Quarkonium transport in strongly coupled plasmas

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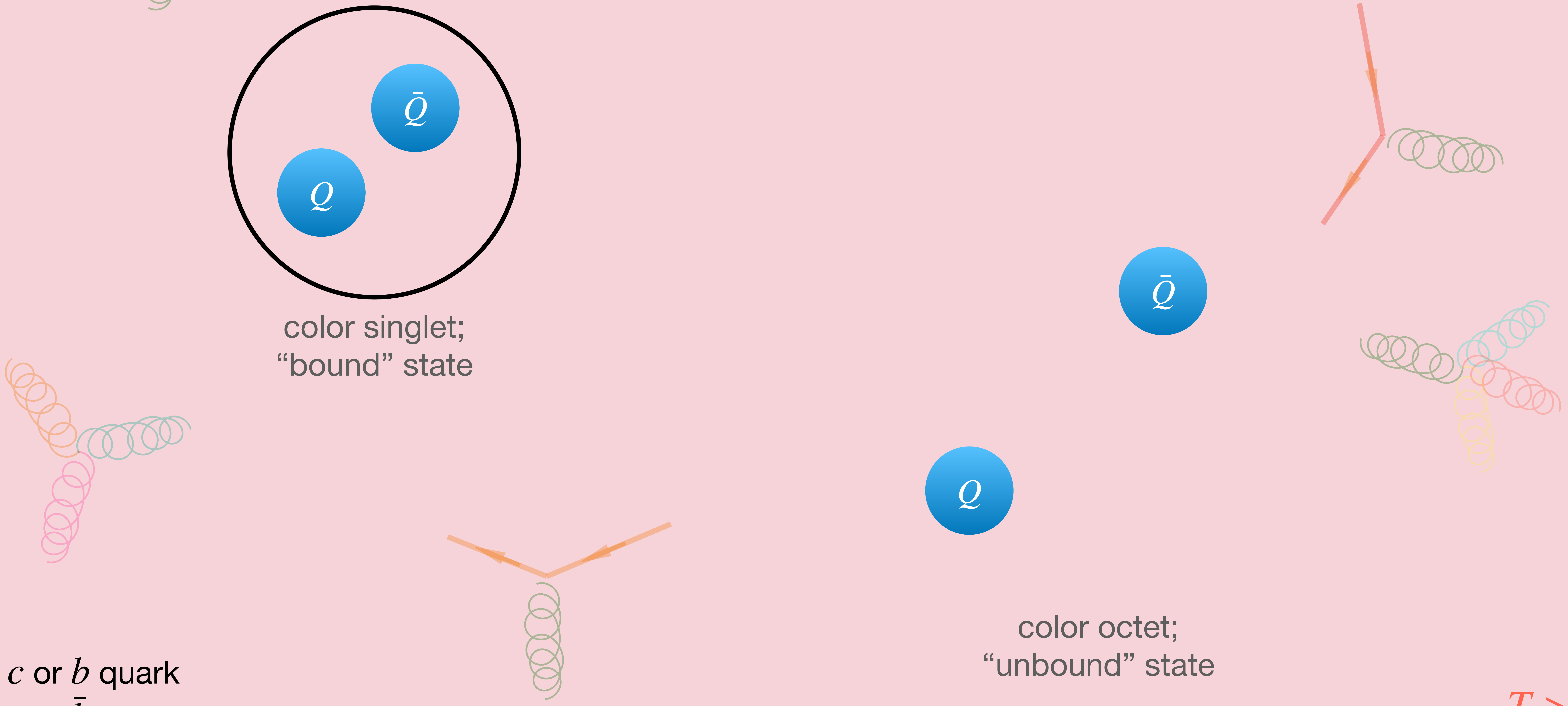
with Xiaojun Yao (UW) and Govert Nijs (MIT)
based on 2107.03945, 2205.04477, 2304.XXXXX

Quarkonium in medium

$M \gg Mv \gg Mv^2$

M: heavy quark mass

v: typical relative speed



Q: c or b quark

$\bar{Q}$: $\bar{c}$ or $\bar{b}$ quark

$T > 0$

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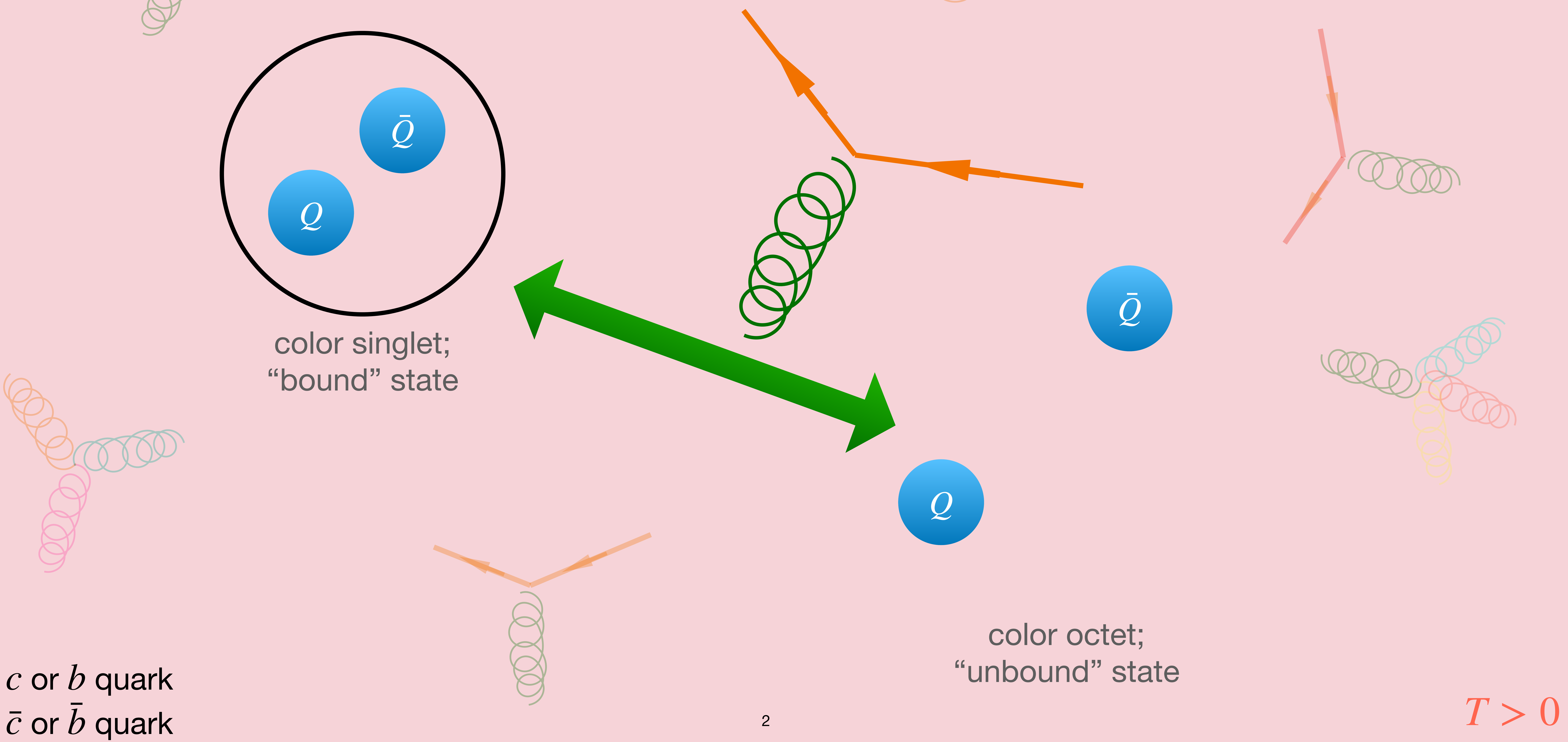

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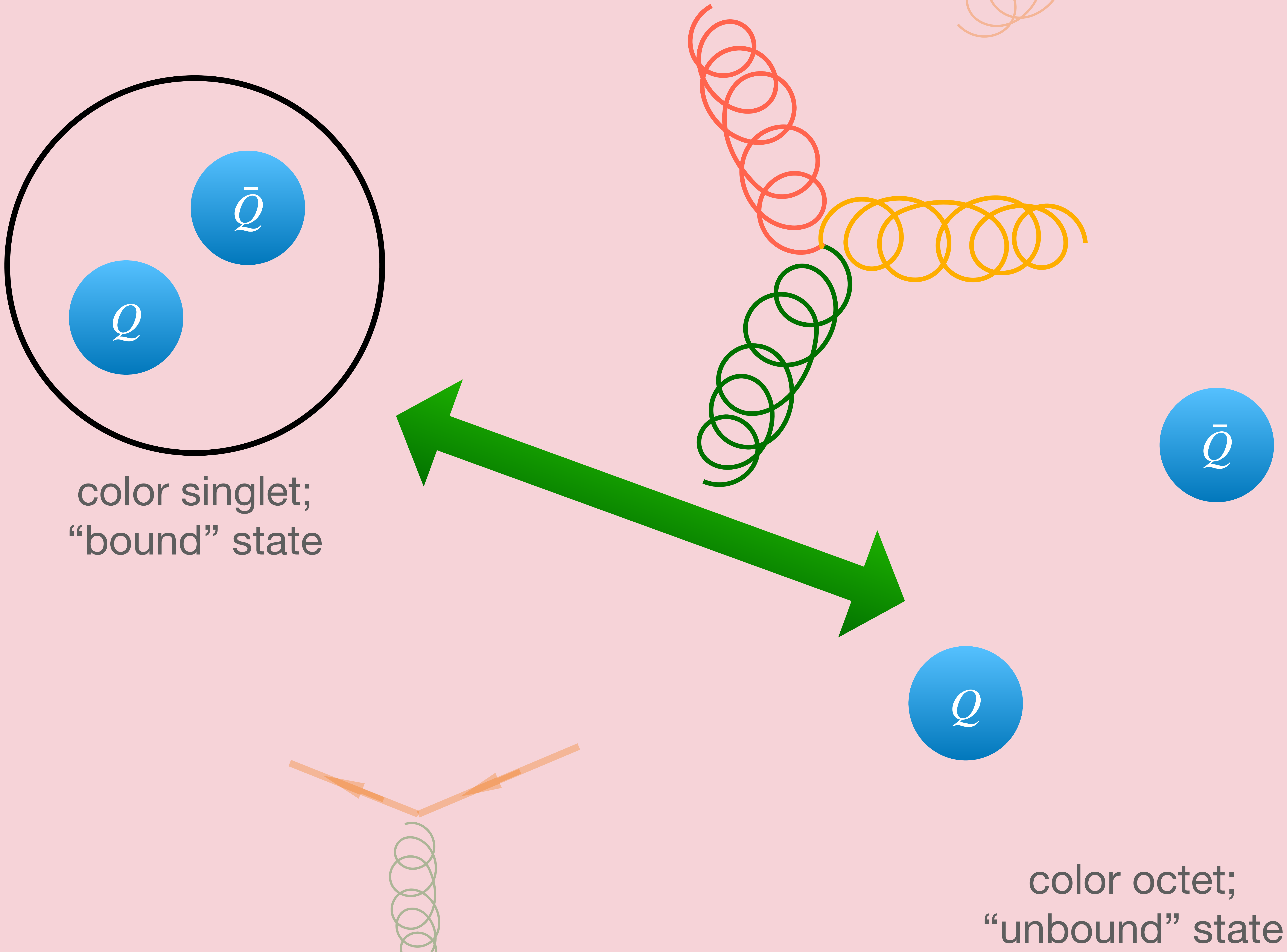

