Radiative energy loss of heavy quark through soft gluon emission in QGP

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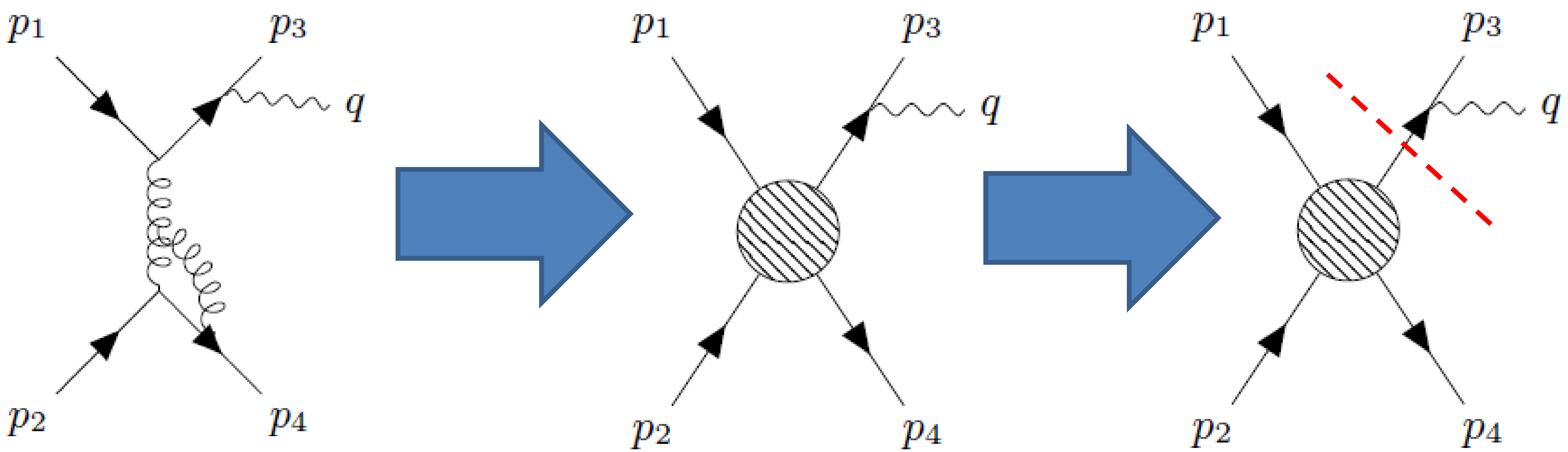
in collaboration with Ilia Grishmanovskii, Olga Soloveva, Elena Bratkovskaya

1. Introduction

Bremsstrahlung photon is emitted from the charged particle which is accelerated or decelerated by scattering (interaction)

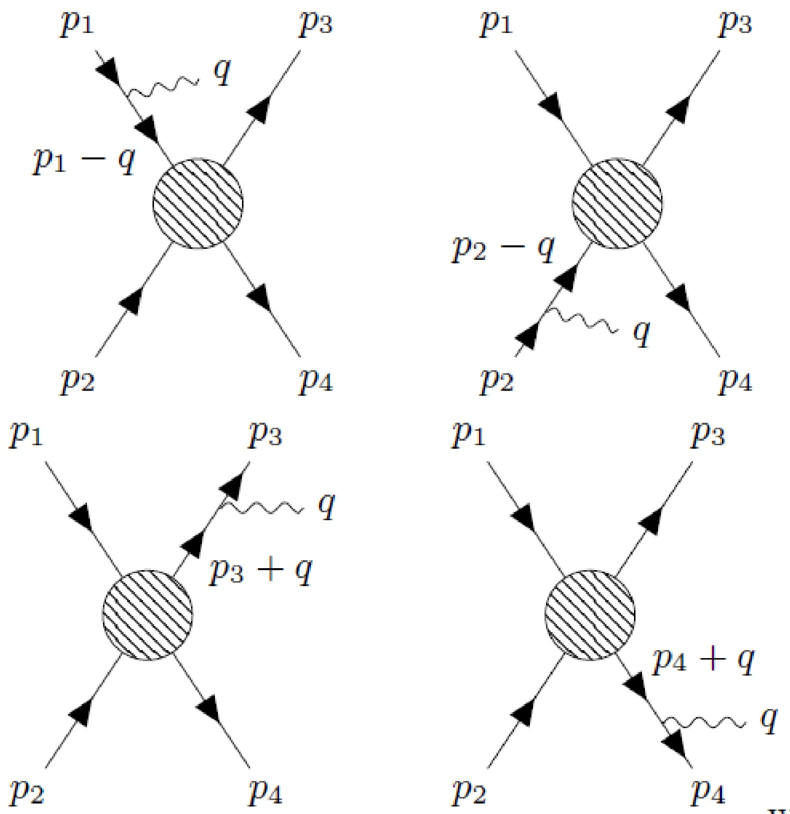
According to Low's theorem (Phy.Rev.110,974)

1. Low-energy Bremsstrahlung photon is emitted from external legs in Feynman diagram (complicated inner structure of scattering can be ignored in the leading order)
2. Elastic scattering and photon emission are separable (factorizable)



2. Low's theorem in QED

π+π→π+π+γ



$$
\begin{aligned}
M_{2\to 2+\gamma}(p_1,p_2;p_3,p_4,q) &= \varepsilon^*_\mu(q) \\
&\times\{M_{2\to 2}(p_1-q,p_2;p_3,p_4)G(p_1-q)V^\mu(p_1;p_1-q) \\
&+M_{2\to 2}(p_1,p_2-q;p_3,p_4)G(p_2-q)V^\mu(p_2;p_2-q) \\
&+V^\mu(p_3+q;p_3)G(p_3+q)M_{2\to 2}(p_1,p_2;p_3+q,p_4) \\
&+V^\mu(p_4+q;p_4)G(p_4+q)M_{2\to 2}(p_1,p_2;p_3,p_4+q)\},
\end{aligned}
$$

$$
\begin{aligned}
G(p) &= \frac{i}{p^2-m^2+i\varepsilon}, \\
V^\mu(p+q,p) &= -iQ(2p+q)^\mu.
\end{aligned}
$$

