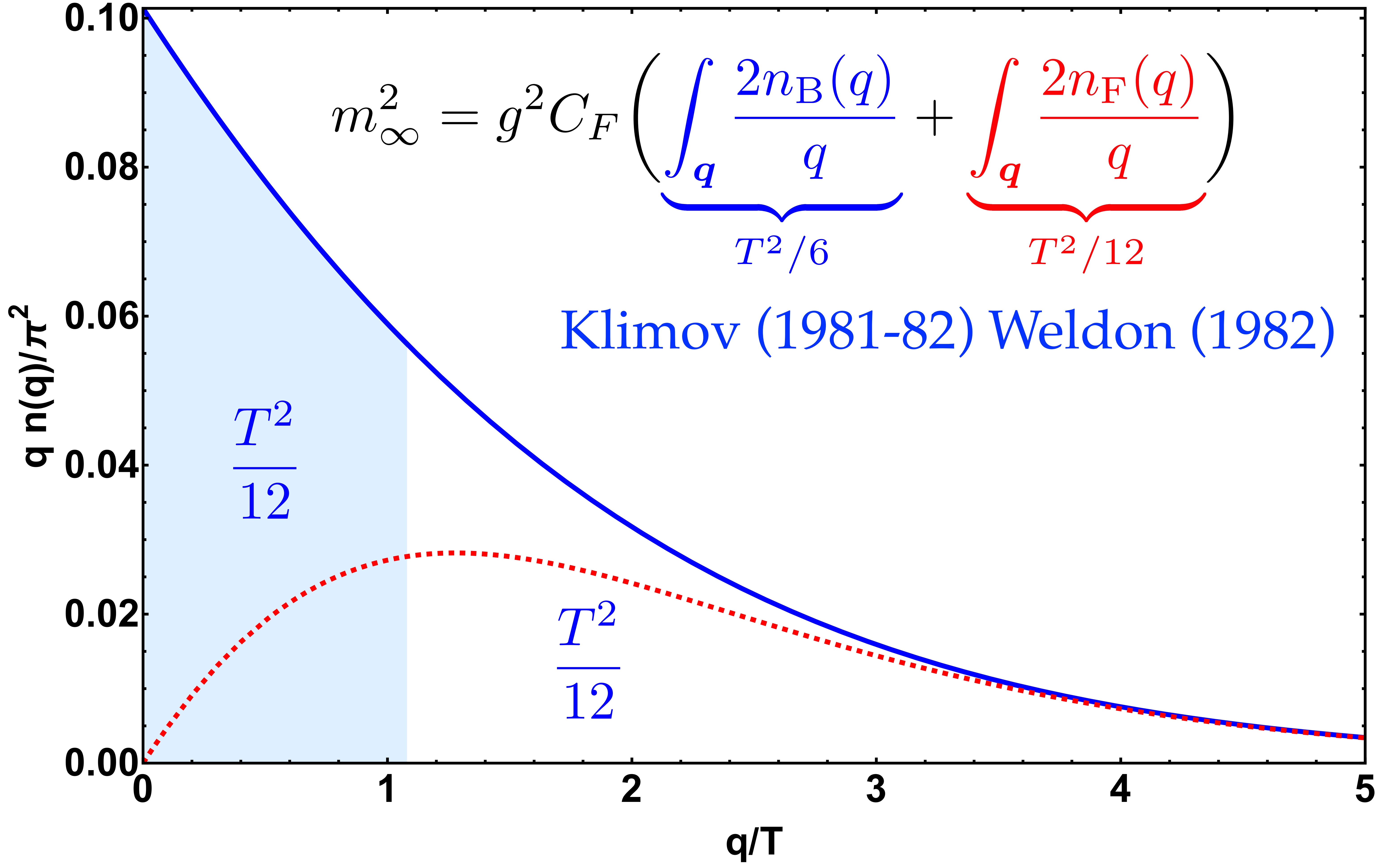
Klimov (1981-82) Weldon (1982)

The asymptotic mass





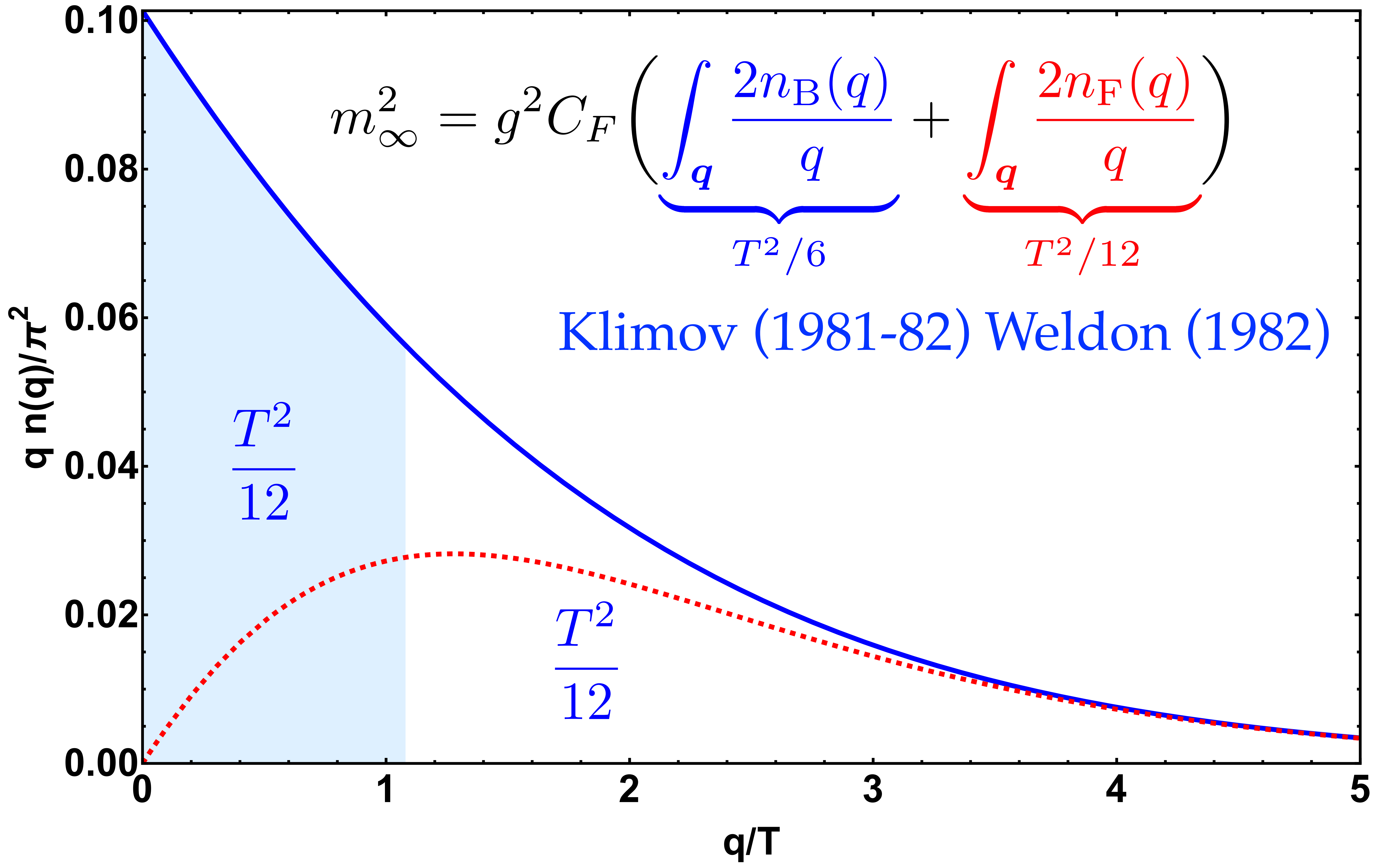
Classical gluons and the asymptotic mass



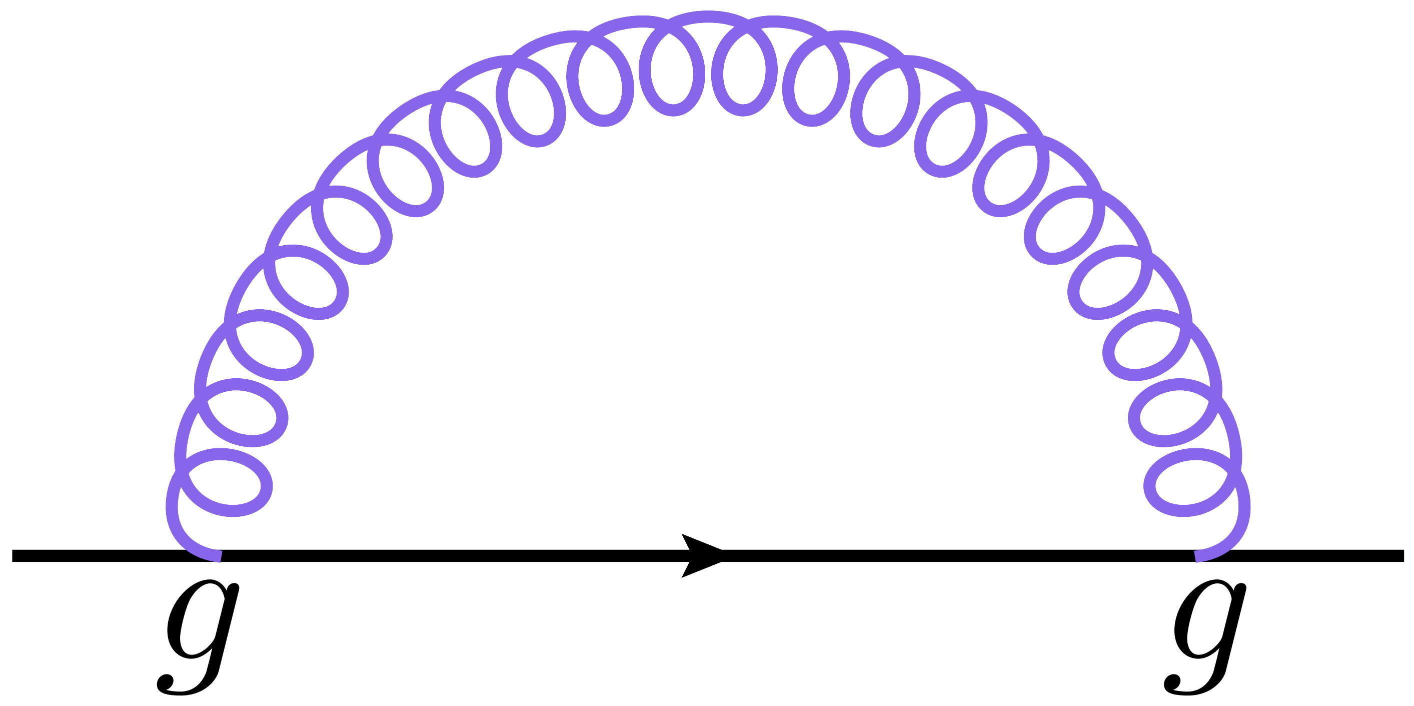
$$n_{\rm B}(q \ll T) \approx \frac{T}{q}$$