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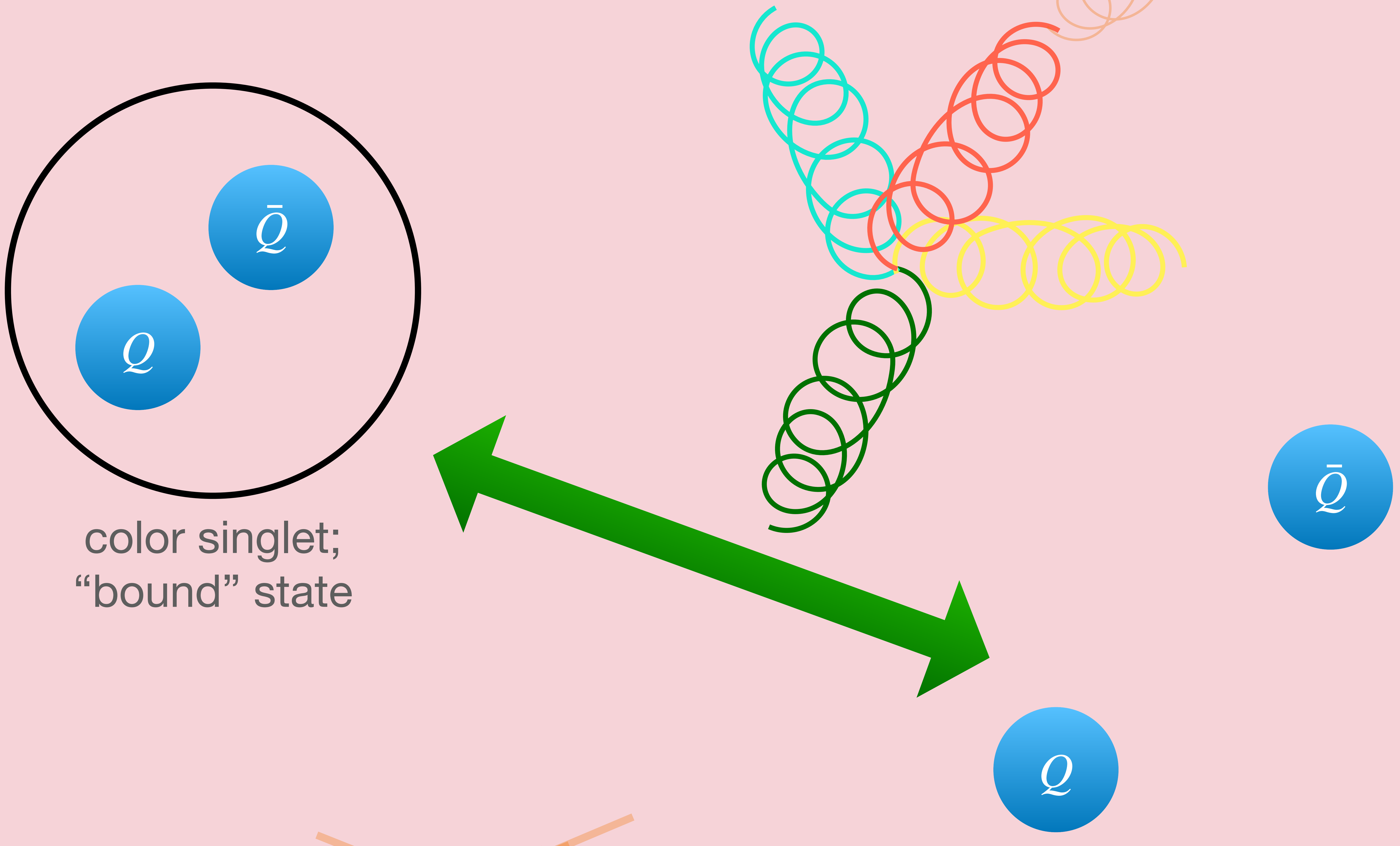

color singlet;
"bound" state

color octet;
"unbound" state

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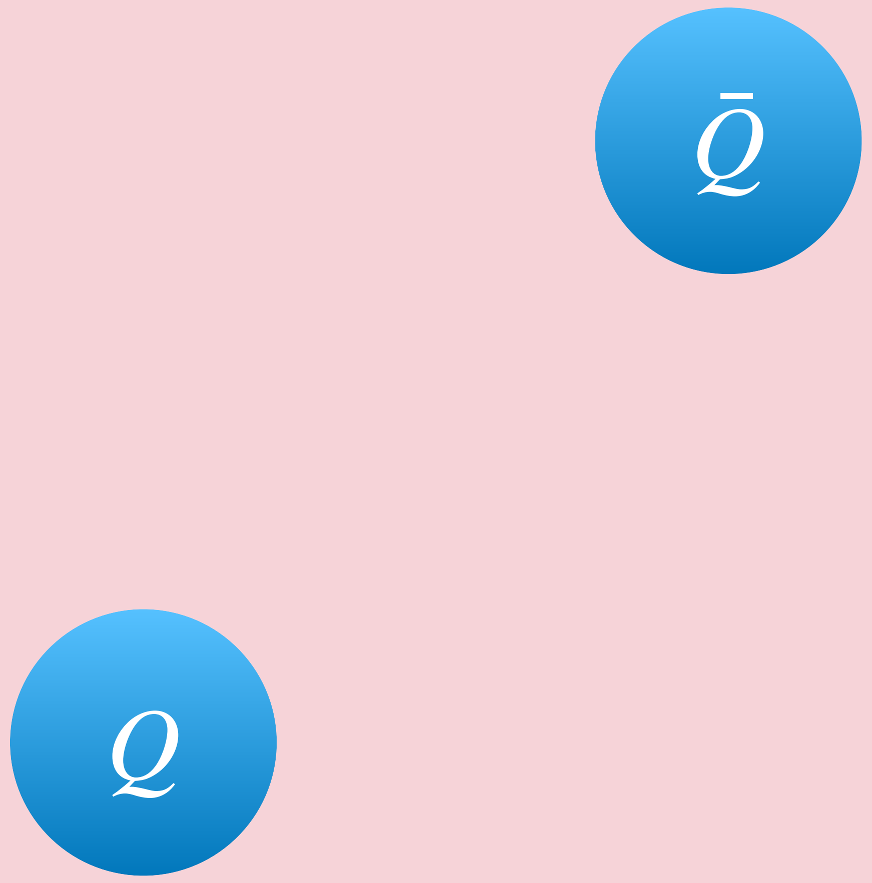
Quarkonium in medium

At high T, quarkonium "melts" because the medium screens the interactions between heavy quarks (Matsui & Satz 1986)

$$Q\bar{Q} \text{ melts if } r \sim \frac{1}{Mv} \gg \frac{1}{T}$$



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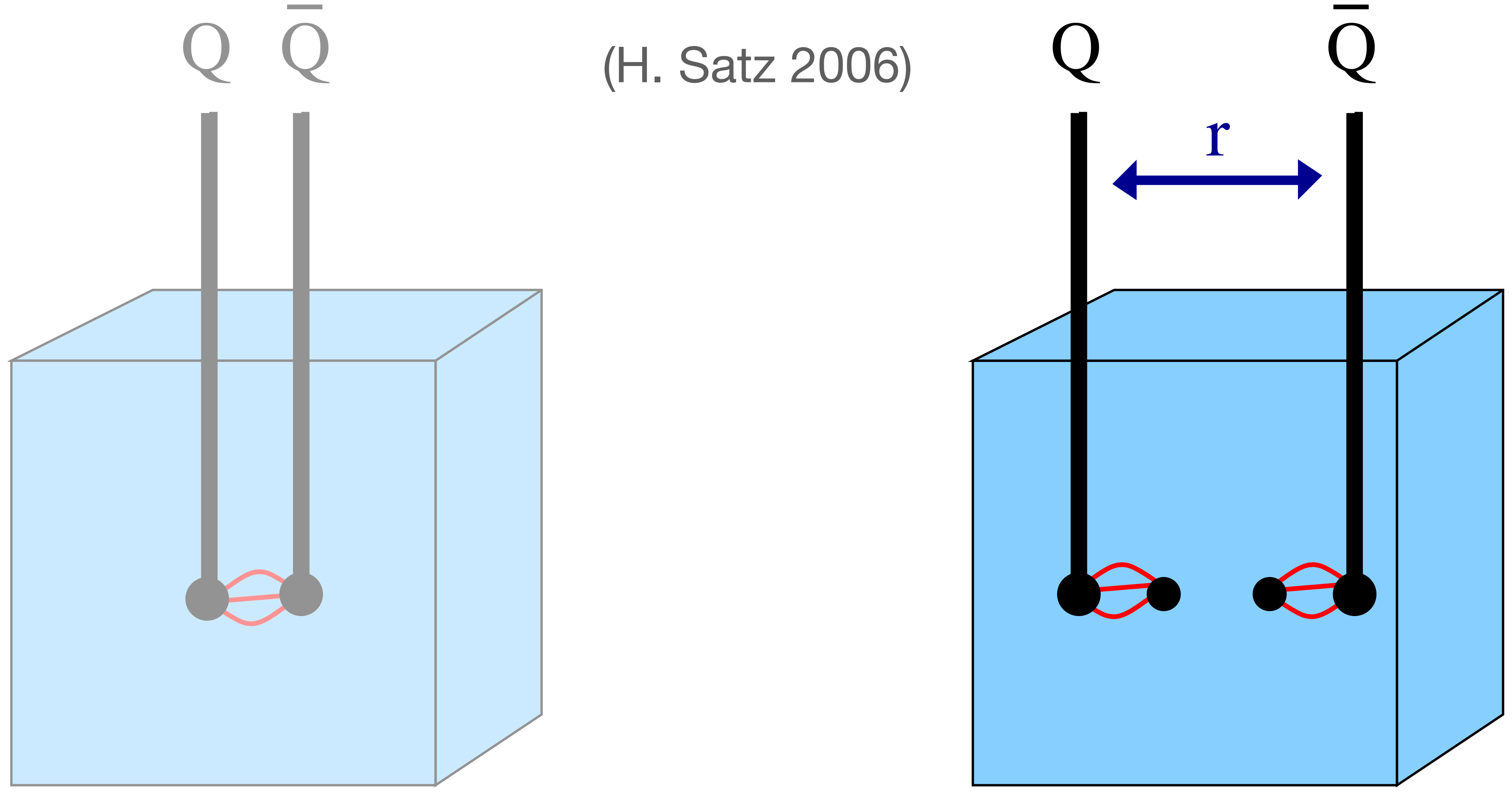