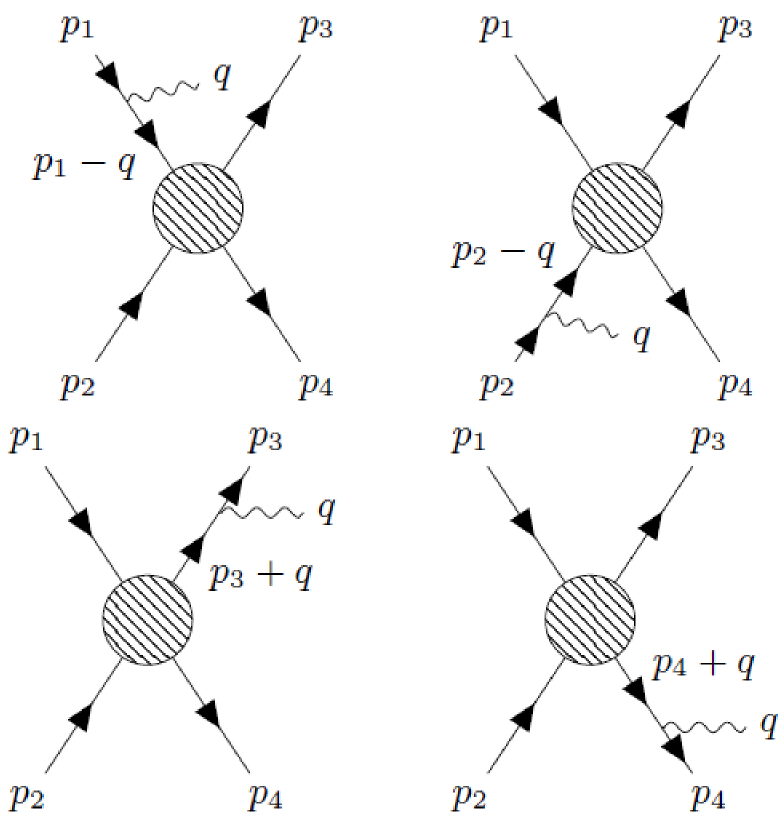
If q << p1, p2, p3, p4

$$
\begin{aligned}
&M_{2\to 2+\gamma}(p_1,p_2;p_3,p_4,q) \\
&= \varepsilon^*_\mu(q)\left\{-\frac{Q_1 p_1^\mu}{p_1\cdot q}-\frac{Q_2 p_2^\mu}{p_2\cdot q}+\frac{Q_3 p_3^\mu}{p_3\cdot q}+\frac{Q_4 p_4^\mu}{p_4\cdot q}\right\} \\
&\quad\times M_{2\to 2}(p_1,p_2;p_3,p_4),
\end{aligned}
$$

which satisfies the Ward's identity: $q^\mu (M_{2\to 2+\gamma})_\mu = 0$

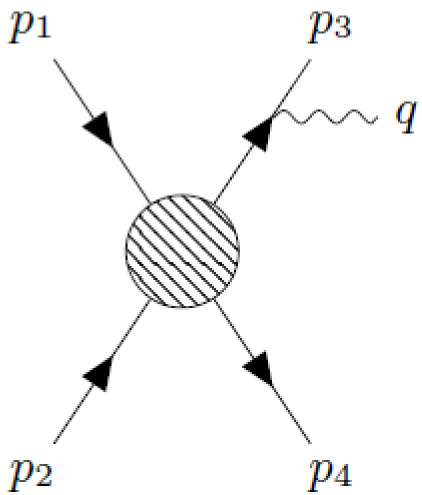
Soft photon emission from the scattering of fermions

p+p→p+p+γ



$$\begin{aligned}
&M_{2\to 2+\gamma}(p_1, p_2; p_3, p_4, q) = \varepsilon^*_\mu(q) \\
&\times\{M_{2\to 2}(p_1 - q, p_2; p_3, p_4) G(p_1 - q) V^\mu(p_1; p_1 - q) \\
&+ M_{2\to 2}(p_1, p_2 - q; p_3, p_4) G(p_2 - q) V^\mu(p_2; p_2 - q) \\
&+ V^\mu(p_3 + q; p_3) G(p_3 + q) M_{2\to 2}(p_1, p_2; p_3 + q, p_4) \\
&+ V^\mu(p_4 + q; p_4) G(p_4 + q) M_{2\to 2}(p_1, p_2; p_3, p_4 + q)\},
\end{aligned}$$

$$\begin{aligned}
G(p) &= i\frac{\not p + m}{p^2 - m^2 + i\varepsilon}, \\
V^\mu(p+q, p) &= -iQ\gamma^\mu.
\end{aligned}$$



$$\bar{u}^s(p_3) \to -\bar{u}^s(p_3) i Q_3 \gamma^\mu i \frac{\not{p}_3 + \not{q} + m}{(p_3+q)^2 - m^2 + i\varepsilon} \varepsilon^*_\mu(q)$$
$$= -\bar{u}^s(p_3) i Q_3 \gamma^\mu i \frac{u^r(p_3+q)\bar{u}^r(p_3+q)}{(p_3+q)^2 - m^2 + i\varepsilon} \varepsilon^*_\mu(q)$$
$$= Q_3 \frac{\bar{u}^s(p_3)\gamma^\mu u^r(p_3+q)}{2p_3 \cdot q} \varepsilon^*_\mu(q) \bar{u}^r(p_3+q).$$

Making use of the Gordon decomposition [5],

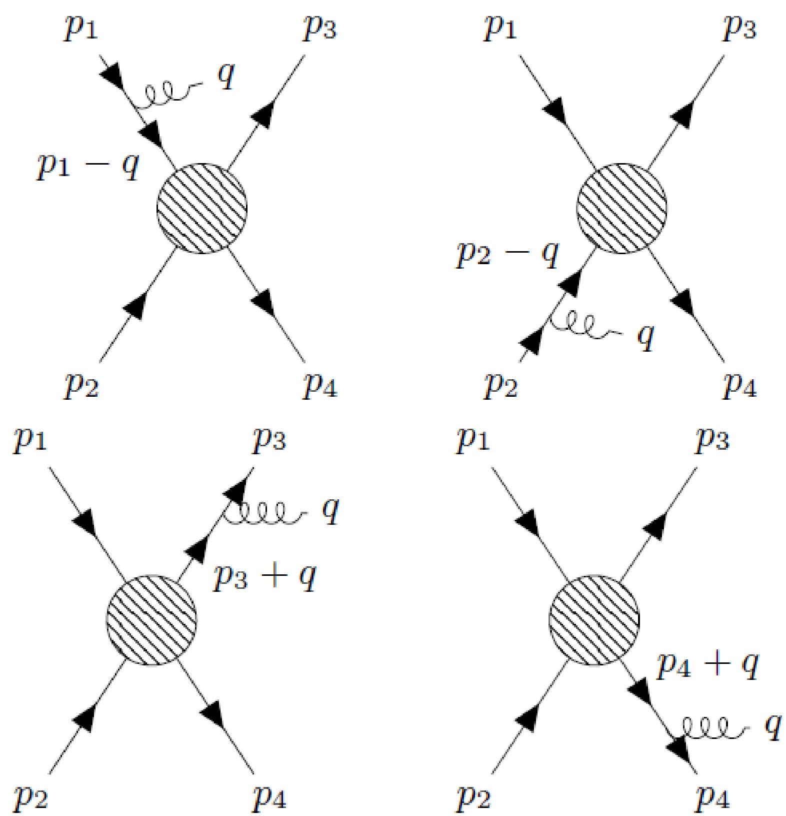
$$\bar{u}^s(p_3)\gamma^\mu u^r(p_3+q)$$
$$= \frac{1}{2m}\bar{u}^s(p_3)\{(2p_3+q)^\mu - i\sigma^{\mu\nu}q_\nu\}u^r(p_3+q)$$
$$\approx \frac{p_3^\mu}{m}\bar{u}^s(p_3)u^r(p_3) = 2p_3^\mu \delta_{sr}$$

$$\bar{u}^s(p_3) \to \varepsilon^*_\mu(q)\frac{Q_3 p_3^\mu}{p_3 \cdot q}\bar{u}^s(p_3+q) \approx \varepsilon^*_\mu(q)\frac{Q_3 p_3^\mu}{p_3 \cdot q}\bar{u}^s(p_3).$$