- Half of the bosonic integral comes from the $q \lesssim T$ region

Classical gluons and the asymptotic mass

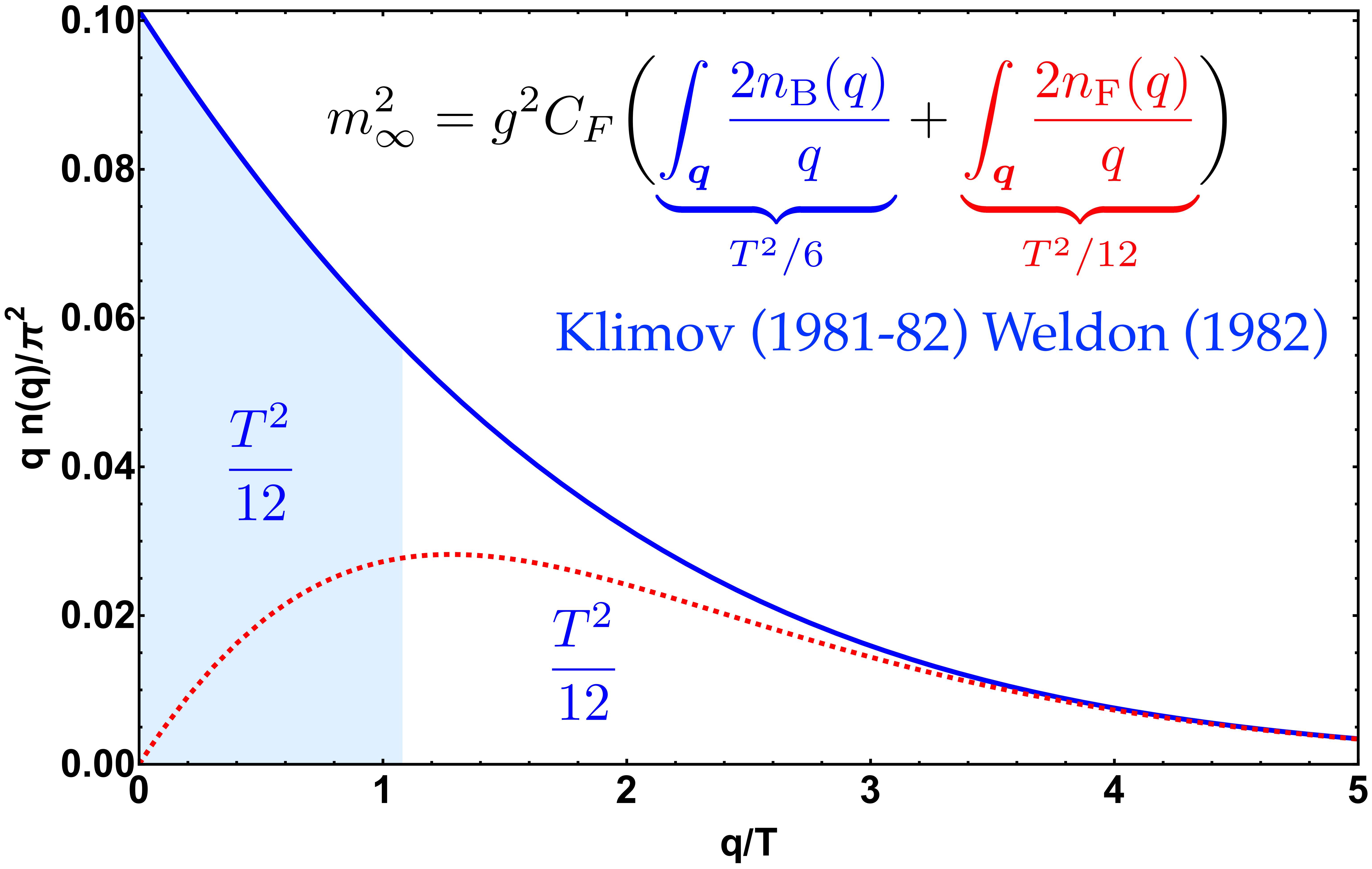


$$n_{\mathrm{B}}(q \ll T) \approx \frac{T}{q}$$

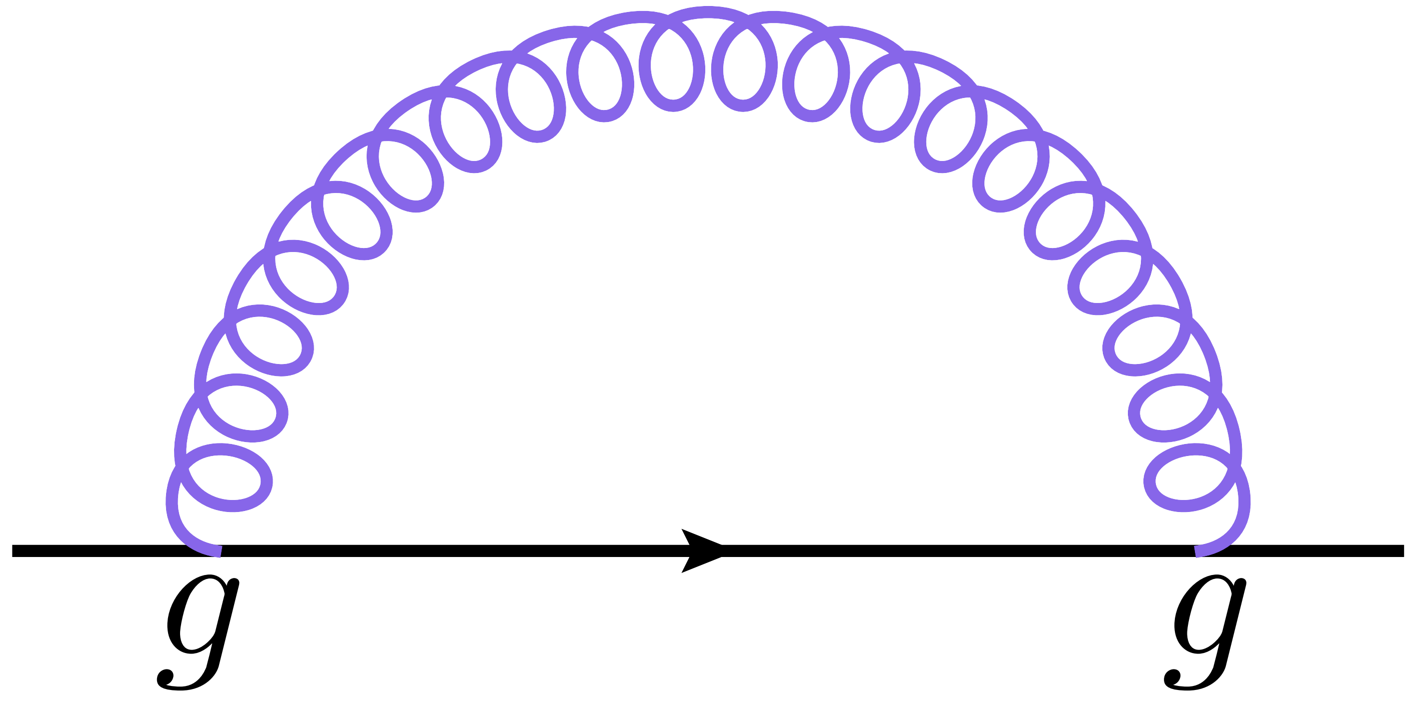


- We can then expect large contributions from soft classical gluons

Classical gluons and the asymptotic mass



$$n_B(q \ll T) \approx \frac{T}{q}$$

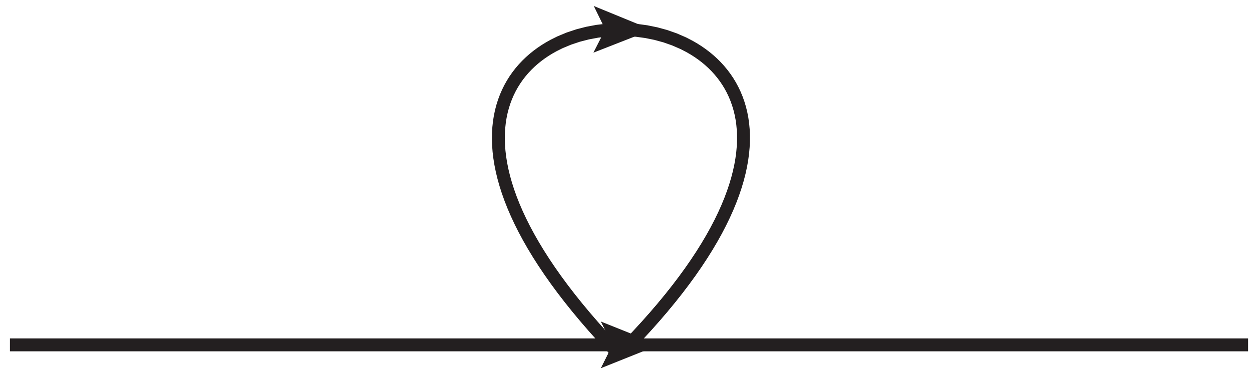


- For $q \lesssim gT$ this contribution becomes non-perturbative, $g^2 n_B(q) \sim 1$