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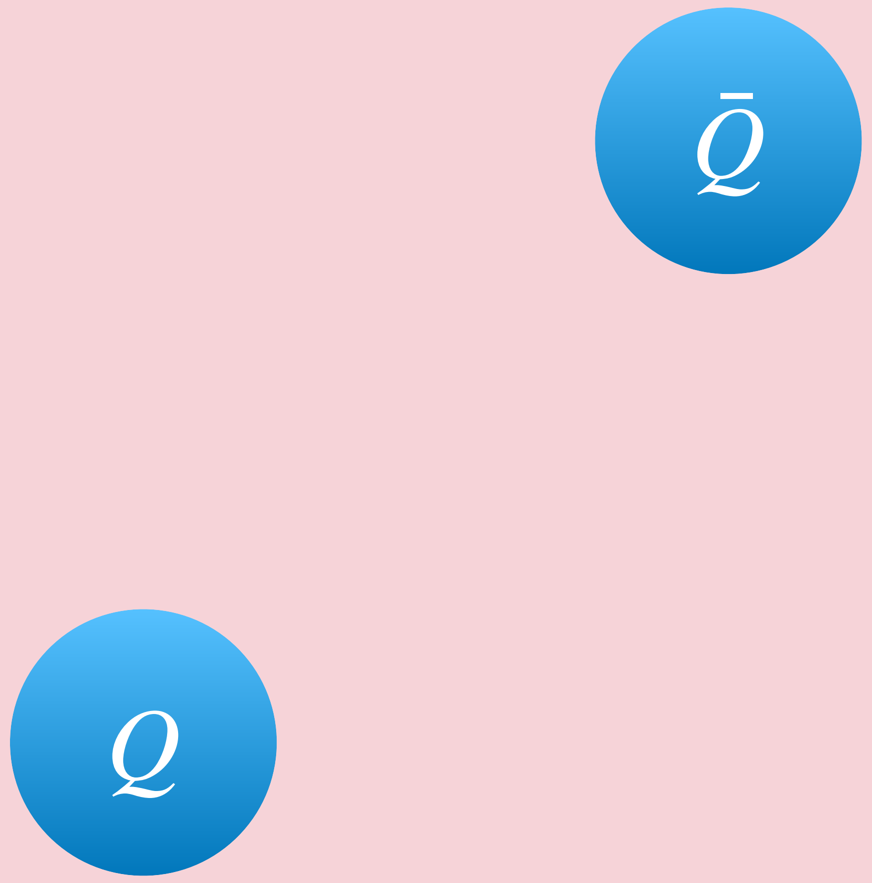

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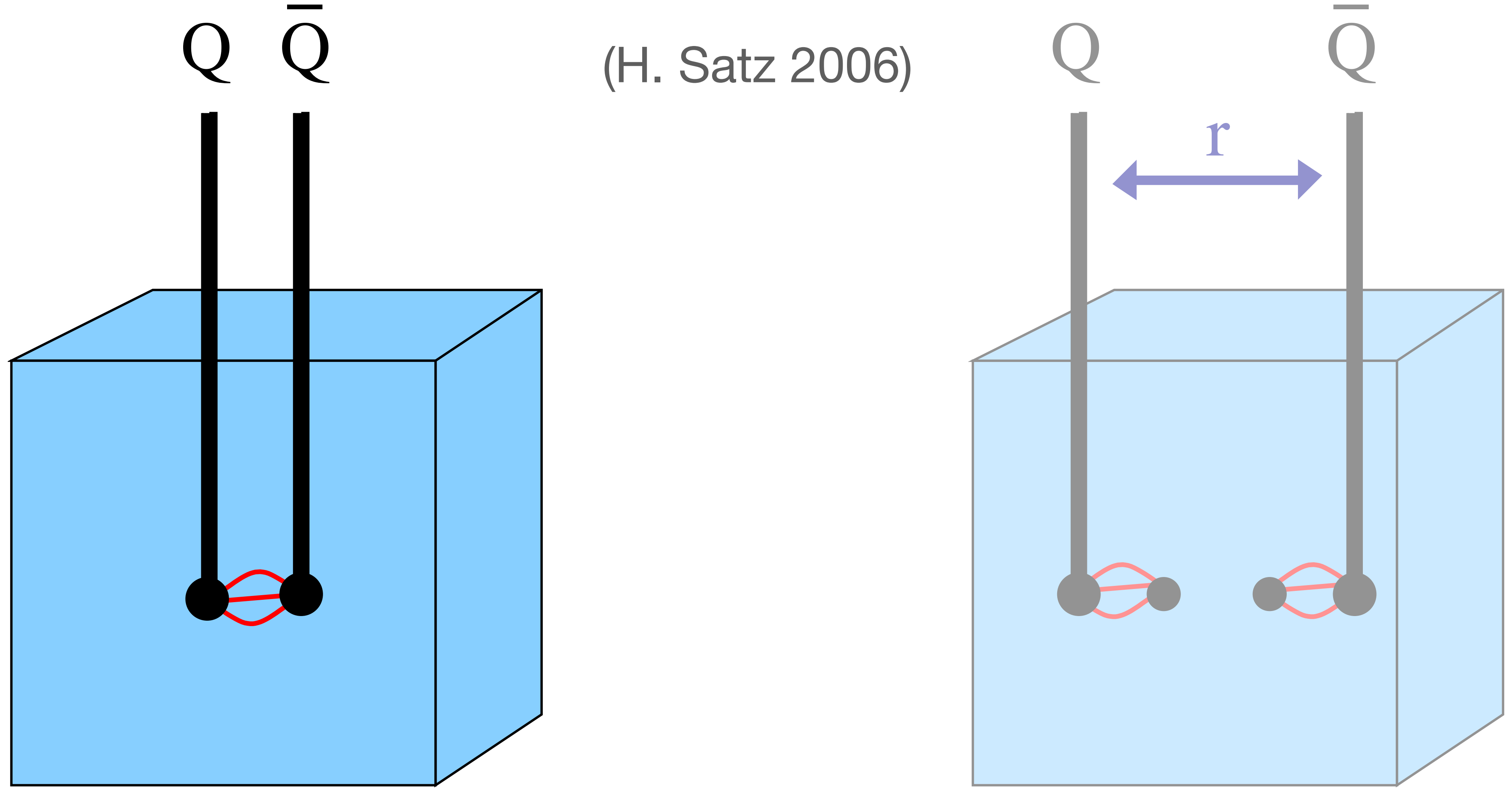
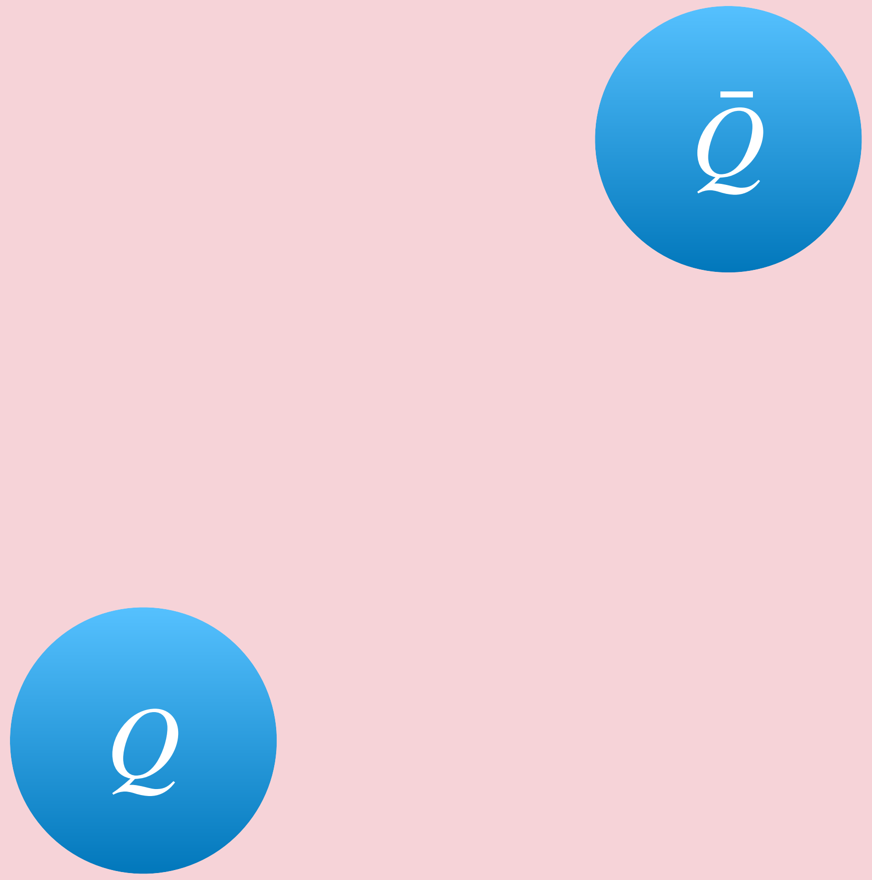
(H. Satz 2006)