Same as in π+π→π+π+γ scattering

3. Low's theorem in QCD
(soft gluon emission from quark)

q+q→q+q+g

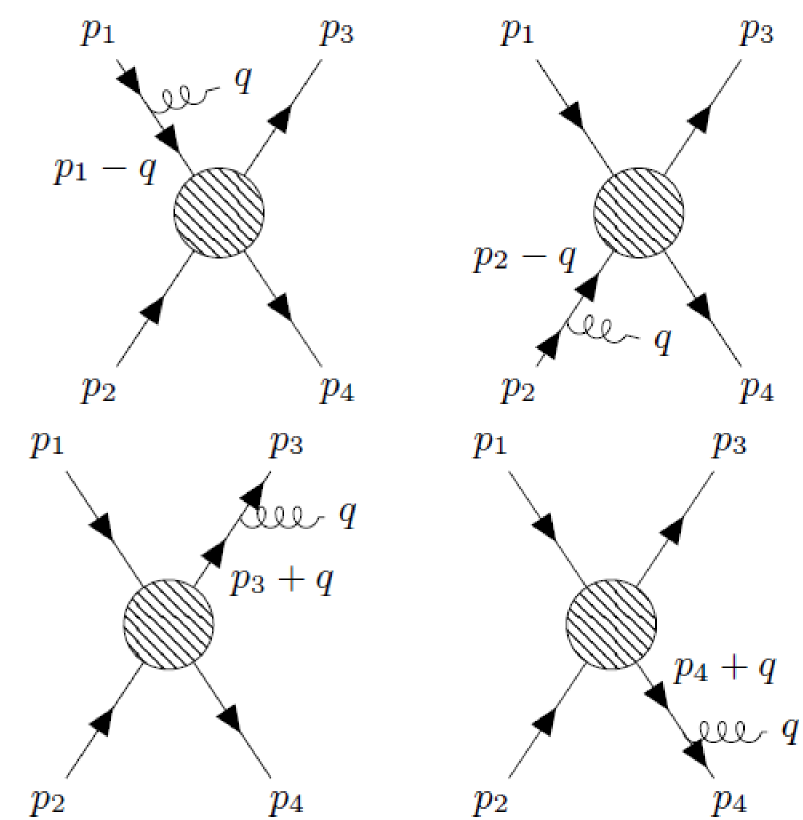


$$
\begin{aligned}
M_{2\to 2+\gamma}(p_1, p_2; p_3, p_4, q) &= \varepsilon^*_\mu(q) \\
&\times\{M_{2\to 2}(p_1 - q, p_2; p_3, p_4) G(p_1 - q) V^\mu(p_1; p_1 - q) \\
&+ M_{2\to 2}(p_1, p_2 - q; p_3, p_4) G(p_2 - q) V^\mu(p_2; p_2 - q) \\
&+ V^\mu(p_3 + q; p_3) G(p_3 + q) M_{2\to 2}(p_1, p_2; p_3 + q, p_4) \\
&\cdot V^\mu(p_4 + q; p_4) G(p_4 + q) M_{2\to 2}(p_1, p_2; p_3, p_4 + q)\},
\end{aligned}
$$

$$
\begin{aligned}
G_{ij}(p) &= i\frac{\not{p} + m}{p^2 - m^2 + i\varepsilon}\delta_{ij}, \\
V^{\mu,a}_{ij}(p+q, p) &= ig\gamma^\mu T^a_{ij},
\end{aligned}
$$

Everything is same as in QED except color factors

q+q→q+q+g



$$M^{kl;ij}_{2q\to 2q+g}(p_1,p_2;p_3,p_4,q)$$
$$= g\varepsilon^{a*}_{\mu}(q)\left\{\frac{p_1^\mu}{p_1\cdot q}M^{kl;mj}_{2q\to 2q}T^a_{mi} + \frac{p_2^\mu}{p_2\cdot q}M^{kl;im}_{2q\to 2q}T^a_{mj} - \frac{p_3^\mu}{p_3\cdot q}T^a_{km}M^{ml;ij}_{2q\to 2q} - \frac{p_4^\mu}{p_4\cdot q}T^a_{lm}M^{km;ij}_{2q\to 2q}\right\},$$

i,j: color indices of incoming quarks
k,l: color indices of outgoing quarks
a: color index of emitted gluon
m: color index of intermediate quark

By using the simplest color structure

$$M^{kl;ij}_{2q\to 2q} \sim T^b_{ki}T^b_{lj} = \frac{1}{2}\left(\delta_{jk}\delta_{il} - \frac{1}{N_c}\delta_{ik}\delta_{jl}\right).$$

one can show Slavnov-Taylor identity

$$q_\mu M^{\mu,a,kl;ij}_{2q\to 2q+g}(p_1,p_2;p_3,p_4,q) = 0,$$

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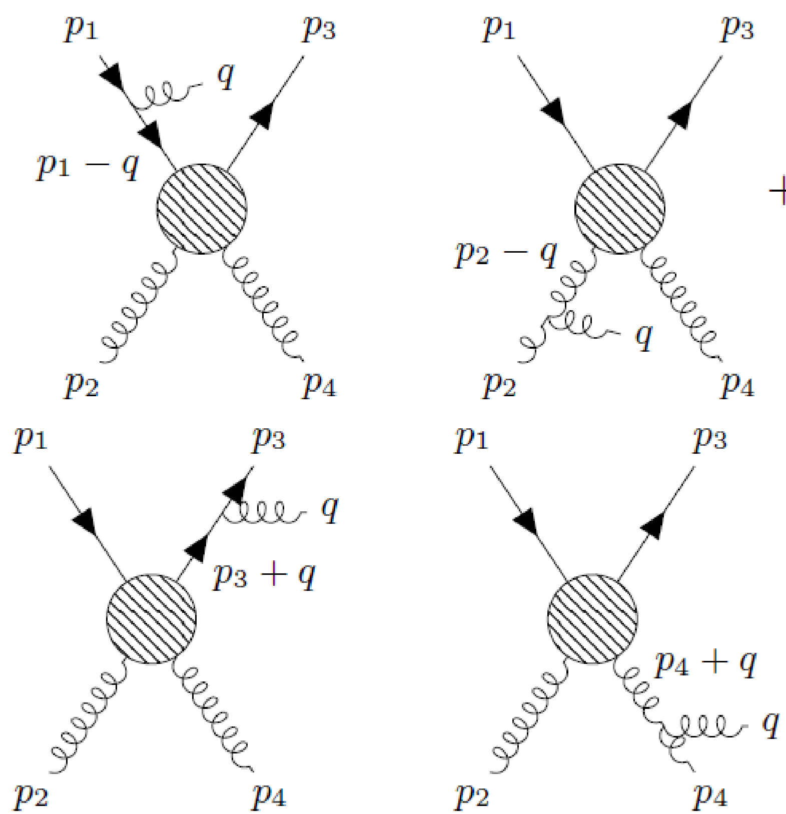
Scattering amplitude square for q+q→q+q+g