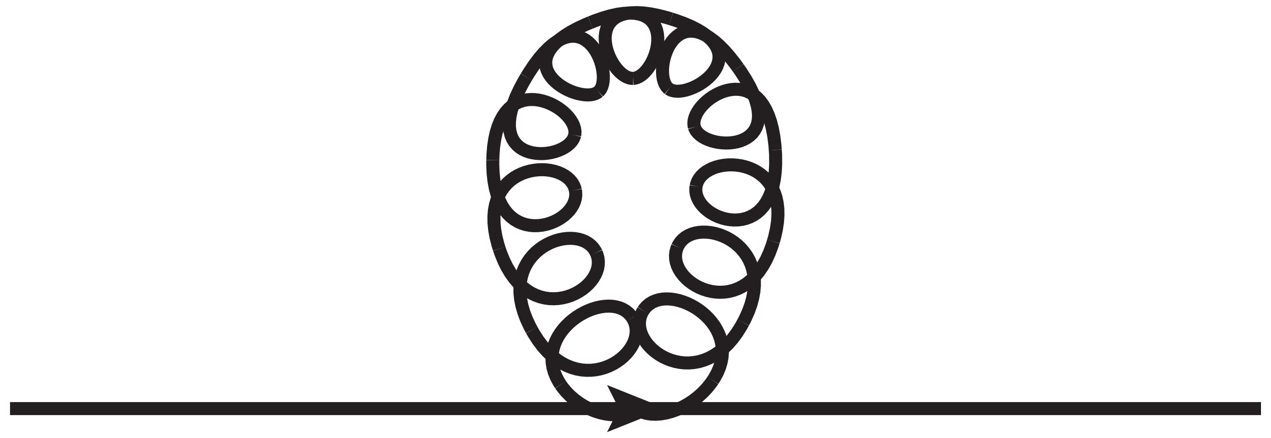
The asymptotic mass, non-perturbatively

$$m_\infty^2 = g^2 C_F \Bigg(\underbrace{\int_{\boldsymbol{q}} \frac{2n_{\rm B}(q)}{q}}_{T^2/6} + \underbrace{\int_{\boldsymbol{q}} \frac{2n_{\rm F}(q)}{q}}_{T^2/12} \Bigg)$$

$$= g^2 C_F \Big(Z_g + Z_f \Big) + \mathcal{O}(1/p^+)$$



$g^2\bar{\psi}\psi/p^+$



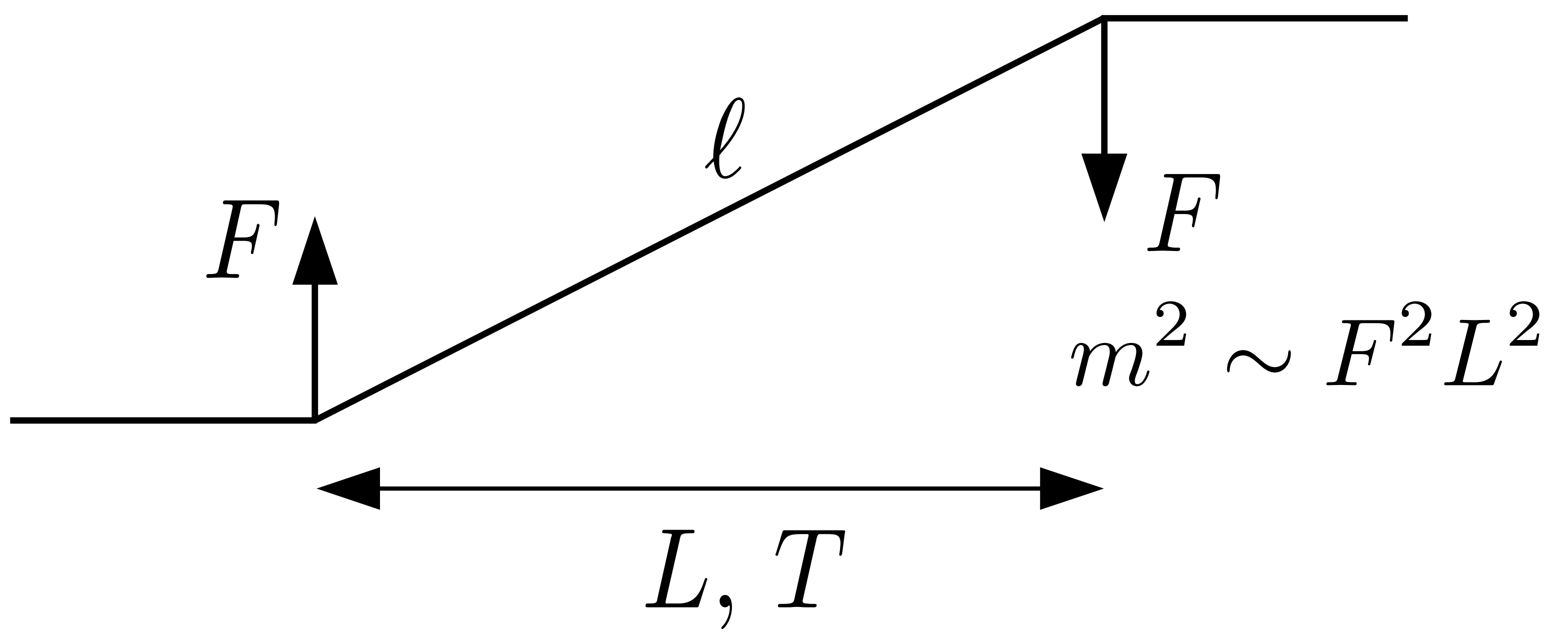
$g^2 A_\perp^2/p^+$

- From Feynman diagrams to EFT operators, concentrate on $Z_{\rm g}$

$$Z_{\rm f} \equiv \frac{1}{2d_R} \left\langle \bar{\psi} \frac{\not{v}}{v\cdot D} \psi \right\rangle \quad \text{with } v^\mu = (1,0,0,1)$$

$$Z_{\rm g} \equiv \frac{1}{d_A} \left\langle v_\alpha F^{\alpha\mu} \frac{1}{(v\cdot D)^2} v_\nu {F^\nu}_\mu \right\rangle$$

Caron-Huot (2008)



Moore Schlusser (2020)

The asymptotic mass, non-perturbatively

- From Feynman diagrams to EFT operators, concentrate on Z_g

$$Z_\mathrm{g} \equiv \frac{1}{d_A}\left\langle v_\alpha F^{\alpha\mu}\frac{1}{(v\cdot D)^2}v_\nu F^{\nu}{}_{\mu}\right\rangle$$
$$= \frac{2}{d_A}\int_0^\infty \mathrm{d}L L \,\mathrm{Tr}\left\langle U(-\infty;L)v_\alpha F^{\alpha\mu}(L)\,U(L;0)\,v_\nu F^{\nu}{}_{\mu}(0)U(0;-\infty)\right\rangle$$

- Breakthrough: soft classical modes at space-like separations become **Euclidean** and **time-independent**. Light-like limit possible, see JG Weitz (2022) for caveats in the case of $\hat{q}$. Talk by Weitz later

- Horrible HTL perturbative calculation or extremely challenging 4D lattice on the light-cone become 3D Electrostatic QCD (EQCD). NLO $\delta Z_\mathrm{g} = -\frac{Tm_D}{2\pi}$ Caron-Huot (2008)

The asymptotic mass, non-perturbatively

- From Feynman diagrams to EFT operators, concentrate on Z_g

$$Z_\mathrm{g} \equiv \frac{1}{d_A}\left\langle v_\alpha F^{\alpha\mu}\frac{1}{(v\cdot D)^2}v_\nu F^\nu{}_\mu\right\rangle$$
$$= \frac{2}{d_A}\int_0^\infty \mathrm{d}L\, L\, \mathrm{Tr}\left\langle U(-\infty;L) v_\alpha F^{\alpha\mu}(L)\, U(L;0)\, v_\nu F^\nu{}_\mu(0) U(0;-\infty)\right\rangle$$

- **Our strategy**: lattice EQCD for $L \gtrsim 1/m_D$, pQCD for $L \lesssim 1/m_D \sim 1/gT$ What does it mean in practice?

- Recently: continuum-extrapolated EQCD lattice data for the scattering kernel and merging with pQCD Moore Schlusser **PRD101** (2020) Moore Schlichting Schlusser Soudi **JHEP2110** (2021) Schlichting Soudi **PRD105** (2022)

EQCD

$$Z_{\rm g} = \frac{2}{d_A}\int_0^\infty {\rm d}L L\,{\rm Tr}\left\langle U(-\infty;L)v_\alpha F^{\alpha\mu}(L)\,U(L;0)\,v_\nu F^{\nu}\ \mu(0)U(0;-\infty)\right\rangle$$

- EQCD is the *dimensionally-reduced* (3D) EFT for the classical modes, which correspond to the Euclidean zero modes. 3D SU(3) + adjoint Higgs ($A_0 \to \Phi$)

$$S_{\rm EQCD} = \int_{\vec{x}}\left\{\frac{1}{2}{\rm Tr}\,F_{ij}F_{ij} + {\rm Tr}\,[D_i,\Phi][D_i,\Phi] + m_D^2{\rm Tr}\,\Phi^2 + \lambda_E({\rm Tr}\,\Phi^2)^2\right\}$$

Kajantie Laine Rummukainen Shaposhnikov (1995-97) Braaten Nieto (1994-95)

- By putting EQCD on the lattice we can get the classical contribution non-perturbatively at all orders. But how?