$Q\bar{Q}$ can bind if $r \sim \dfrac{1}{Mv} < \dfrac{1}{T}$



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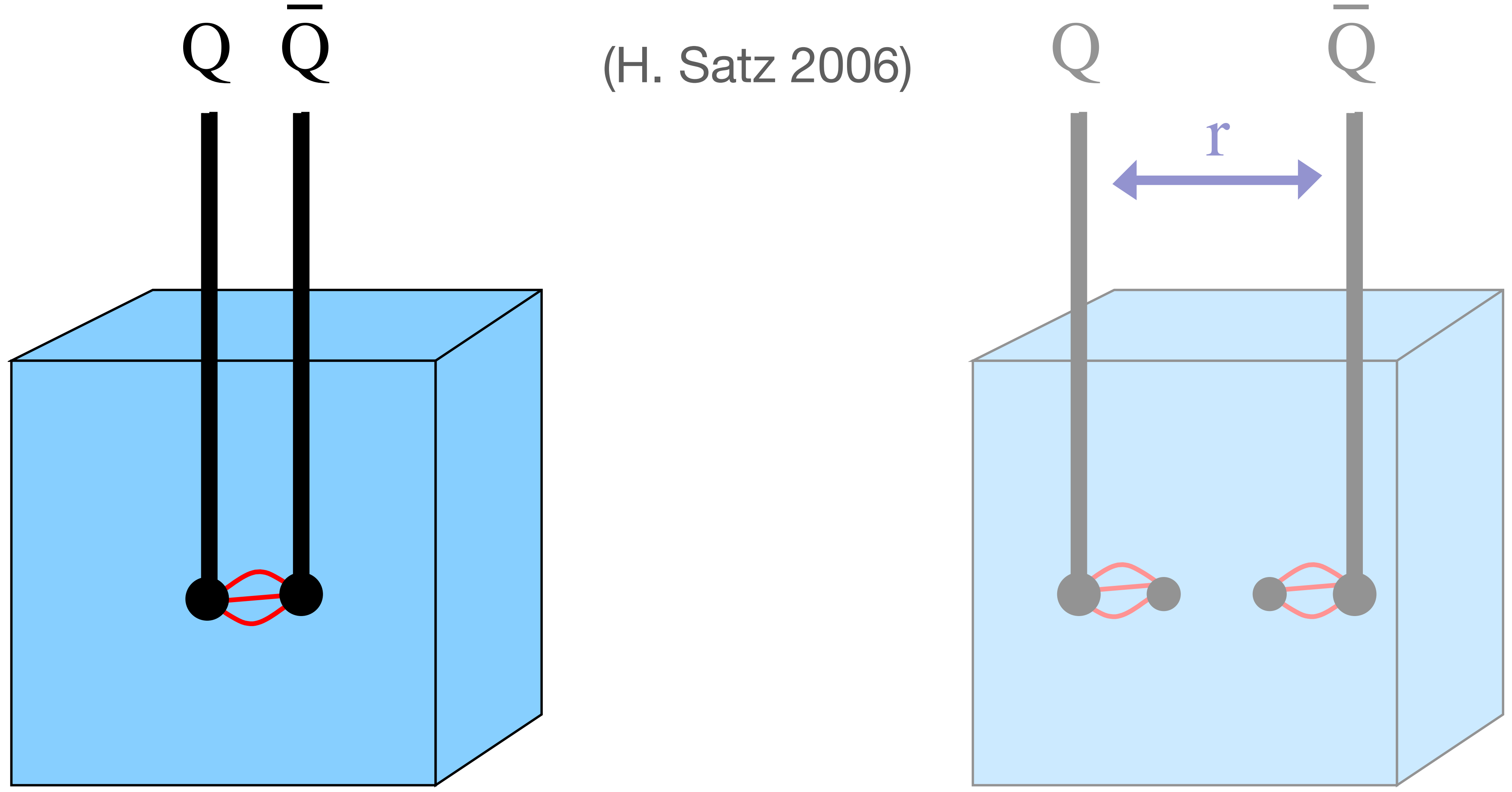
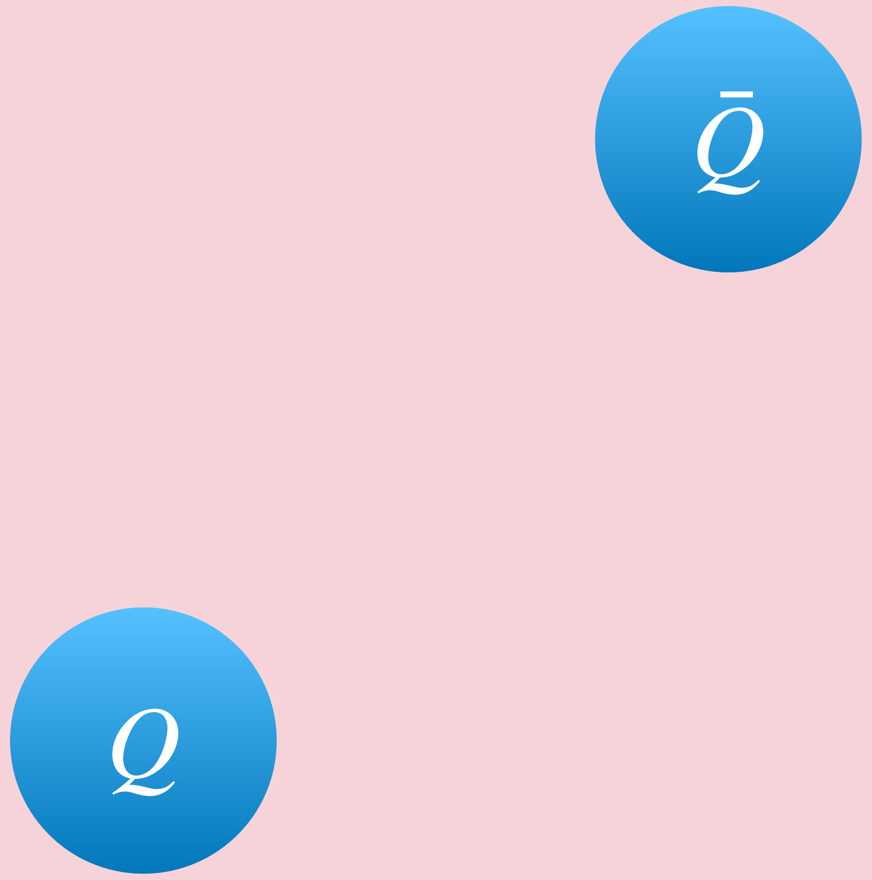


$Q\bar{Q}$ can bind if $r \sim \dfrac{1}{Mv} < \dfrac{1}{T}$

$\Longrightarrow$ most of quarkonium starts to form when $Mv \gtrsim T$



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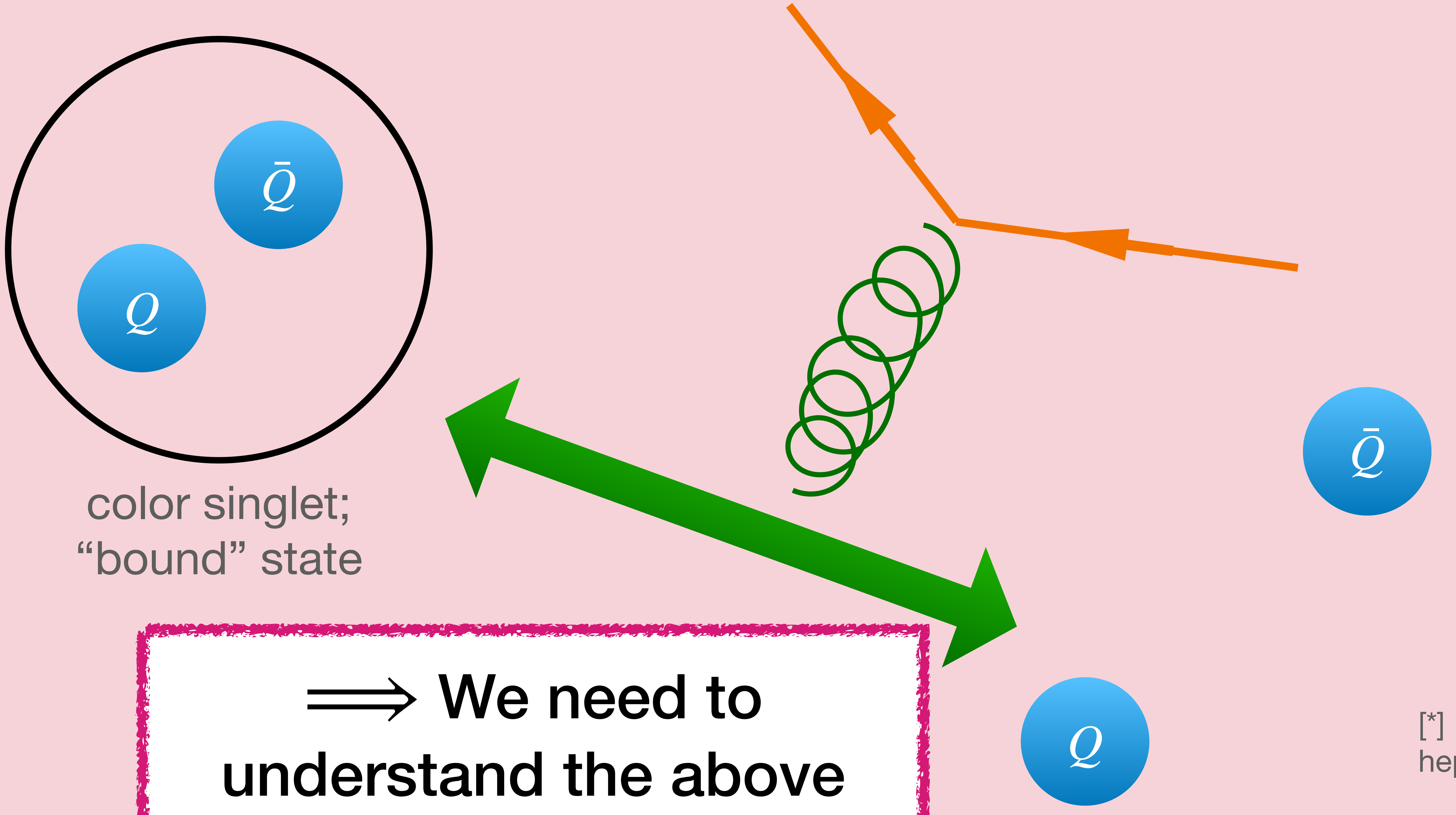

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$\Longrightarrow$ We need to understand the above dynamics in the hierarchy

$$Mv \gg T$$

$\Longrightarrow$ pNRQCD [*]