$$
\begin{aligned}
|M_{2q\to 2q+g}|^2 = {}& 2\,\frac{g^2}{N_c}\Bigg[(N_c^2-1)\left\{\frac{m_1^2}{(p_1\cdot q)^2}+\frac{m_2^2}{(p_2\cdot q)^2}\right.\\
&\left.+\frac{m_3^2}{(p_3\cdot q)^2}+\frac{m_4^2}{(p_4\cdot q)^2}\right\}-\frac{4p_1\cdot p_2}{(p_1\cdot q)(p_2\cdot q)}\\
&-\frac{4p_3\cdot p_4}{(p_3\cdot q)(p_4\cdot q)}+\frac{2p_1\cdot p_3}{(p_1\cdot q)(p_3\cdot q)}+\frac{2p_2\cdot p_4}{(p_2\cdot q)(p_4\cdot q)}\\
&-2(N_c^2-2)\left\{\frac{p_1\cdot p_4}{(p_1\cdot q)(p_4\cdot q)}+\frac{p_2\cdot p_3}{(p_2\cdot q)(p_3\cdot q)}\right\}\Bigg]\\
&\times|M_{2q\to 2q}|^2.
\end{aligned}
$$

(soft gluon emission from gluon)

q+g→q+g+g

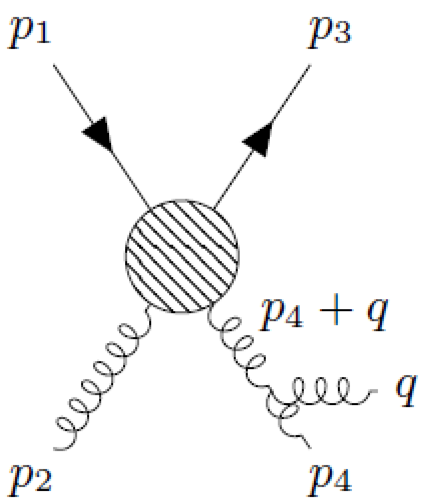


$$\begin{aligned}
M_{2\to2+\gamma}(p_1,p_2;p_3,p_4,q) &= \varepsilon^*_\mu(q) \\
&\times\{M_{2\to2}(p_1-q,p_2;p_3,p_4)G(p_1-q)V^\mu(p_1;p_1-q) \\
&+M_{2\to2}(p_1,p_2-q;p_3,p_4)G(p_2-q)V^\mu(p_2;p_2-q) \\
&+V^\mu(p_3+q;p_3)G(p_3+q)M_{2\to2}(p_1,p_2;p_3+q,p_4) \\
&+V^\mu(p_4+q;p_4)G(p_4+q)M_{2\to2}(p_1,p_2;p_3,p_4+q)\},
\end{aligned}$$

$$G^{\mu\nu,ab}(p) = \frac{-ig^{\mu\nu}}{p^2+i\varepsilon}\delta_{ab},$$

$$\begin{aligned}
V^{\mu\nu\lambda,abc}(p+q,p) &= gf^{abc}[g^{\nu\mu}(p+2q)^\lambda \\
&+g^{\lambda\nu}(-2p-q)^\nu + g^{\mu\lambda}(p-q)^\mu]
\end{aligned}$$

for gluon propagator and
3-gluon vertex



$$\varepsilon_\lambda^{b*}(p_4)M_{2\to 2}^{\lambda,b} \to -igf^{bdc}\varepsilon_\lambda^{b*}(p_4)\varepsilon_\mu^{c*}(q)\Big[g_\nu^\mu(p_4+2q)^\lambda + g^{\lambda\mu}(p_4-q)_\nu + g_\nu^\lambda(-2p_4-q)^\mu\Big]\frac{1}{2p_4\cdot q}M_{2\to 2}^{\nu,d}$$

$$\approx igf^{bdc}\varepsilon_\mu^{c*}(q)\frac{p_4^\mu}{p_4\cdot q} \times \varepsilon_\nu^{b*}(p_4)M_{2\to 2}^{\nu,d},$$

$$M_{q+g\to q+g+g}^{\mu,jbc;ia}(p_1,p_2;p_3,p_4,q)$$
$$= g\left\{\frac{p_1^\mu}{p_1\cdot q}M_{q+g\to q+g}^{jb;ma}T_{mi}^c - \frac{p_3^\mu}{p_3\cdot q}T_{jm}^c M_{q+g\to q+g}^{mb;ia} + i\frac{p_2^\mu}{p_2\cdot q}f^{adc}M_{q+g\to q+g}^{jb;id} + i\frac{p_4^\mu}{p_4\cdot q}f^{bdc}M_{q+g\to q+g}^{jd;ia}\right\},$$

Considering that the color structure of $M_{q+g\to q+g}^{jb;ia}$ is given by $[T^a,T^b]_{ji}$ or $if^{abc}T_{ji}^c$,

$$q_\mu M_{q+g\to q+g+g}^{\mu,jbc;ia}(p_1,p_2;p_3,p_4,q) = 0,$$

Scattering amplitude square for q+g→q+g+g

$$|M_{q+g\to q+g+g}|^2 = -g^2\Bigg[\frac{N_c^2-1}{2N_c}\left(\frac{m_1^2}{(p_1\cdot q)^2}+\frac{m_3^2}{(p_3\cdot q)^2}\right)
+\frac{1}{N_c}\frac{p_1\cdot p_3}{(p_1\cdot q)(p_3\cdot q)}-\frac{N_c}{2}\Bigg(\frac{2p_2\cdot p_4}{(p_2\cdot q)(p_4\cdot q)}
+\frac{p_1\cdot p_2}{(p_1\cdot q)(p_2\cdot q)}+\frac{p_3\cdot p_4}{(p_3\cdot q)(p_4\cdot q)}+\frac{p_1\cdot p_4}{(p_1\cdot q)(p_4\cdot q)}
+\frac{p_2\cdot p_3}{(p_2\cdot q)(p_3\cdot q)}\Bigg)\Bigg]\times|M_{q+g\to q+g}|^2.$$