EQCD

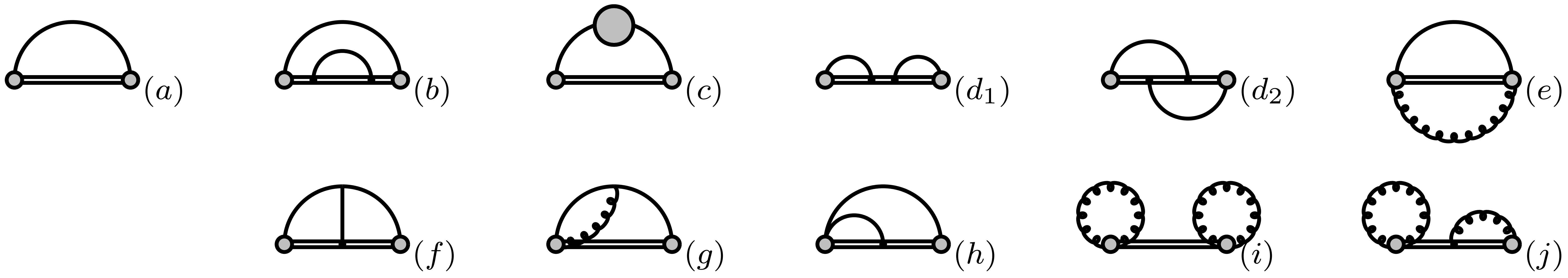
$$Z_{\rm g} = \frac{2}{d_A}\int_0^\infty {\rm d}L L\,{\rm Tr}\left\langle U(-\infty;L)v_\alpha F^{\alpha\mu}(L)\,U(L;0)\,v_\nu F^{\nu}{}_\mu(0)U(0;-\infty)\right\rangle$$

- In practice, we get continuum-extrapolated results for $\mathrm{Tr}\left\langle U(-\infty;L)F(L)\,U(L;0)\,F(0)U(0;-\infty)\right\rangle_{\rm EQCD}$ at a few discrete values of L.

  Moore Schlusser **PRD102** (2020) JG Moore Schicho Schlusser **JHEP02** (2021)

- We need to match to the 4D continuum, since EQCD has the wrong UV

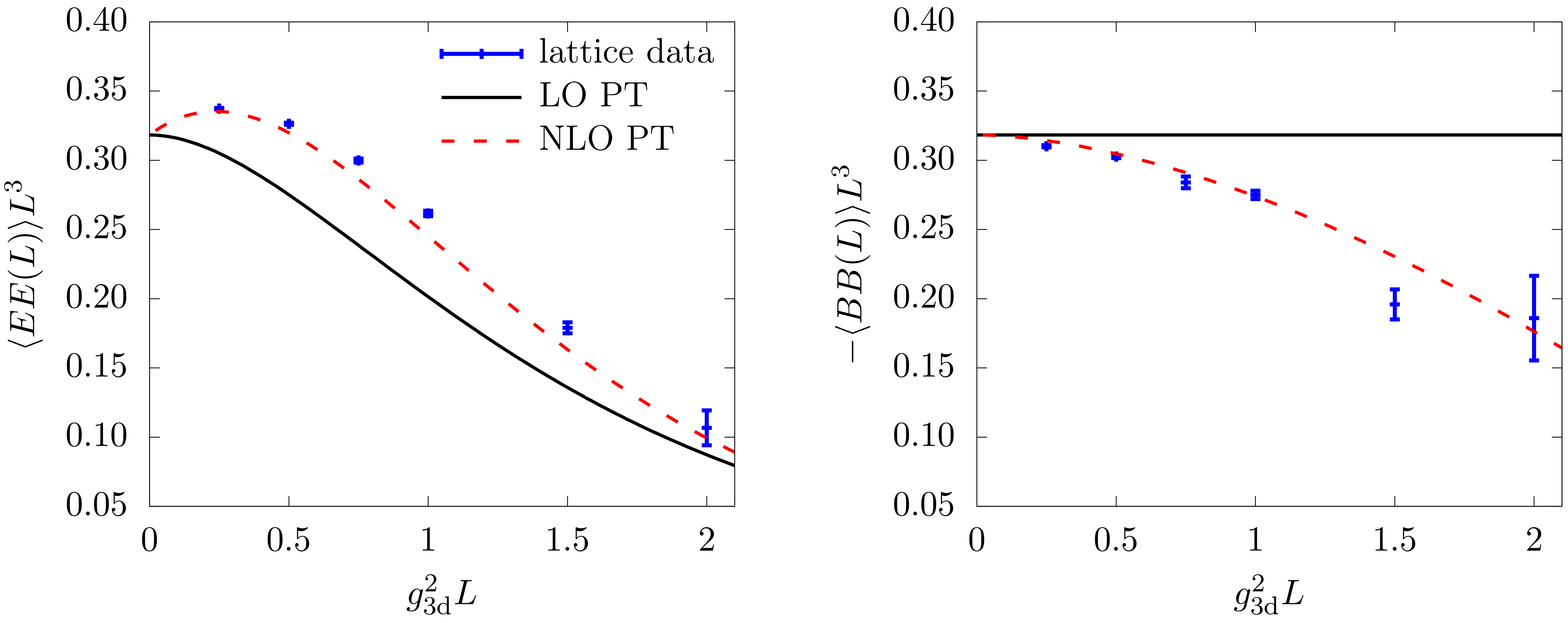
- Start by computing the EQCD correlator to NLO



EQCD results

- Good agreement in the UV, excellent at high $T = 100$ GeV

$$Z_{\rm g}^{\rm EQCD} = \frac{T}{2}\int_0^\infty dL\, L\left(\langle EE\rangle - \langle BB\rangle - \langle EB\rangle\right)$$



JG Moore Schicho Schlusser (2021)