[*] N. Brambilla, A. Pineda, J. Soto. A. Vairo
hep-ph/9907240, hep-ph/0410047

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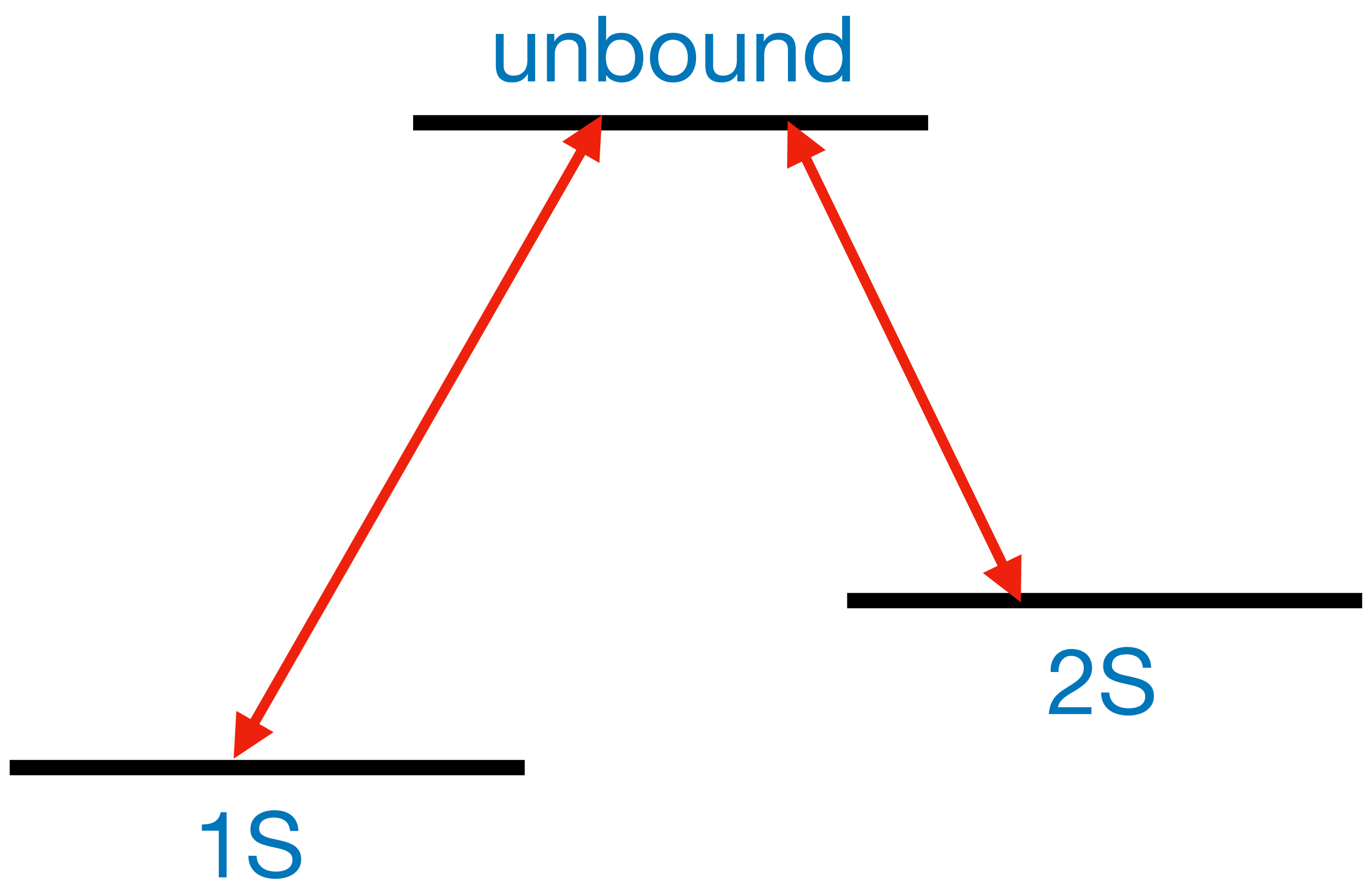
X. Yao, hep-ph/2102.01736

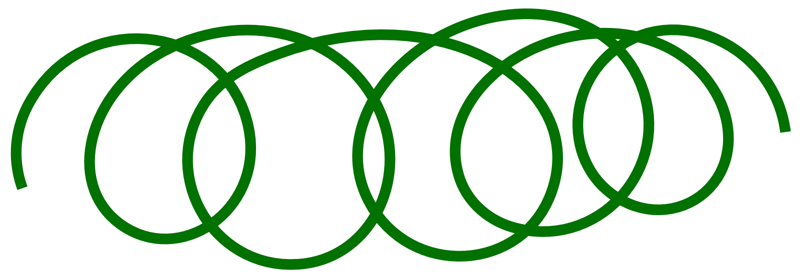
Quarkonium as an open quantum system

Transitions between quarkonium energy levels (the system)

Interaction with the environment

QGP (the environment)





$$\frac{1}{\tau_S} \sim \Delta E_n \sim Mv^2$$

$$\frac{1}{\tau_I} \sim \frac{H_{\rm int}^2}{T} \sim T\frac{T^2}{(Mv)^2}$$

$$\frac{1}{\tau_E} \sim T$$

$$\mathscr{L}_{\rm pNRQCD} = \mathscr{L}_{\rm light\,quarks} + \mathscr{L}_{\rm gluon} + \int d^3r \mathrm{Tr}_{\rm color}\Big[S^\dagger(i\partial_0 - H_s)S + O^\dagger(iD_0 - H_o)O$$

$$+V_A\left(O^\dagger \mathbf{r}\cdot g\mathbf{E}S + \mathrm{h.c.}\right) + \frac{V_B}{2}O^\dagger\{\mathbf{r}\cdot g\mathbf{E}, O\} + \cdots\Big]$$

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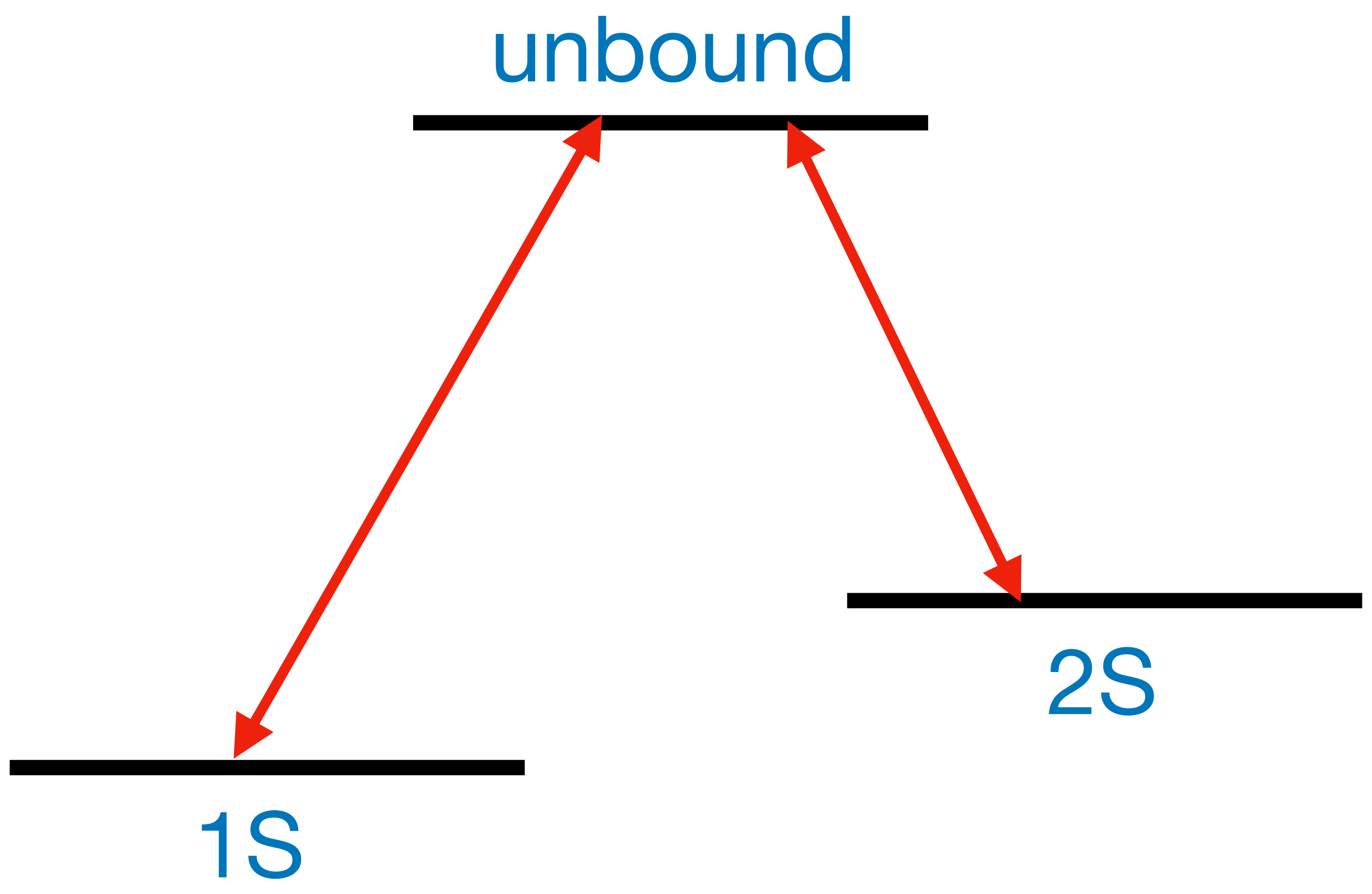

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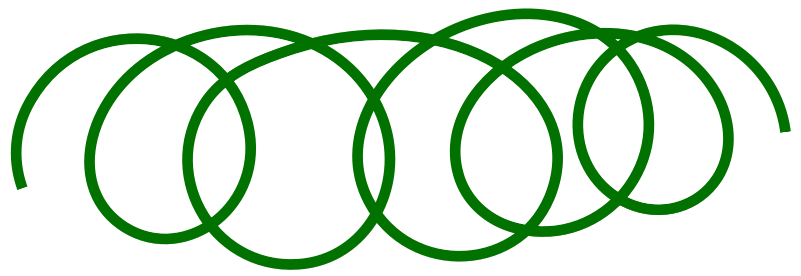