4. Soft gluon emission from sQGP

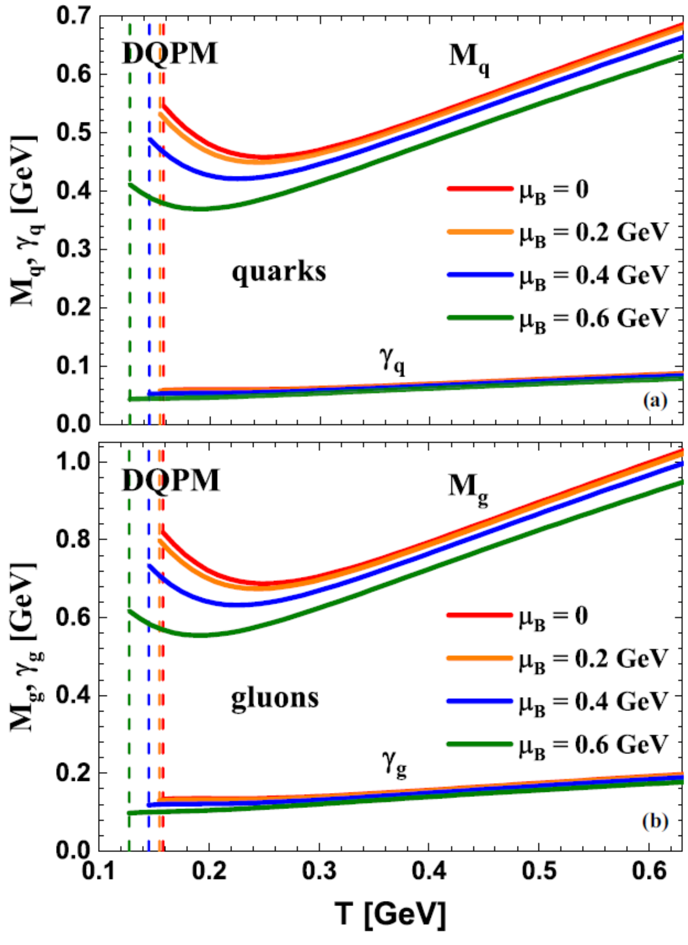
- Propagators of dressed quark/gluon

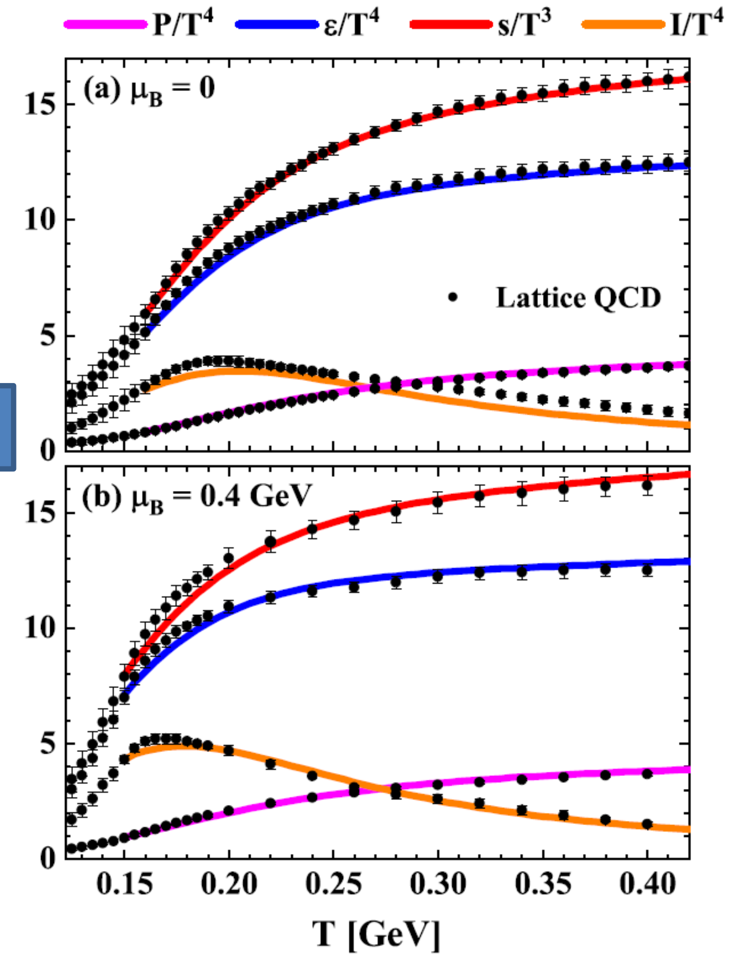
$$G_{ij}(p) = i\frac{\not{p} + m_q(T,\mu)}{p^2 - m_q^2(T,\mu) + i|p_0|\Gamma_q(T,\mu)}\delta_{ij},$$
$$G^{\mu\nu,ab}(p) = \frac{-ig^{\mu\nu}}{p^2 - m_g^2(T,\mu) + i|p_0|\Gamma_g(T,\mu)}\delta_{ab}.$$

based on Dynamical Quasi-Particle Model (DQPM)

Dynamical Quasi-Particle Model (DQPM)

Lattice EOS is well reproduced both at $\mu_B = 0$ and $\mu_B \neq 0$





Pierre Moreau, et al. PRC 100 (2018) 014911

Final state phase space for cross section

$$\begin{aligned} p_1 &= (E_1,\ 0,\ 0,\ p_1), \\ p_2 &= (E_2,\ 0,\ 0,\ -p_1), \\ p_3 &= (E_3,\ 0,\ p_3\sin\theta,\ p_3\cos\theta) \\ p_4 &= (E_4,\ 0,\ -p_3\sin\theta,\ -p_3\cos\theta) \\ q &= (E_q,\ q\sin\theta'\cos\phi',\ q\sin\theta'\sin\phi',\ q\cos\theta') \end{aligned}$$

assume back-to-back

$$\frac{d\sigma_{2\to 3}}{d\cos\theta} \approx \frac{1}{32\pi p_i\sqrt{s}} \int \frac{d^3q}{(2\pi)^3 2E_q} \frac{|\mathbf{p}_3|}{\sqrt{s_2}} \overline{|M_{2\to 3}|^2}$$

$$\approx \frac{d\sigma_{2\to 2}}{d\cos\theta} \int \frac{d^3q}{(2\pi)^3 2E_q} |\epsilon\cdot J|^2 \frac{|\mathbf{p}_3|\sqrt{s}}{p_f\sqrt{s_2}},$$

$$\begin{aligned} \overline{|M_{2\to 3}|^2} &\equiv |\epsilon\cdot J|^2 \times \overline{|M_{2\to 2}|^2} \\ &= 32\pi s \frac{p_i}{p_f} \frac{d\sigma_{2\to 2}}{d\cos\theta} |\epsilon\cdot J|^2 \end{aligned}$$

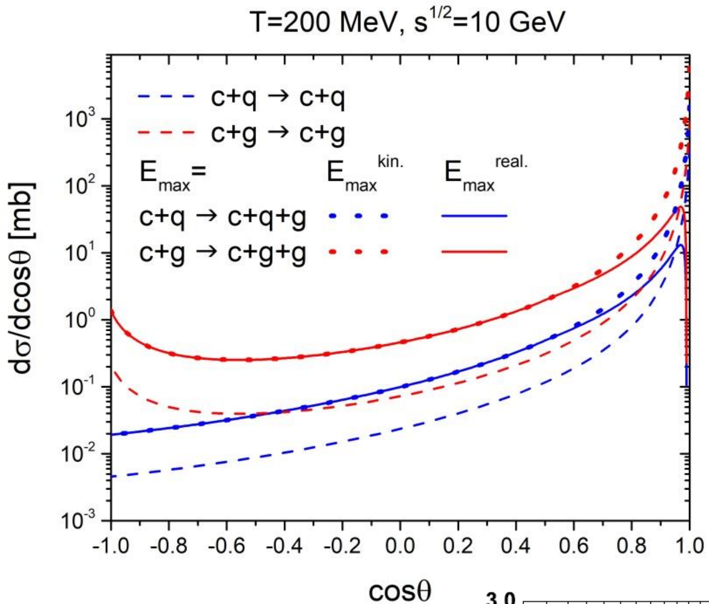
Maximum soft gluon energy:

kinetically allowed maximum from $q^2_{\max} = \frac{\{s-(m_3+m_4+m_g)^2\}\{s-(m_3+m_4-m_g)^2\}}{4s}$