EQCD results

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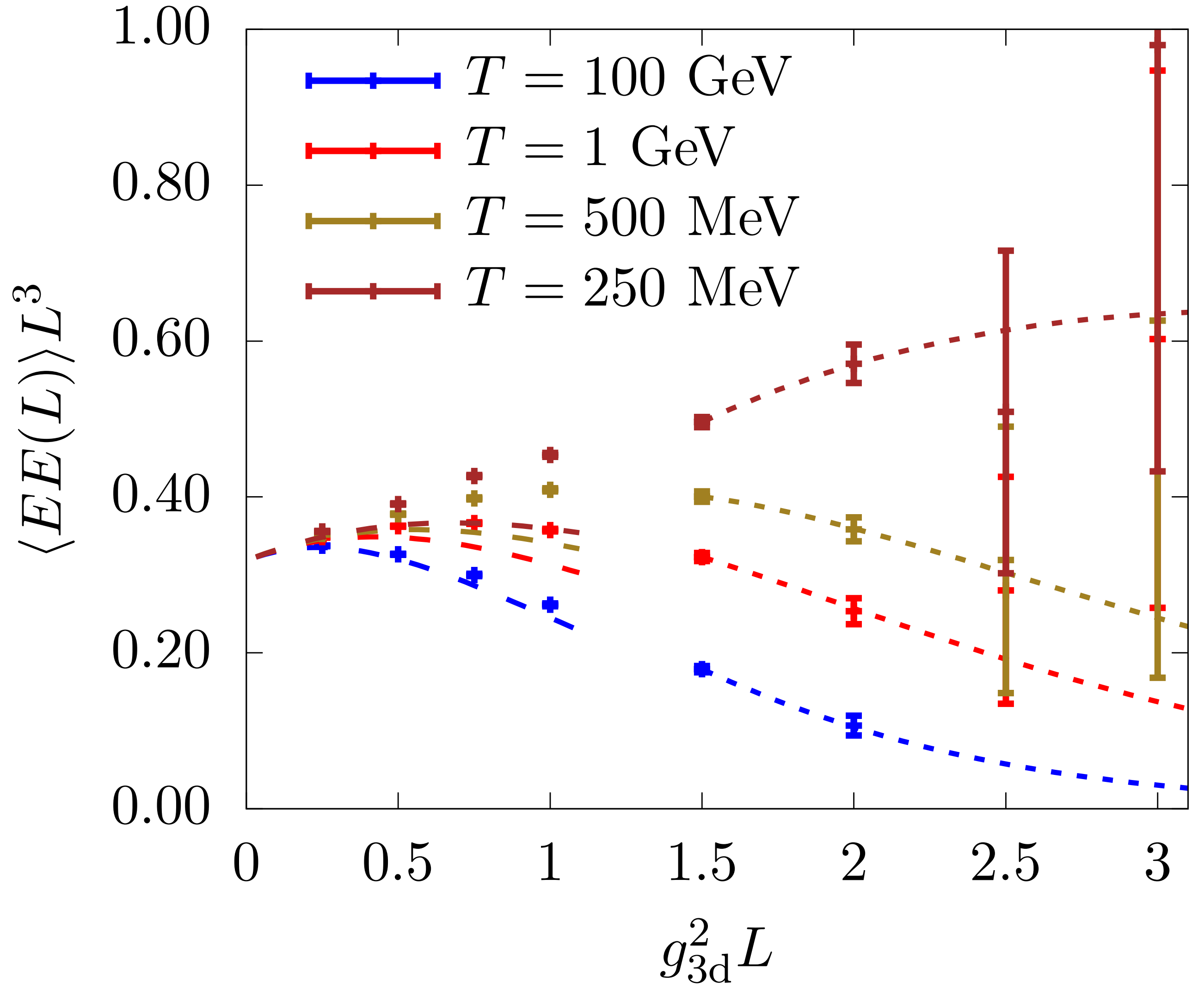
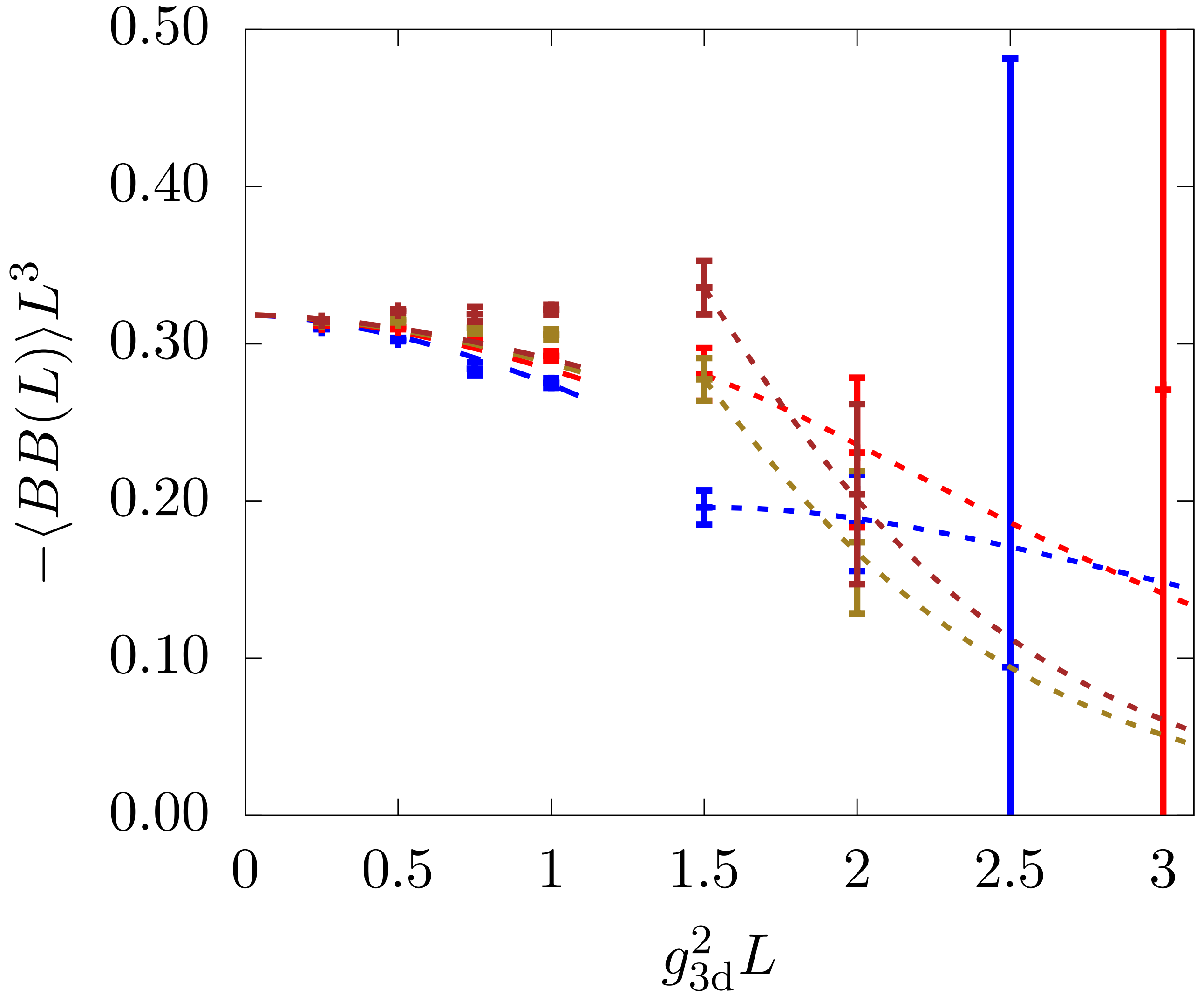
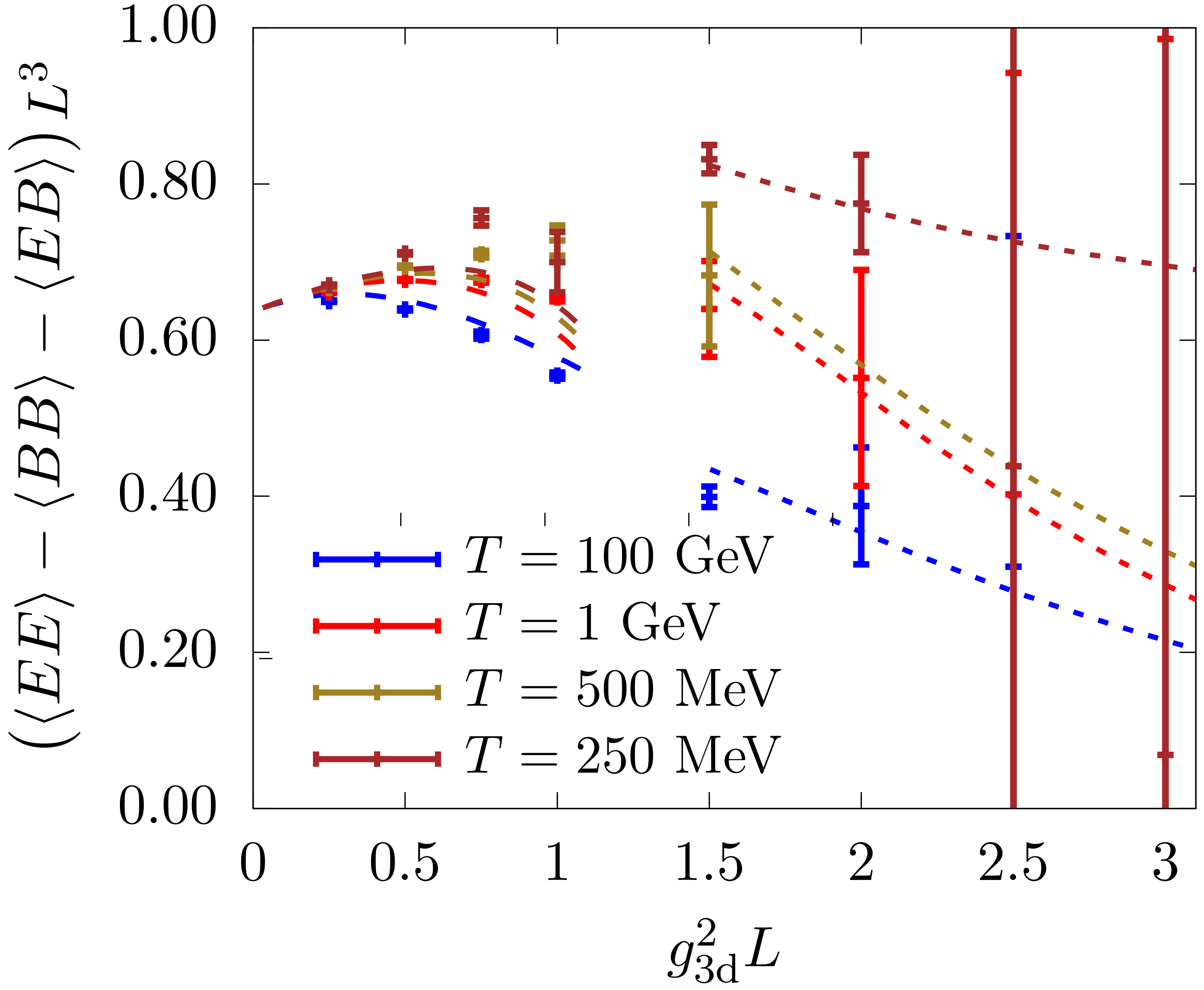






- IR tails modeled by non-perturbative exp. falloff (magnetic screening)
- UV tails handled by perturbative EQCD

JG Moore Schicho Schlusser (2021)

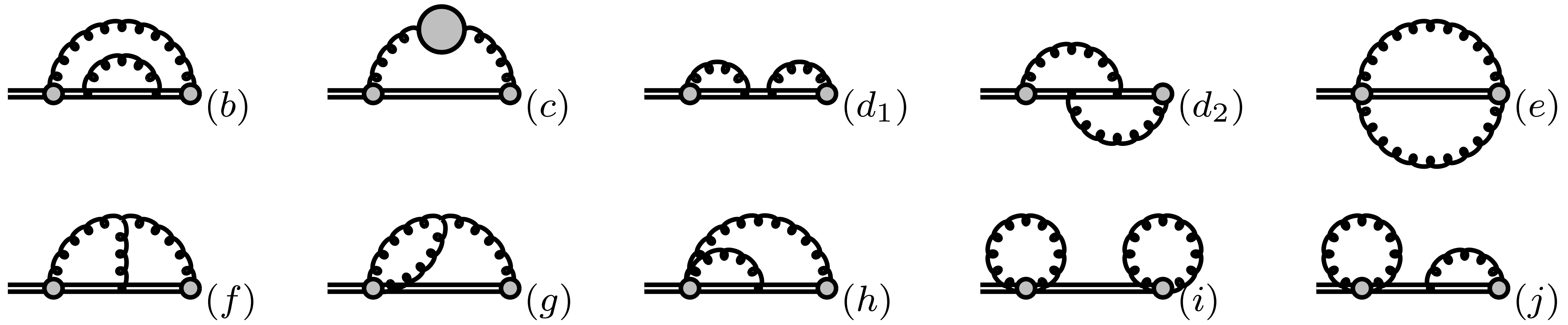
Matching to full QCD

- Integration UV-divergent ($L \to 0$) $$Z_{\rm g}^{\rm EQCD} = \frac{T}{2}\int_0^\infty dL\, L\left(\langle EE\rangle - \langle BB\rangle - \langle EB\rangle\right)$$
- EQCD super-renormalizable, $\langle FF(L \to 0)\rangle = c_0 \frac{1}{L^3} + c_2 \frac{g^2 T}{L^2} + \ldots$
- Only the first two terms give rise to power-law and log divergences. They must cancel with the IR limits of a bare calculation in full thermal QCD. This is easily verified for the power law, that can simply be subtracted
- For the log in a first stage we introduce an intermediate cutoff regulator $-c_2 \frac{g^2 T}{L^2}\theta(L_0 - L)$ and integrate numerically the UV-subtracted EQCD data

JG Moore Schicho Schlusser (2021)