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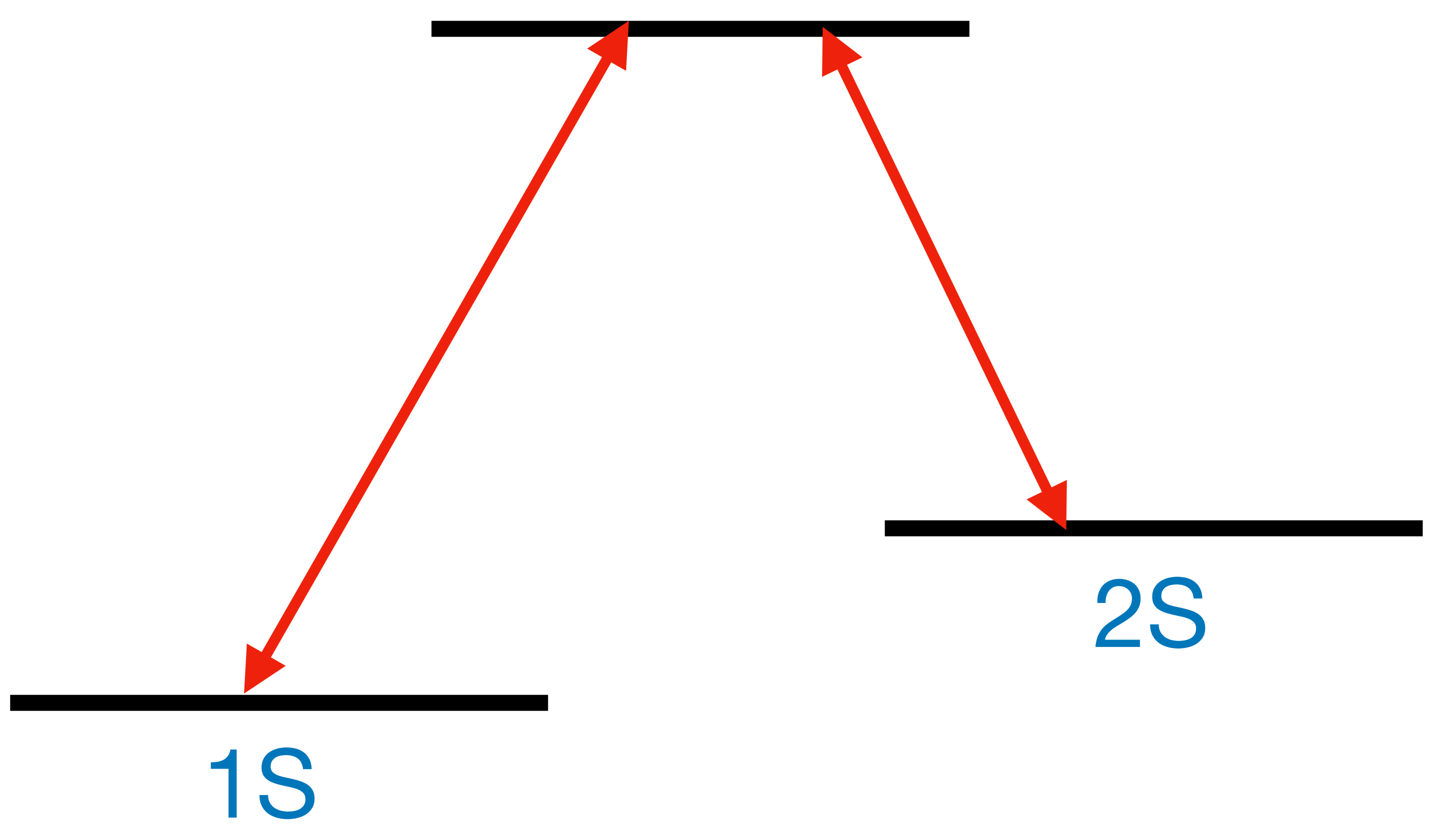

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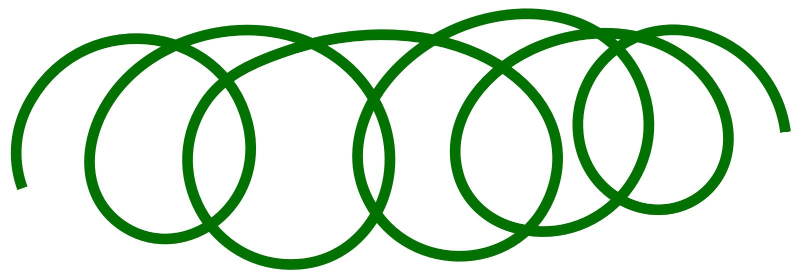

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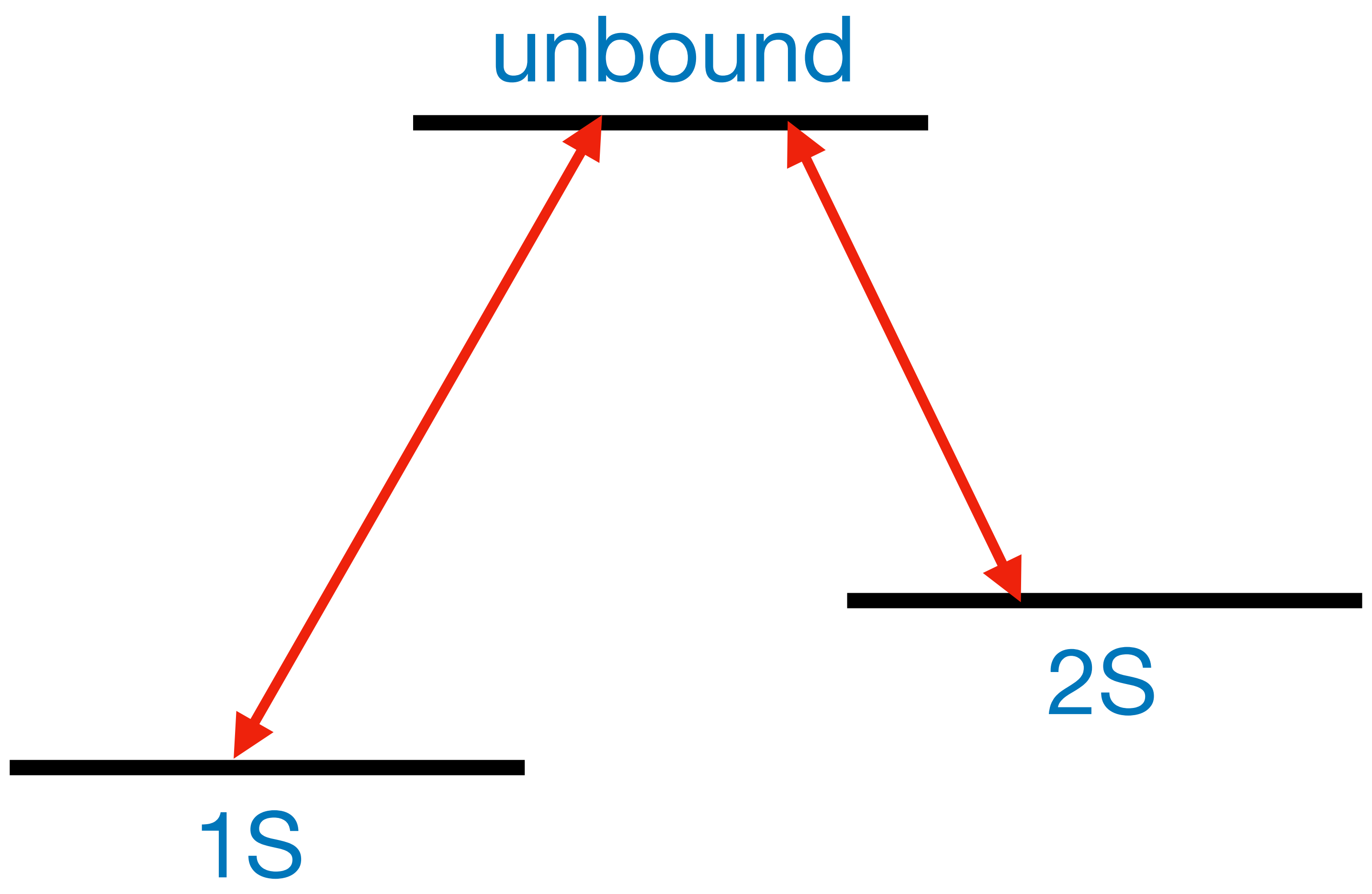

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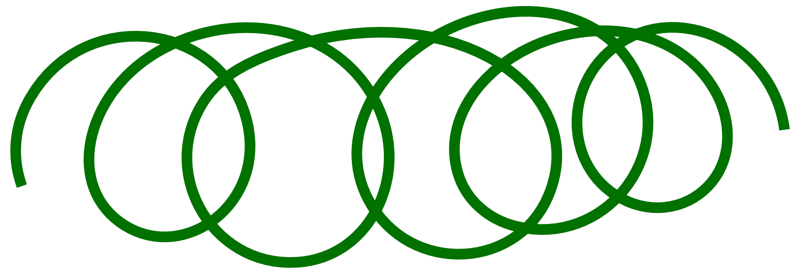