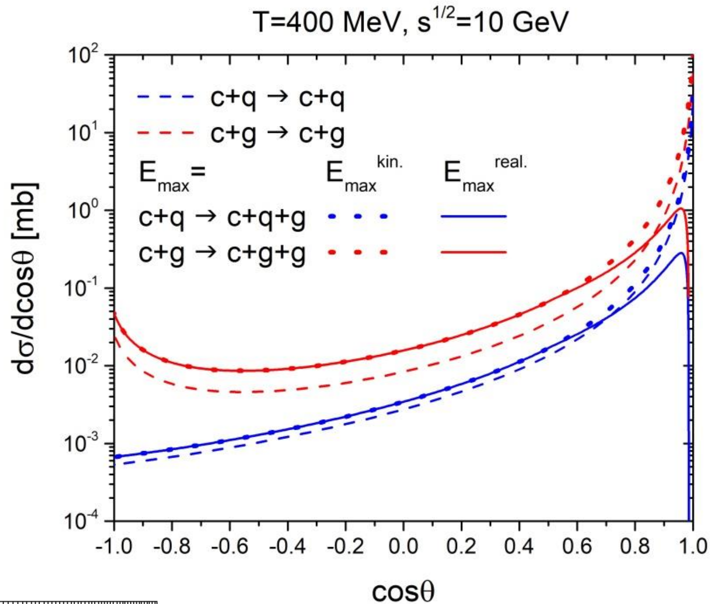
or a limit for the soft gluon approximation $E_q = \sqrt{-t}$

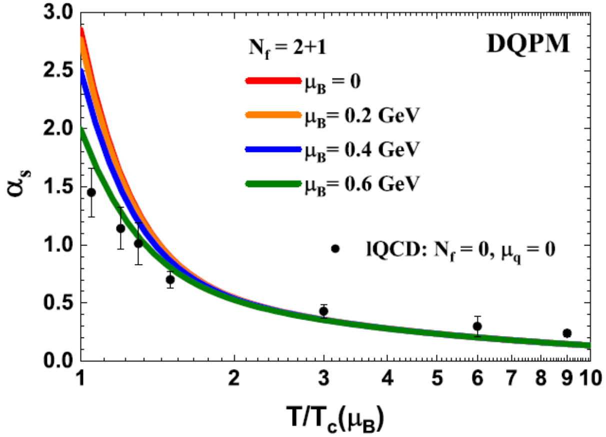
Differential charm cross sections

T=200 MeV



T=400 MeV





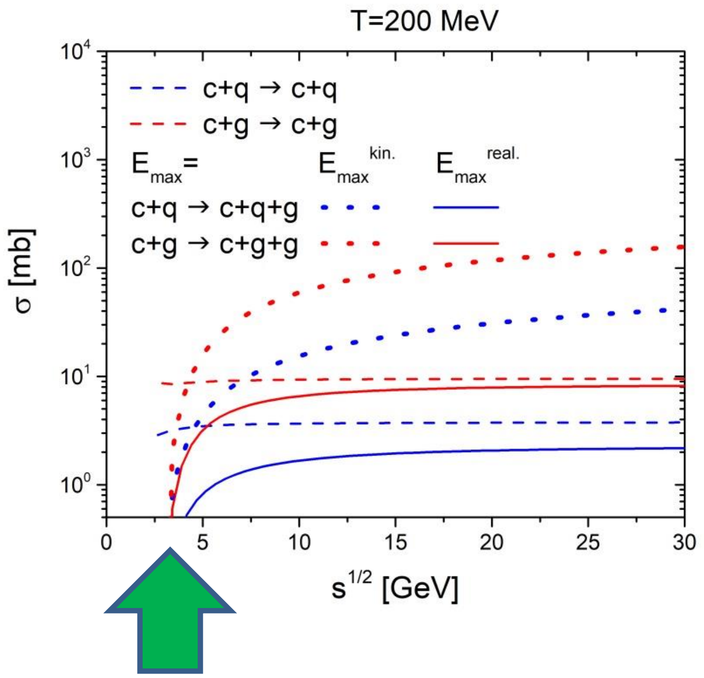
Strong coupling α_s(T) is large near Tc

$$c + q(g) \to c + q(g): \sim\alpha_s^2$$

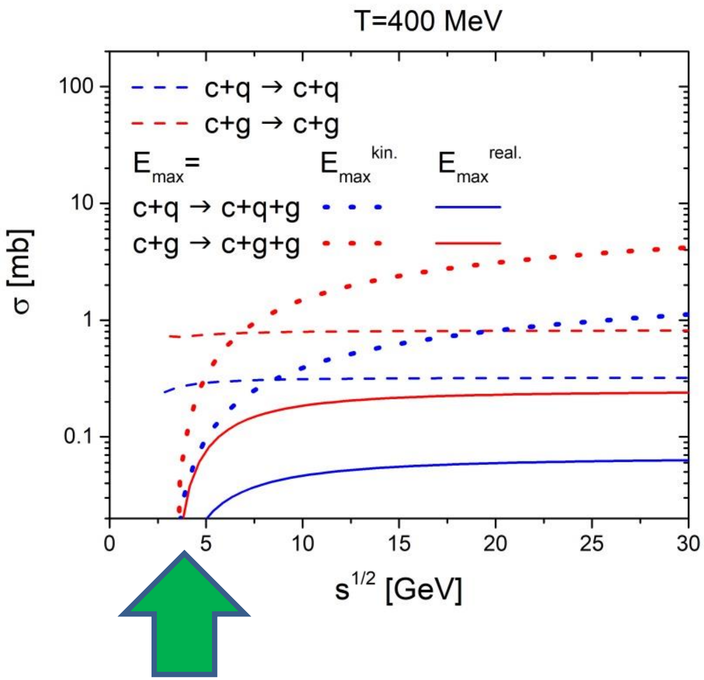
$$c + q(g) \to c + q(g) + g: \sim\alpha_s^3$$