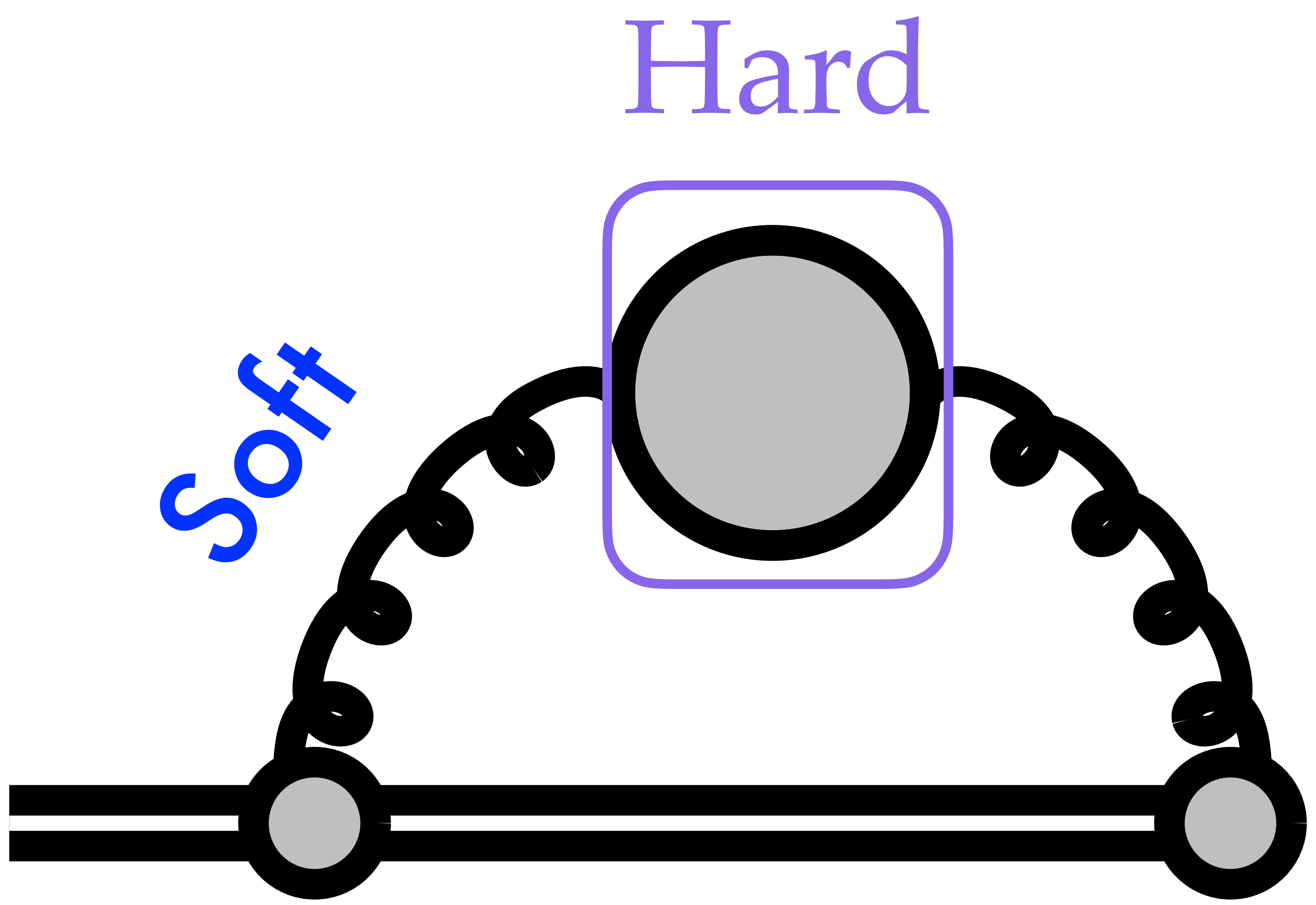
Matching to full QCD

- Proper handling of the log divergence requires the two-loop calculation in thermal QCD



- Only diagram *c* matters in Feynman gauge

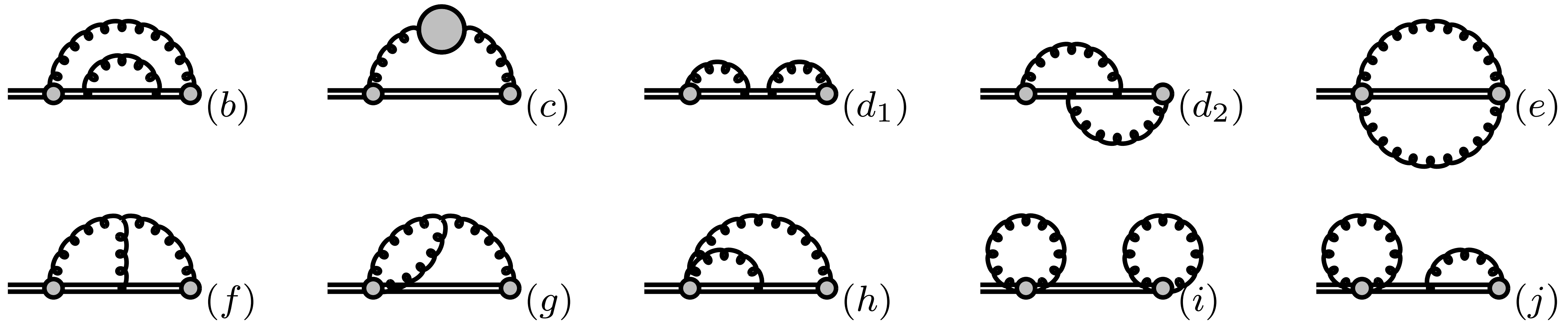
- Translated the cutoff to dimensional regularisation. UV pole of EQCD cancels IR pole of QCD, leaving behind a $g^2T^2\ln(T/m_D)$ term. Regulator dependence gone! Regulator-independent classical contribution negative



JG Schicho Schlusser Weitz in progress

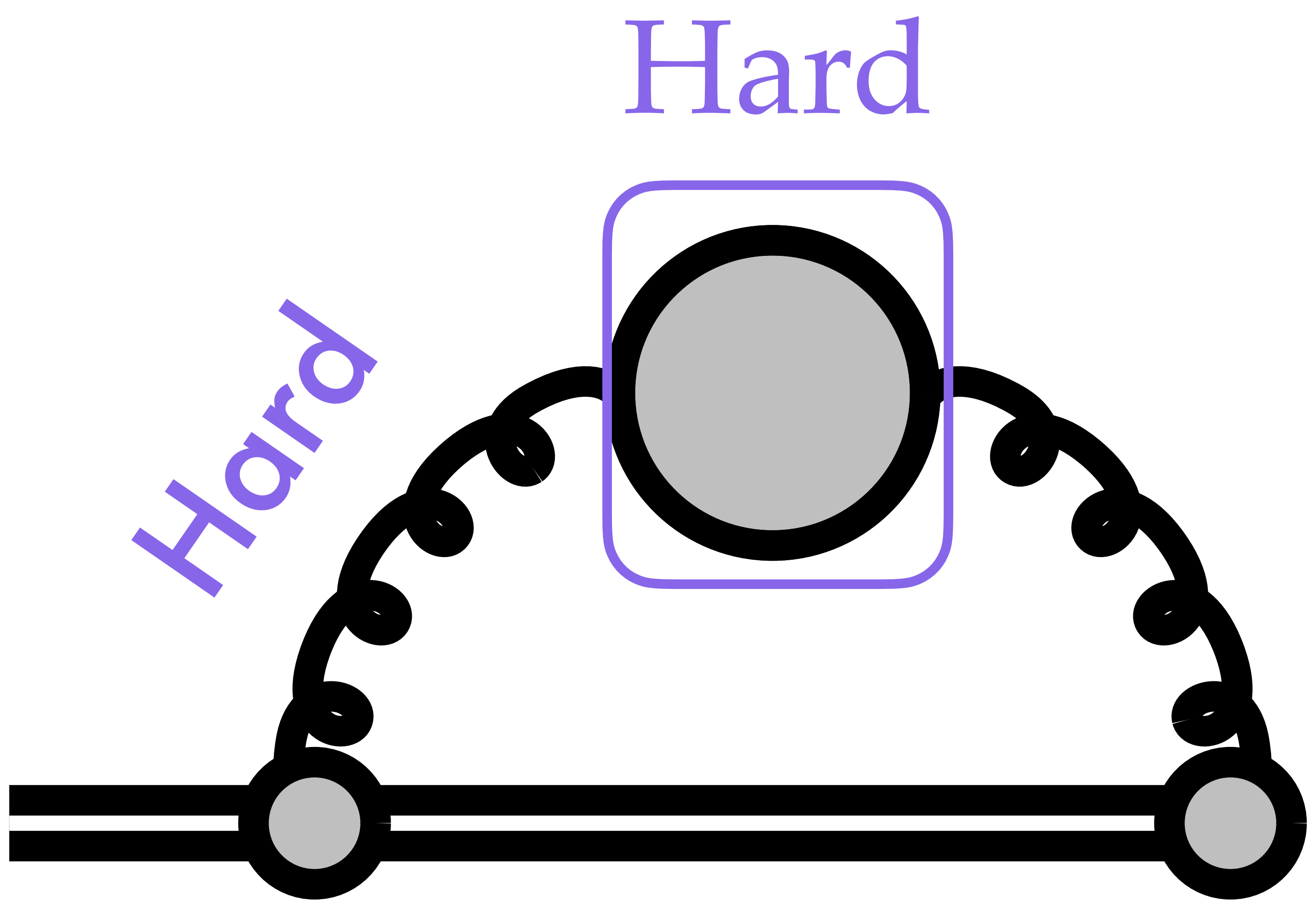
Matching to full QCD

- Proper handling of the log divergence requires the two-loop calculation in thermal QCD



- Only diagram *c* matters in Feynman gauge

- Remainder of the calculation suggests emergence of possible (double-)logarithmic enhancements in the jet's energy