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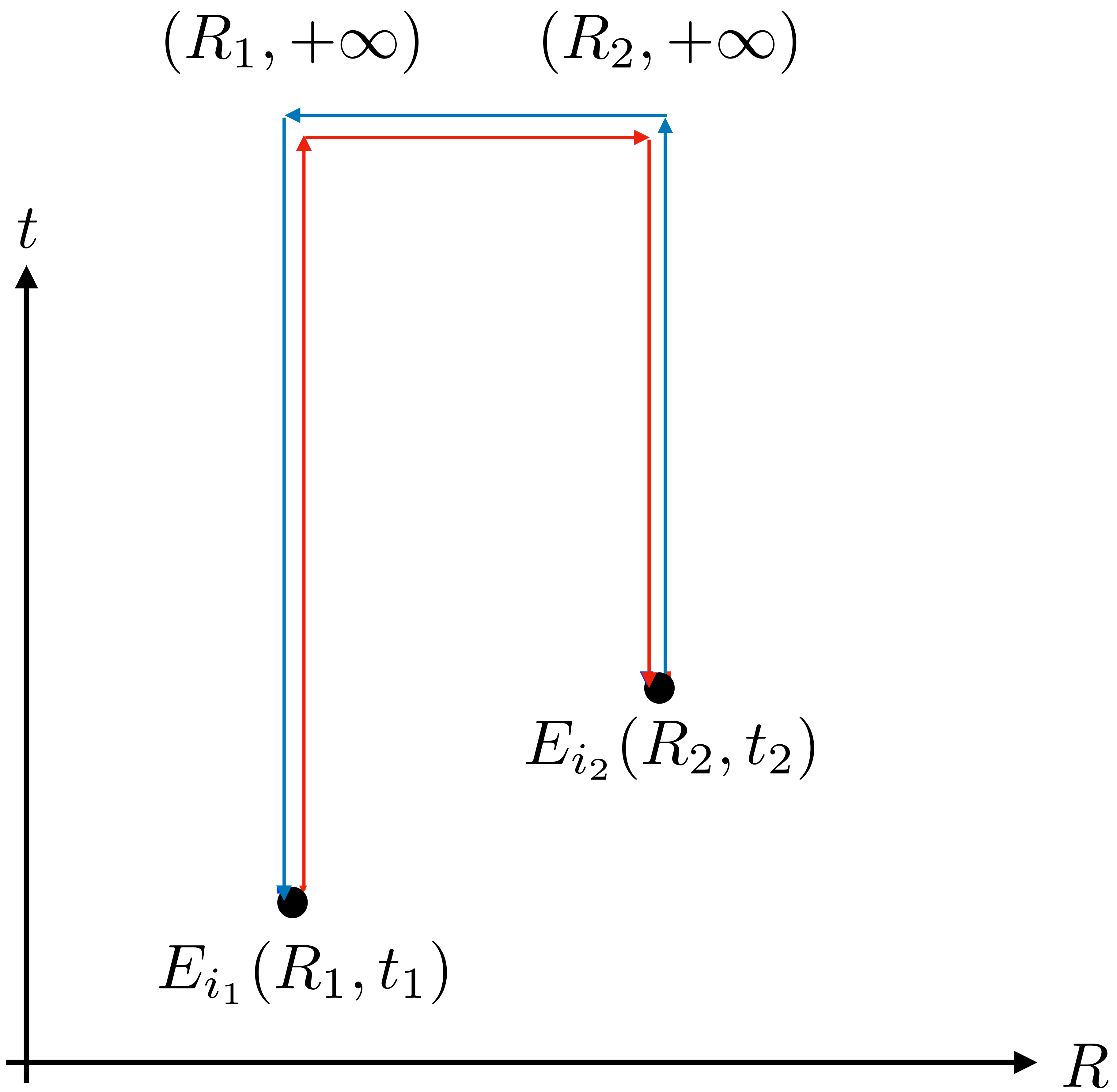
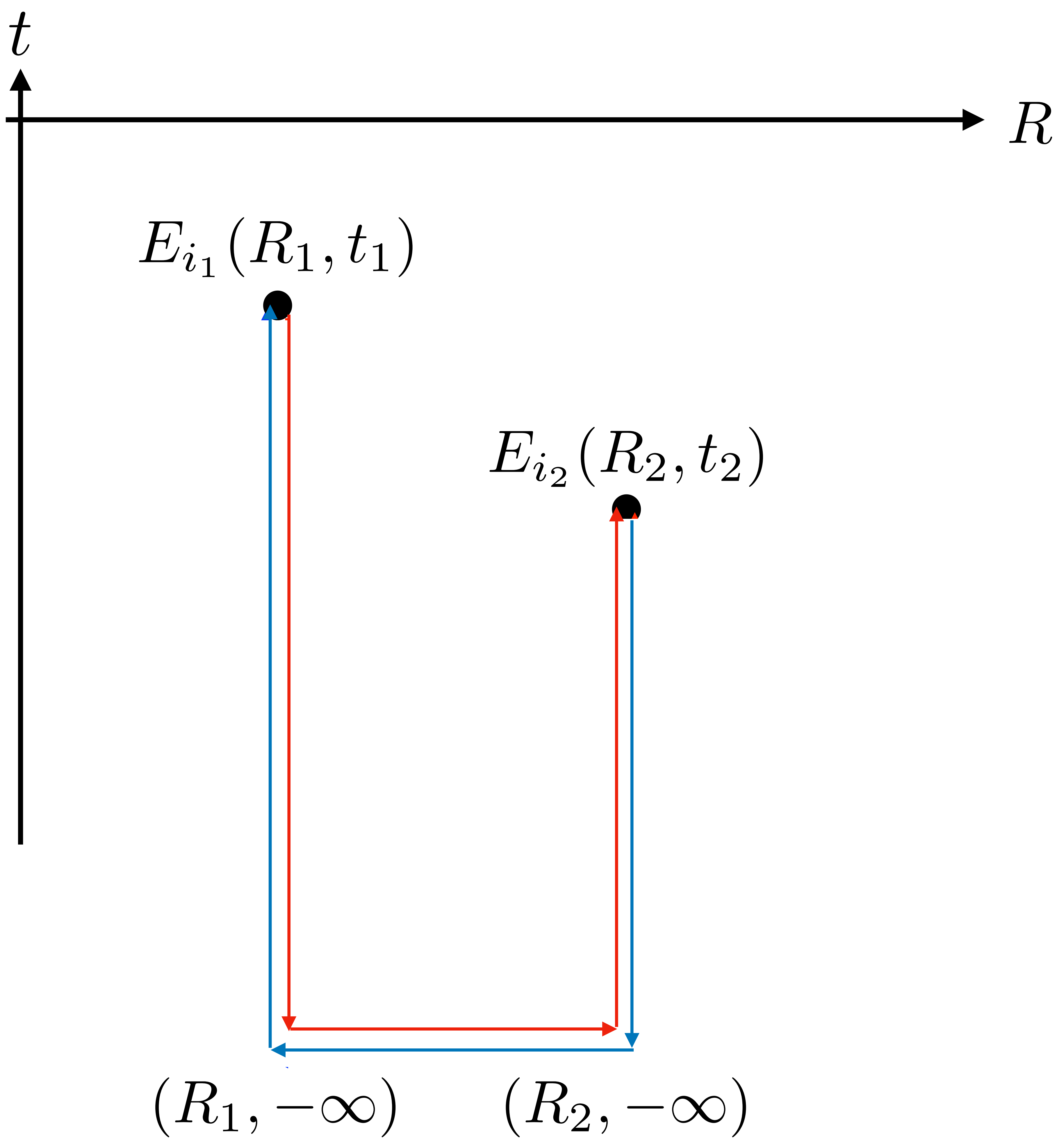
How does the QGP enter the dynamics?

QGP chromoelectric correlators

for quarkonia transport

$$[g_E^{--}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(\mathscr{W}_{2'} E_{i_2}(\mathbf{R}_2, t_2)\right)^a \left(E_{i_1}(\mathbf{R}_1, t_1)\mathscr{W}_{1'}\right)^a \right\rangle_T$$





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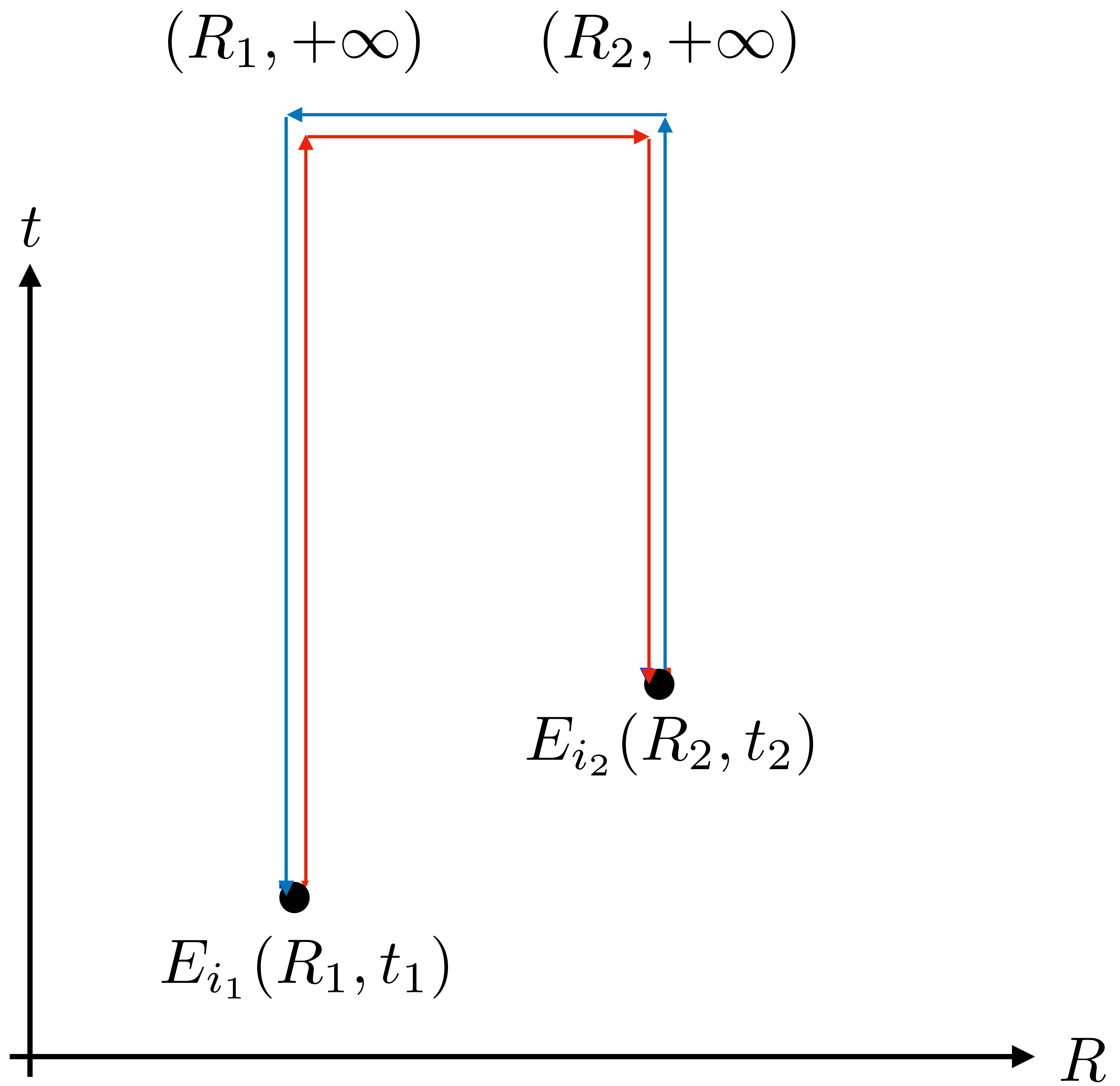