Integrated charm cross sections

T=200 MeV

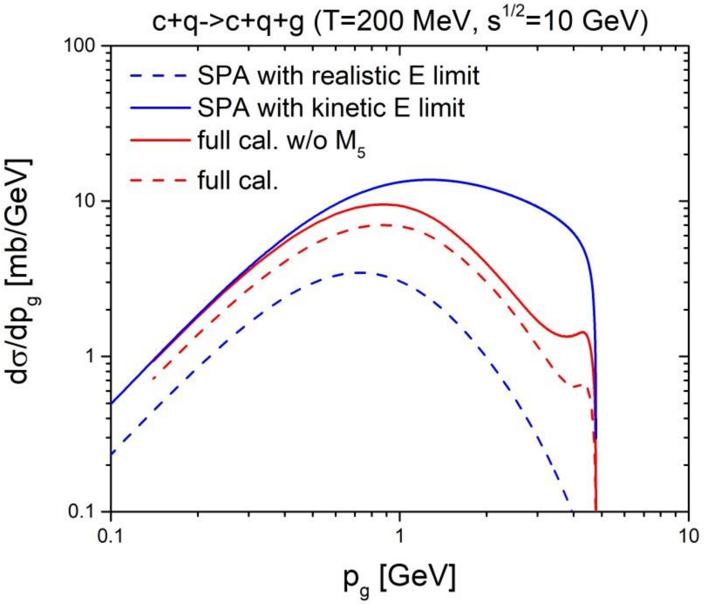


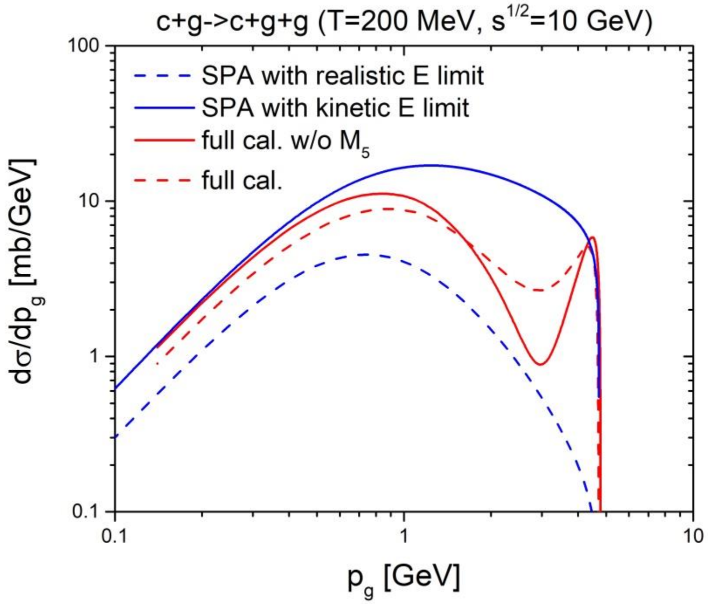
T=400 MeV

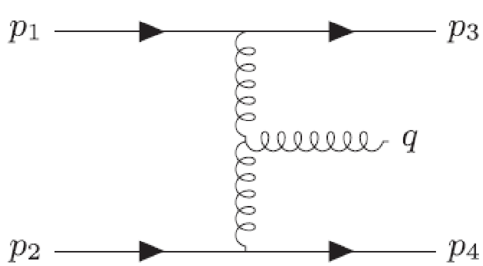


Massive gluon cannot be produced in low energy scattering
That is why 2-to-3 scattering is suppressed at low $s^{1/2}$

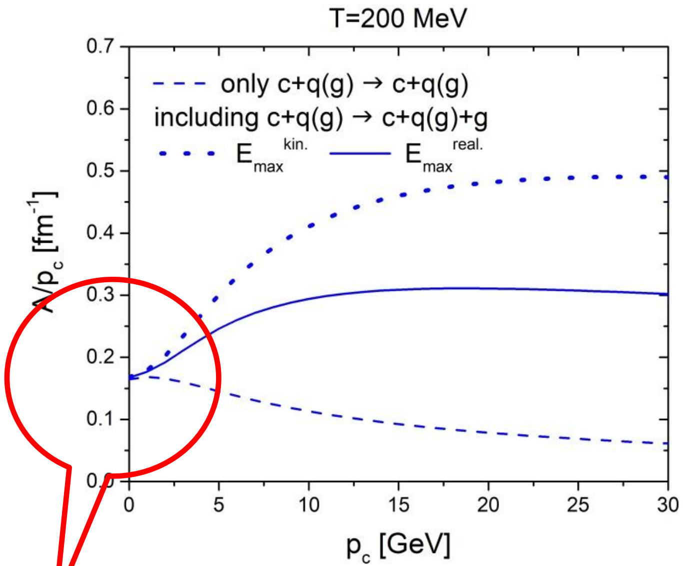
Comparison with full calculations
from Ilia Grishmanovskii (14:20, today)

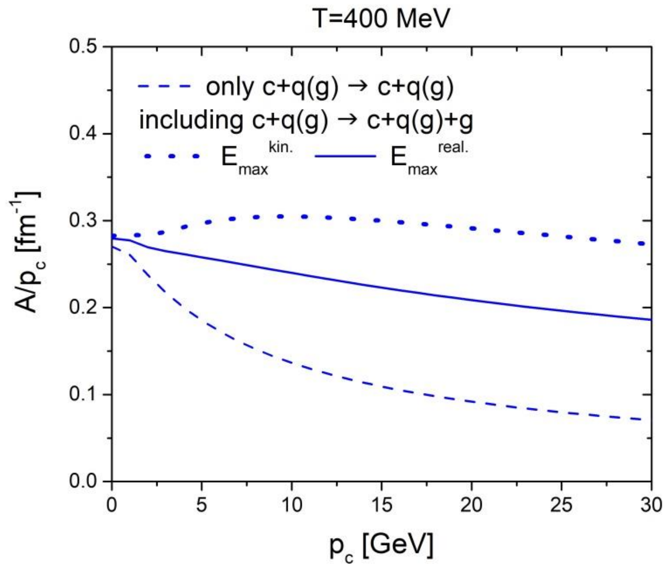




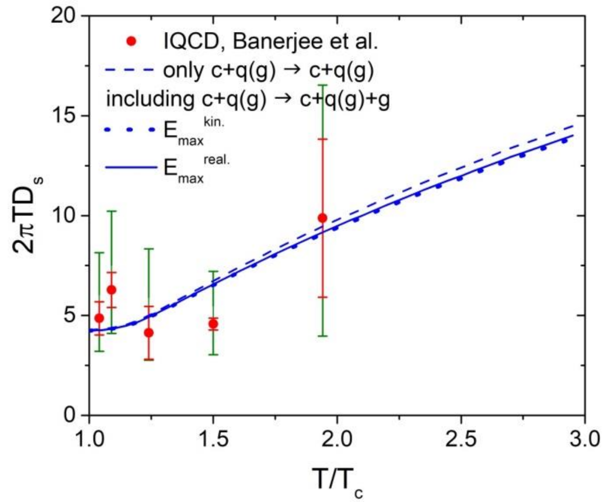
$M_5 =$ 

Drag coefficients of charm $-\frac{d\langle \Delta p \rangle}{dt}$





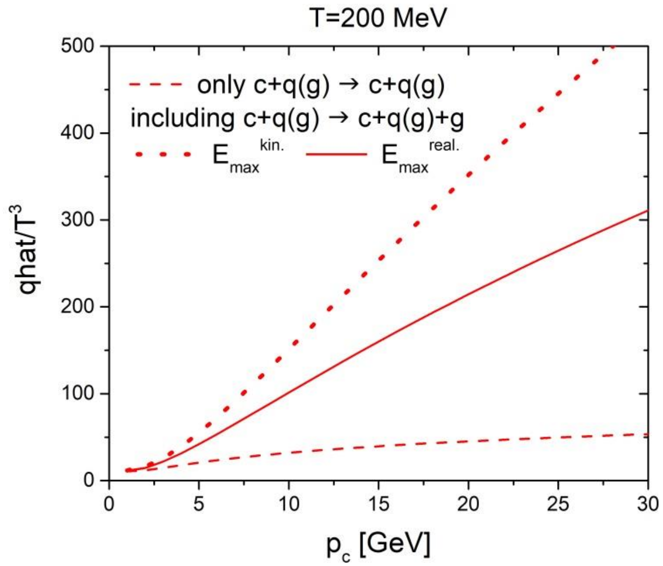
Massive gluon cannot be produced in low energy scattering