JG Schicho Schlusser Weitz in progress

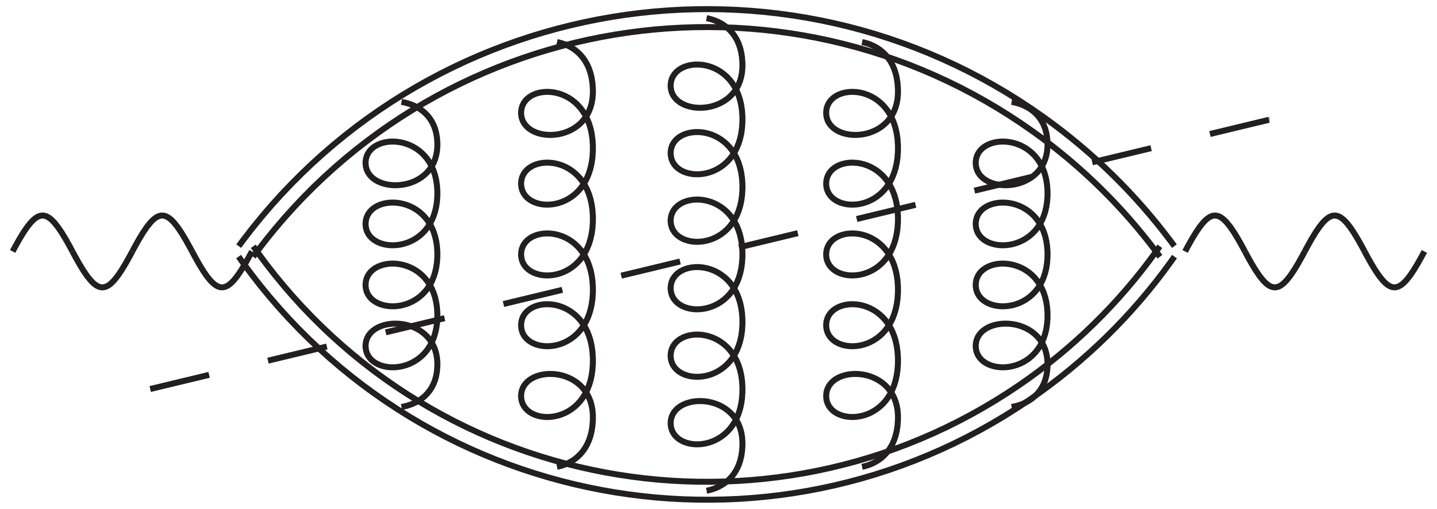
Conclusions

- Large classical contributions very important for many observables
- Methodology of pQCD+lattice EQCD to capture hard modes perturbatively and classical modes at all orders. Successful for $\hat{q}$
- Work in progress for m_∞^2, important ingredient for medium-induced emissions, kinetic theory&transport, photon production

Extra slides

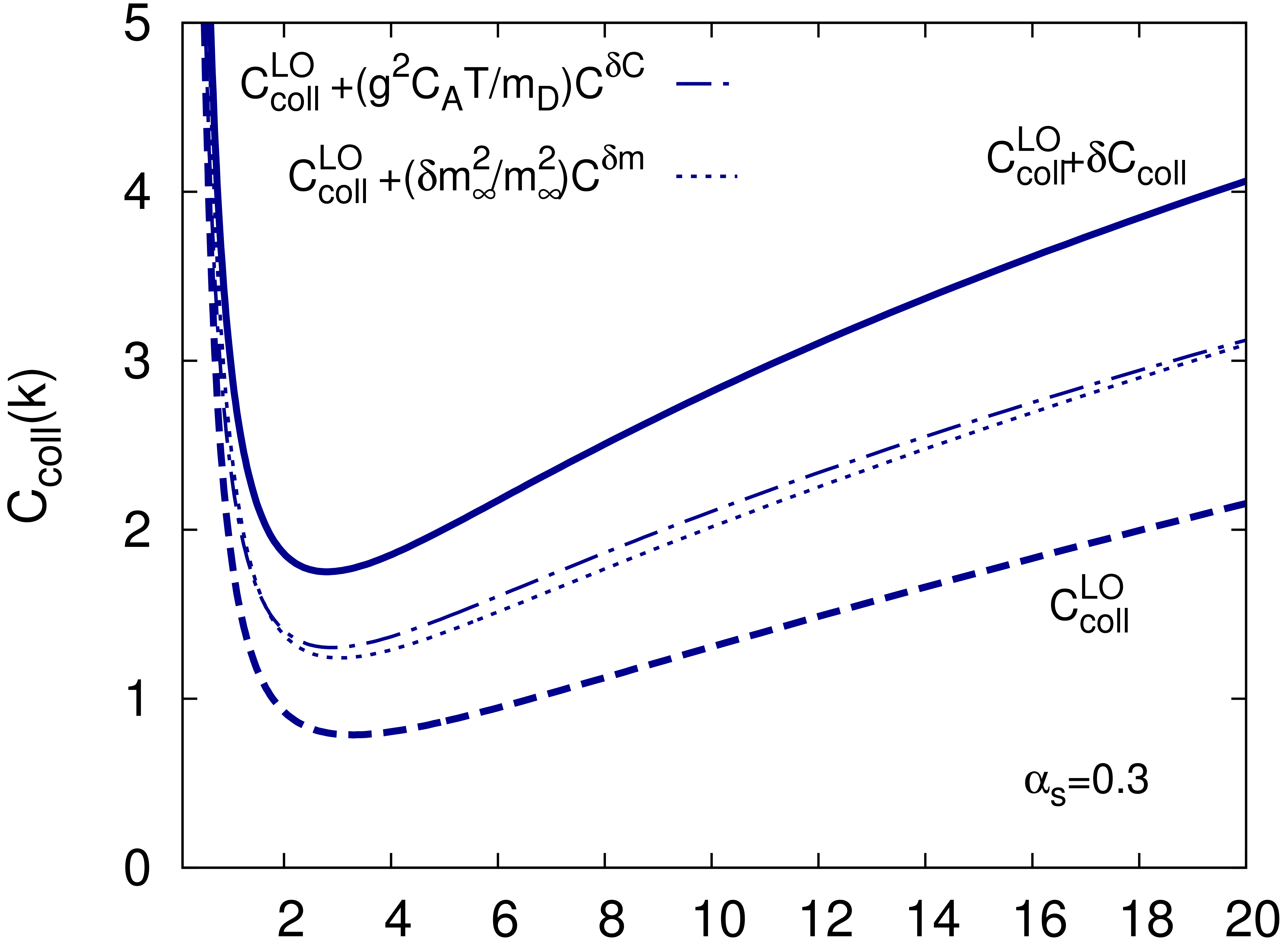
The asymptotic mass and the photon rate

$$\left.\frac{d\Gamma_\gamma}{d^3k}\right|_{\rm coll} =$$



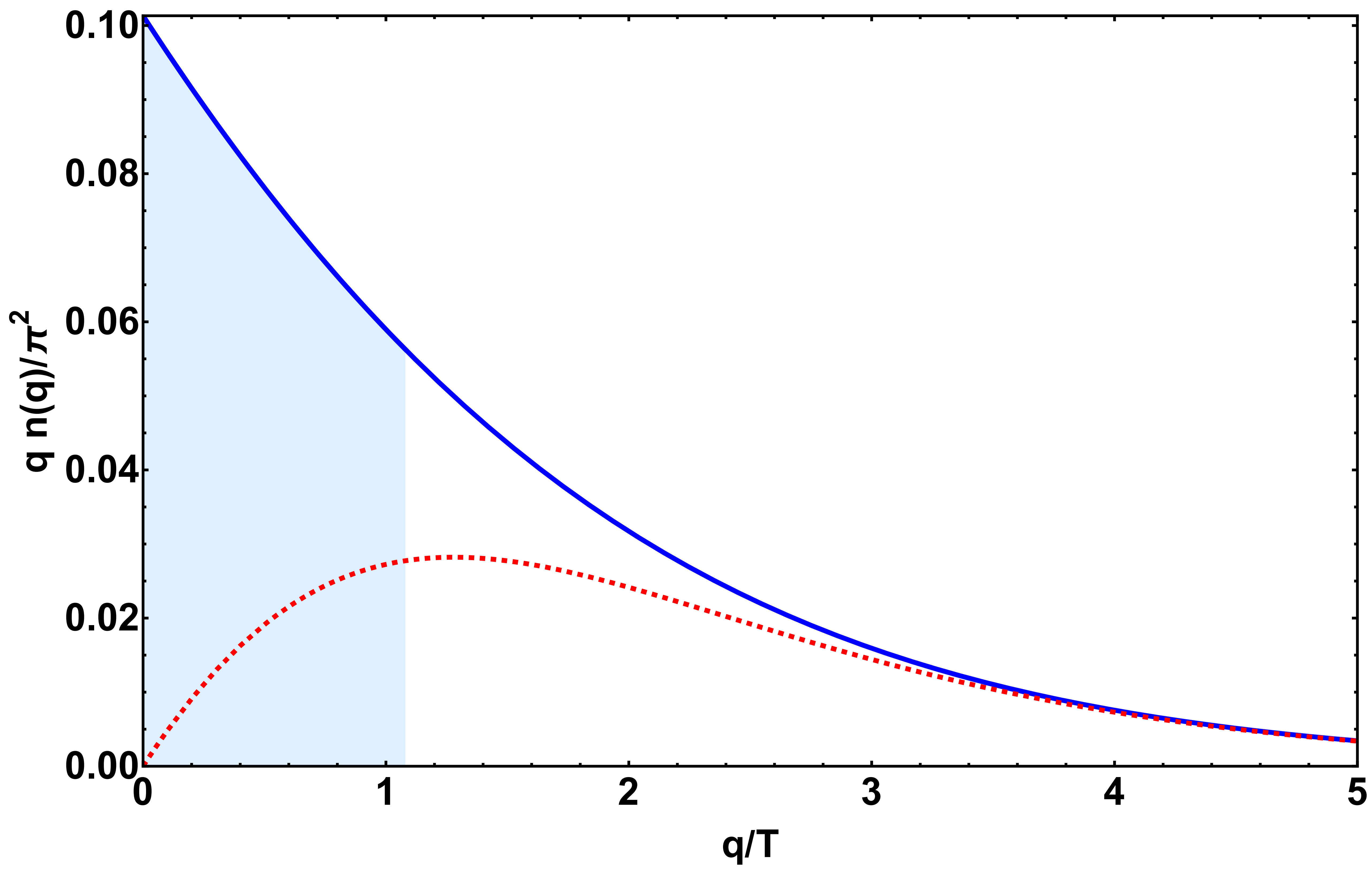
$$(2\pi)^3 \left.\frac{d^3\Gamma_\gamma}{d^3k}\right|_{\rm coll} = \mathcal{A}(k) C_{\rm coll}(k)$$