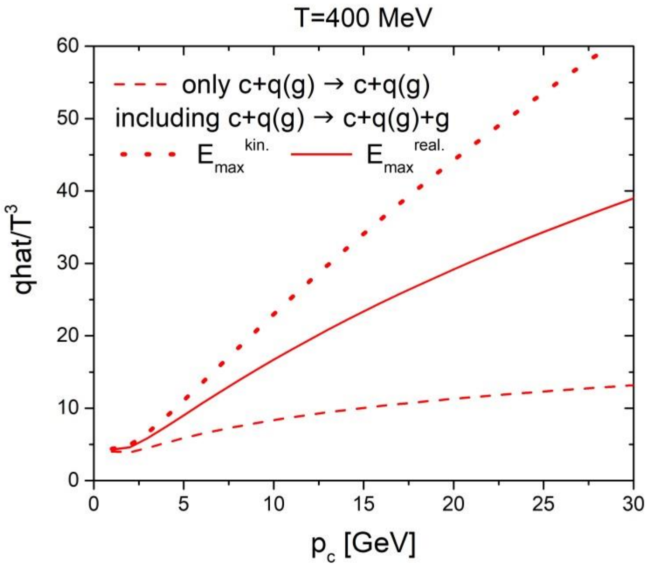


Spatial diffusion coefficient Ds

$$D_s = \lim_{p_c \to 0} \frac{T p_c}{m_c A}$$

qhat/T^3 of charm $\frac{d\langle(\Delta p_T)^2\rangle}{dz}$



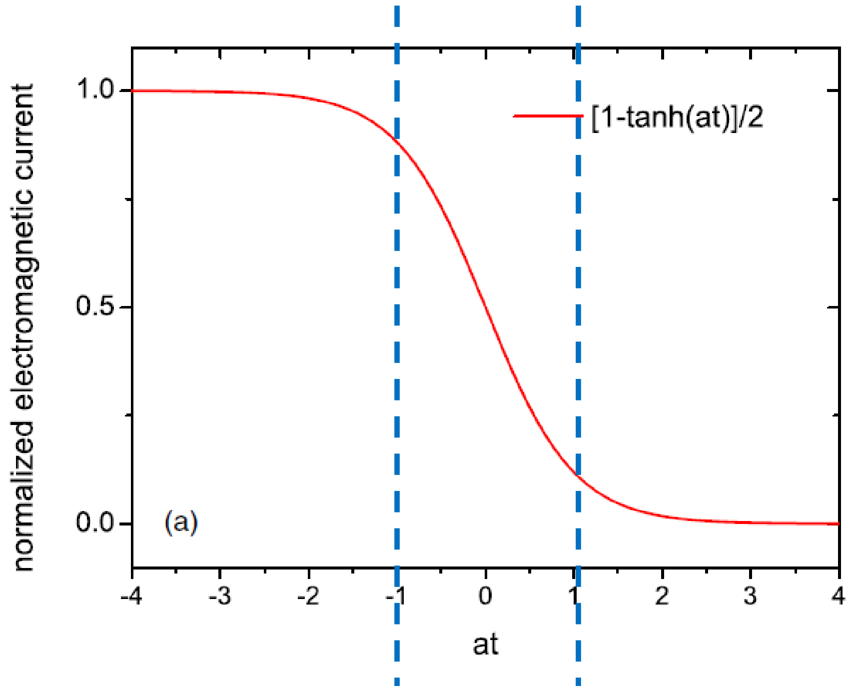


5. Summary

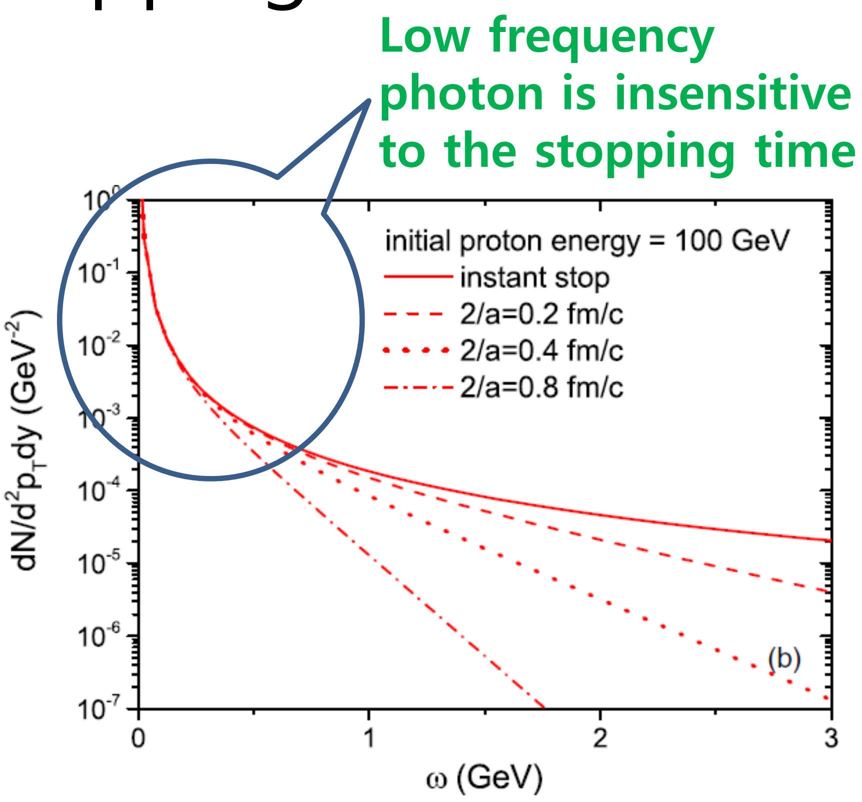
- We have applied the soft photon approximation (Low's theorem) to soft gluon emission from heavy quark scattering.
- It makes the scattering amplitude (or cross section) separable into the elastic scattering of heavy quark and soft gluon emission.
- We have found that the approximation satisfies Slavnov-Taylor identity (Ward identity in QED)
- The soft gluon approximation is applied to sQGP within the Dynamical Quasi-Particle Model (DQPM) which is fitted to the lattice EoS.
- Taking $\sqrt{-t}$ for the upper limit of emitted gluon energy in the soft gluon approximation, **heavy quark transport coefficients little change at low heavy quark momentum but enhance at large heavy quark momentum.**

Thank you for your attention!

Bremsstrahlung photon from charged particle stopping



Stopping time, 2/a



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Low frequency photon cannot provide the details of the stopping