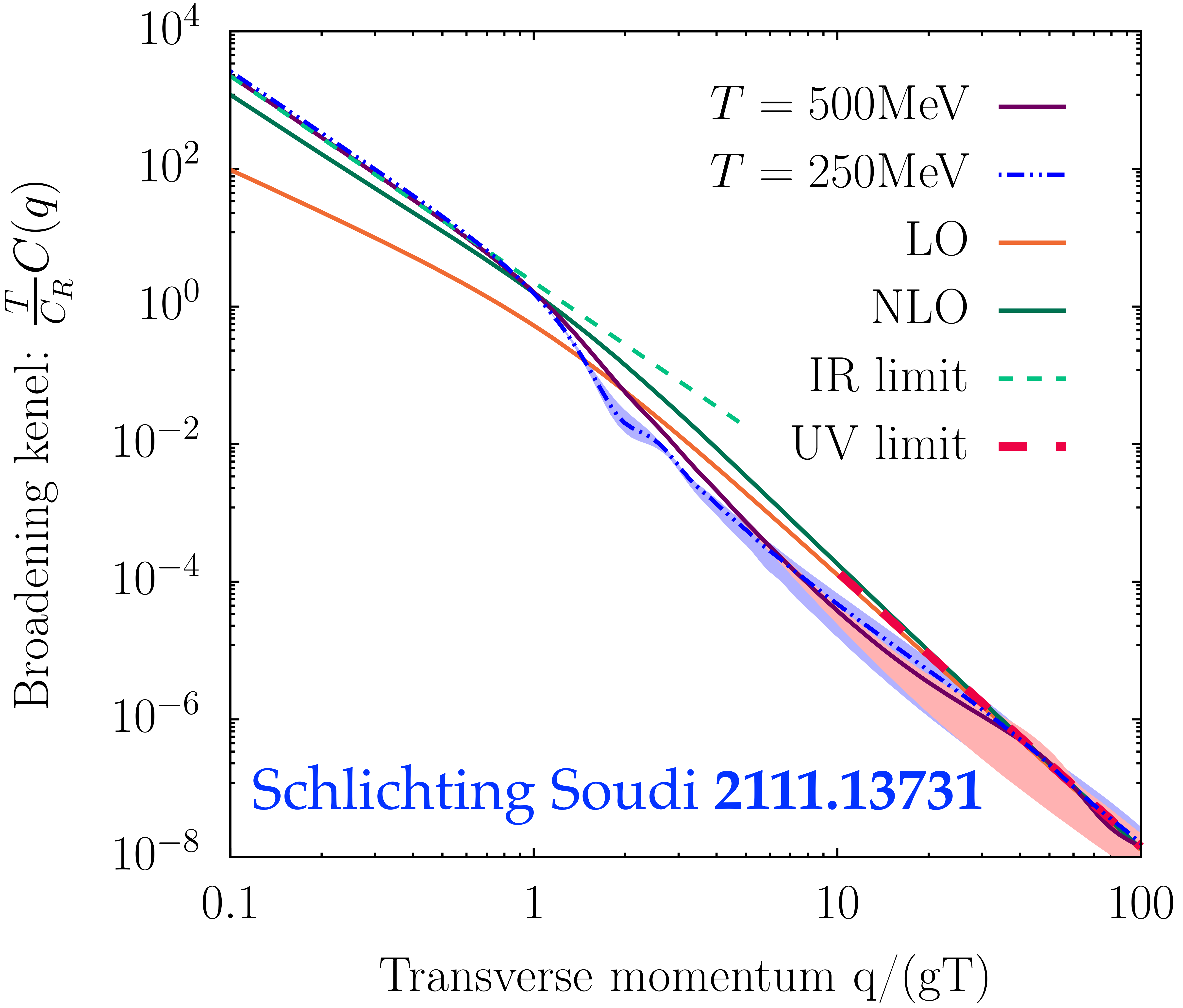
$$\mathcal{A}(k) = \alpha_{\rm EM}\, g^2\ C_F T^2 \frac{n_{\rm F}(k)}{2k} \sum_f Q_f^2 d_f$$



JG Hong Kurkela Lu Moore Teaney (2013)

The asymptotic mass and $\hat{q}$

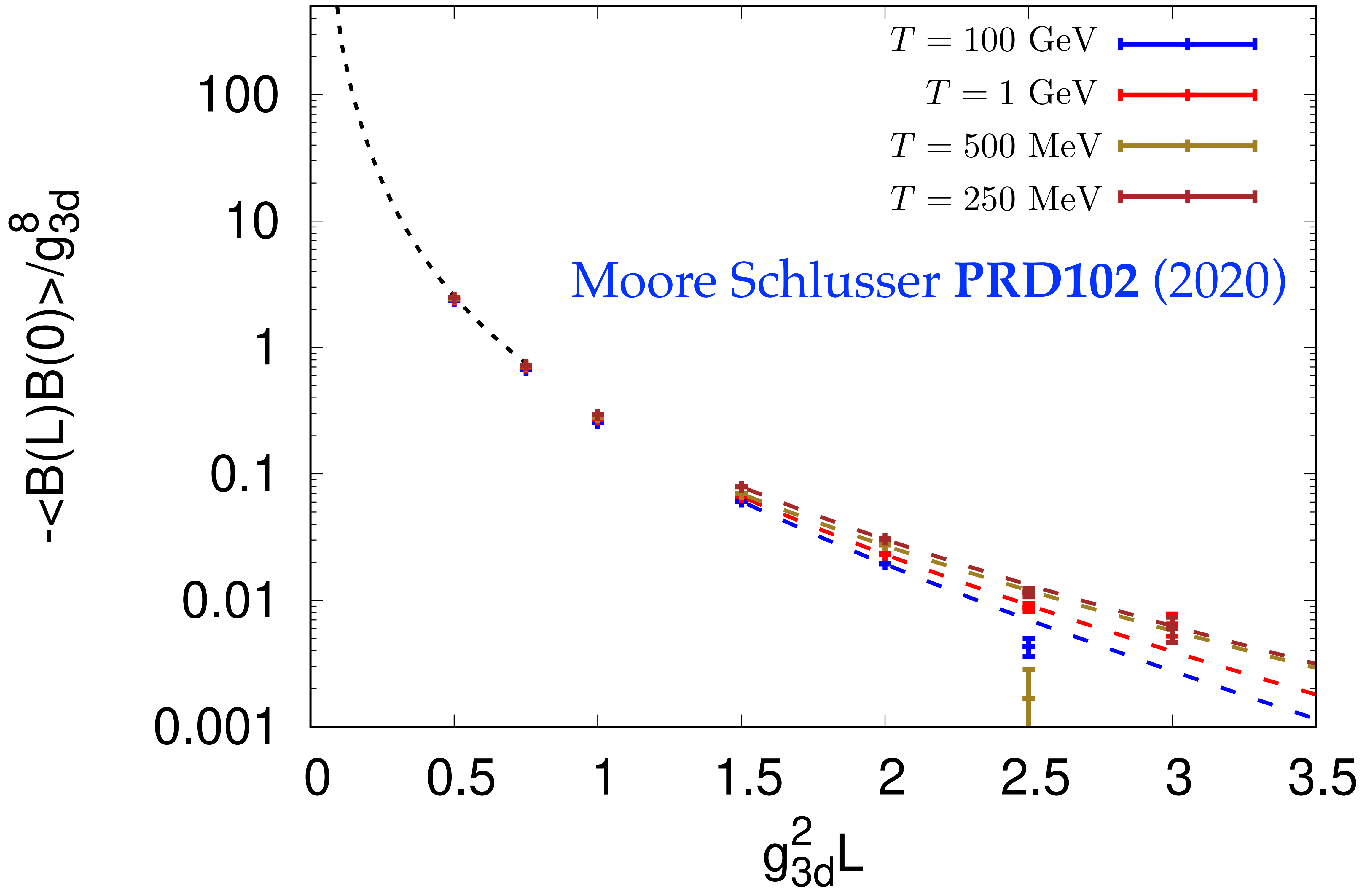




EQCD

$$Z_{\rm g} = \frac{2}{d_A}\int_0^\infty {\rm d}L L \,{\rm Tr}\left\langle U(-\infty;L) v_\alpha F^{\alpha\mu}(L)\, U(L;0)\, v_\nu F^{\nu}\;\mu(0) U(0;-\infty)\right\rangle$$

$$S_{\rm EQCD} = \int_{\vec{x}} \left\{ \frac{1}{2}{\rm Tr}\, F_{ij}F_{ij} + {\rm Tr}\,[D_i,\Phi][D_i,\Phi] + m_D^2 {\rm Tr}\,\Phi^2 + \lambda_E({\rm Tr}\,\Phi^2)^2 \right\}$$



Perturbative expansions

scale T scale gT scale g^2T

$$Z_{\rm g} = \frac{T^2}{6} - \frac{T\mu_{\rm h}}{\pi^2}$$
$$+ \qquad - \frac{Tm_{\rm D}}{2\pi} + \frac{T\mu_{\rm h}}{\pi^2}$$
$$+g^2T^2\left[c^{\rm ln}_{\rm hard}\ln\frac{T}{\mu_{\rm h}} + c_T \quad + c^{\rm ln}_{\rm hard}\ln\frac{\mu_{\rm h}}{m_{\rm D}} + c^{\rm ln}_{\rm soft}\ln\frac{m_D}{\mu_{\rm s}} + c_{gT} \quad + c^{\rm ln}_{\rm soft}\ln\frac{\mu_{\rm s}}{g^2T} + c_{gT^2}\right]$$

scale T scale gT scale g^2T

$$\hat{q}(\mu_{\hat{q}} \sim T) = \alpha_s C_F T m_D^2\left(\ln\frac{T^2}{\mu_h^2} + c_T^{(0)}\right) + \alpha_s C_F T m_D^2 \ln\frac{\mu_h^2}{m_D^2}$$
$$+ \qquad + \alpha_s^2 C_F C_A T^2 m_D c_{gT}^{(1)}$$