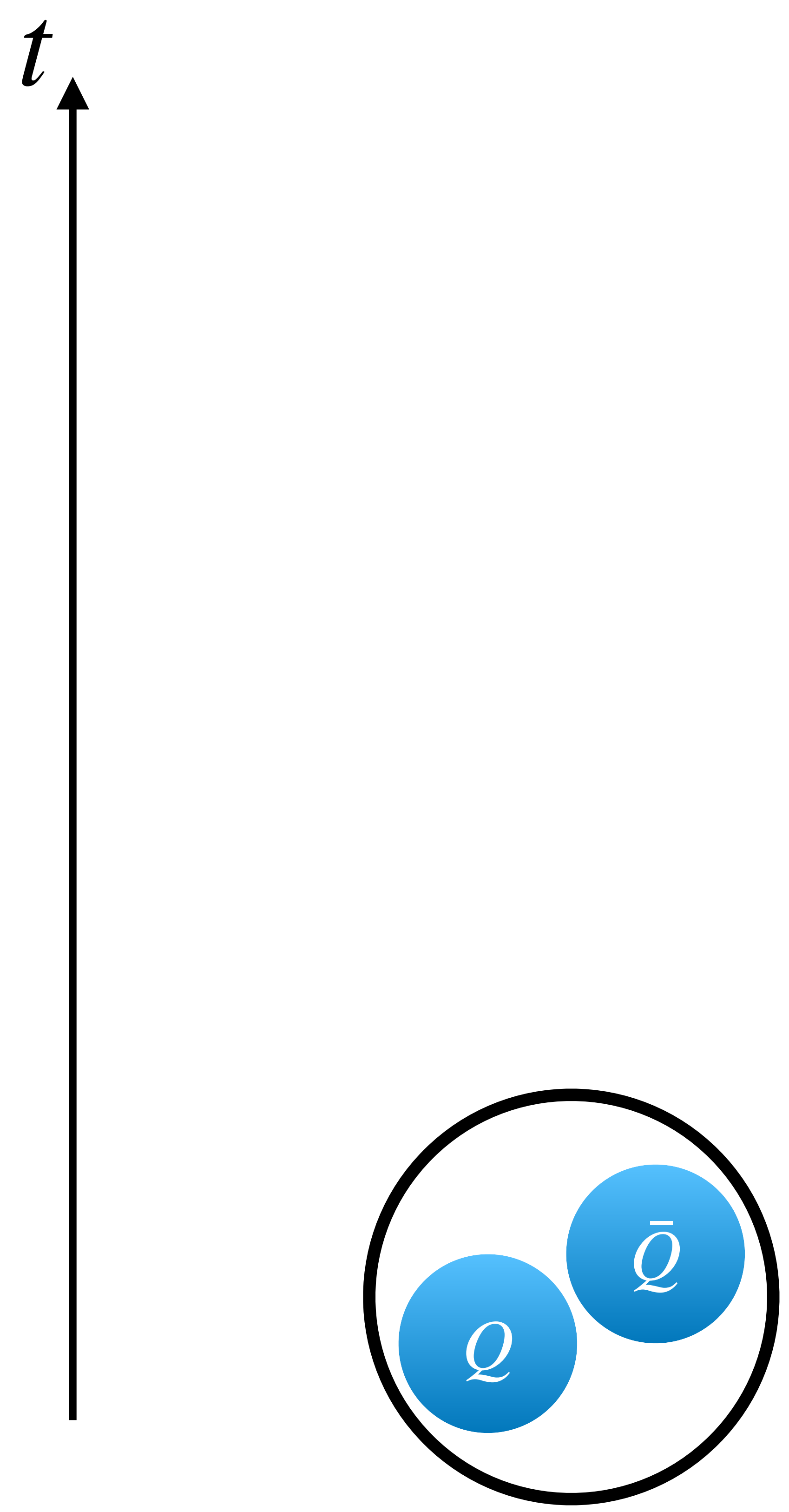

"bound" state:
color singlet

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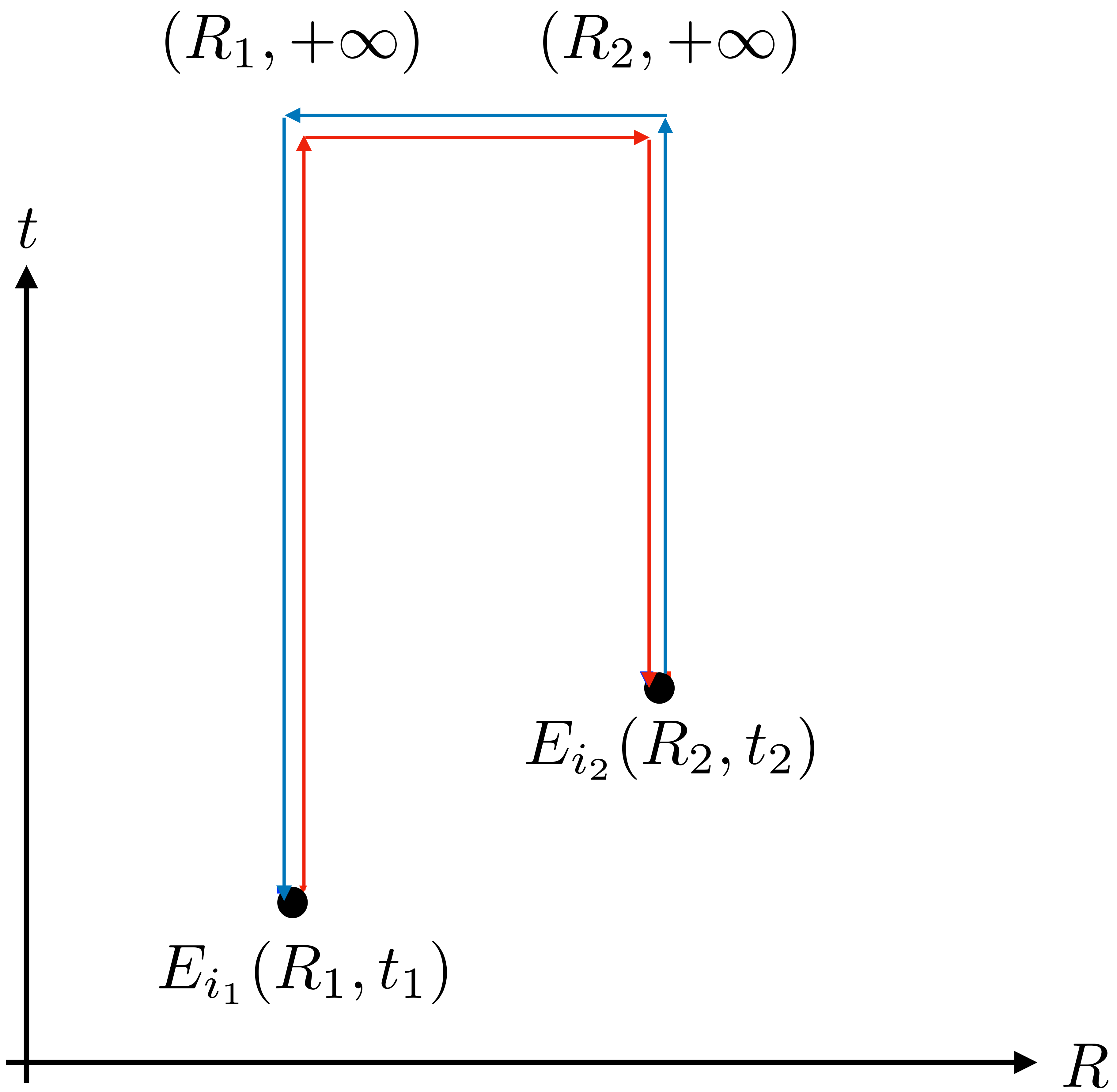

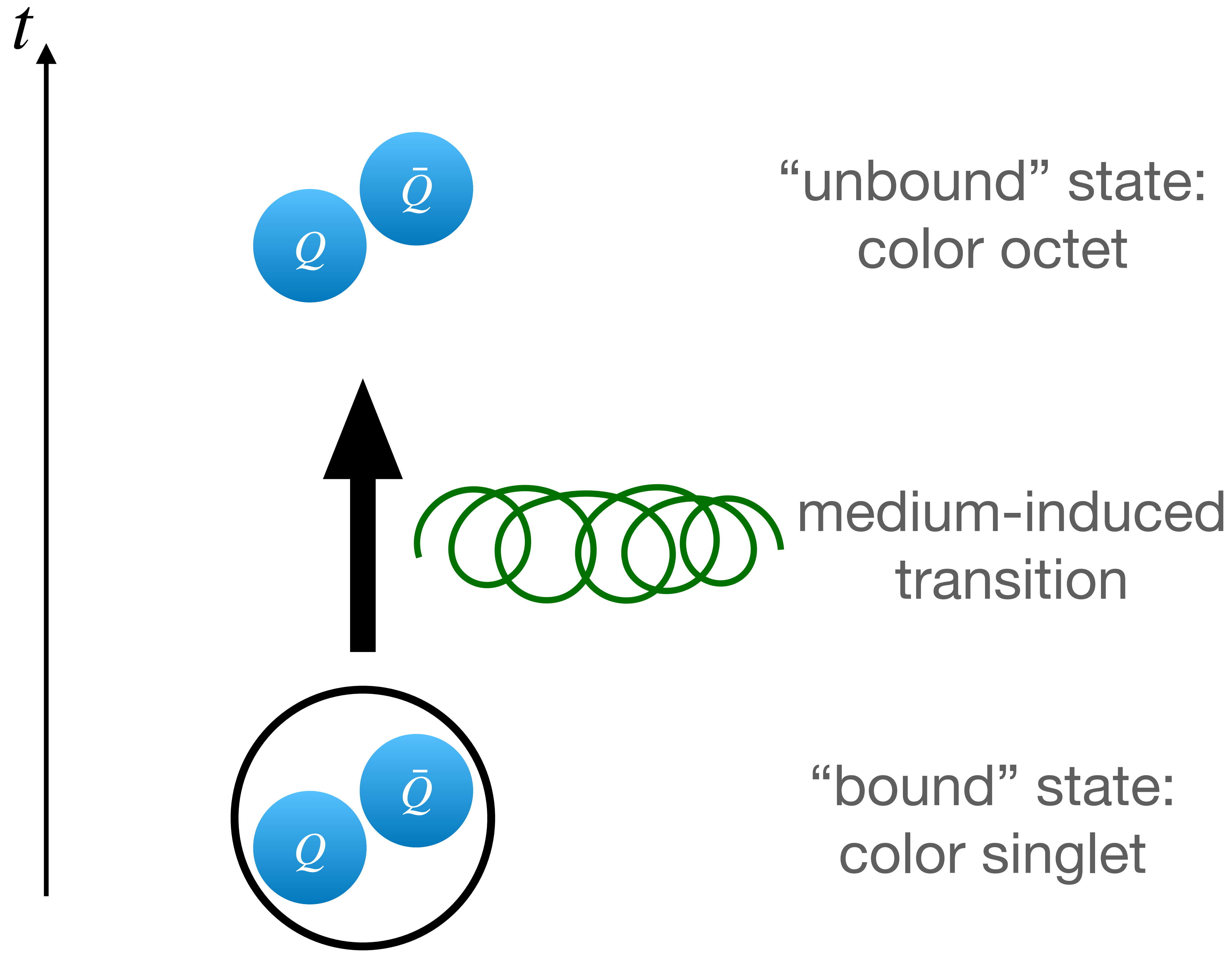

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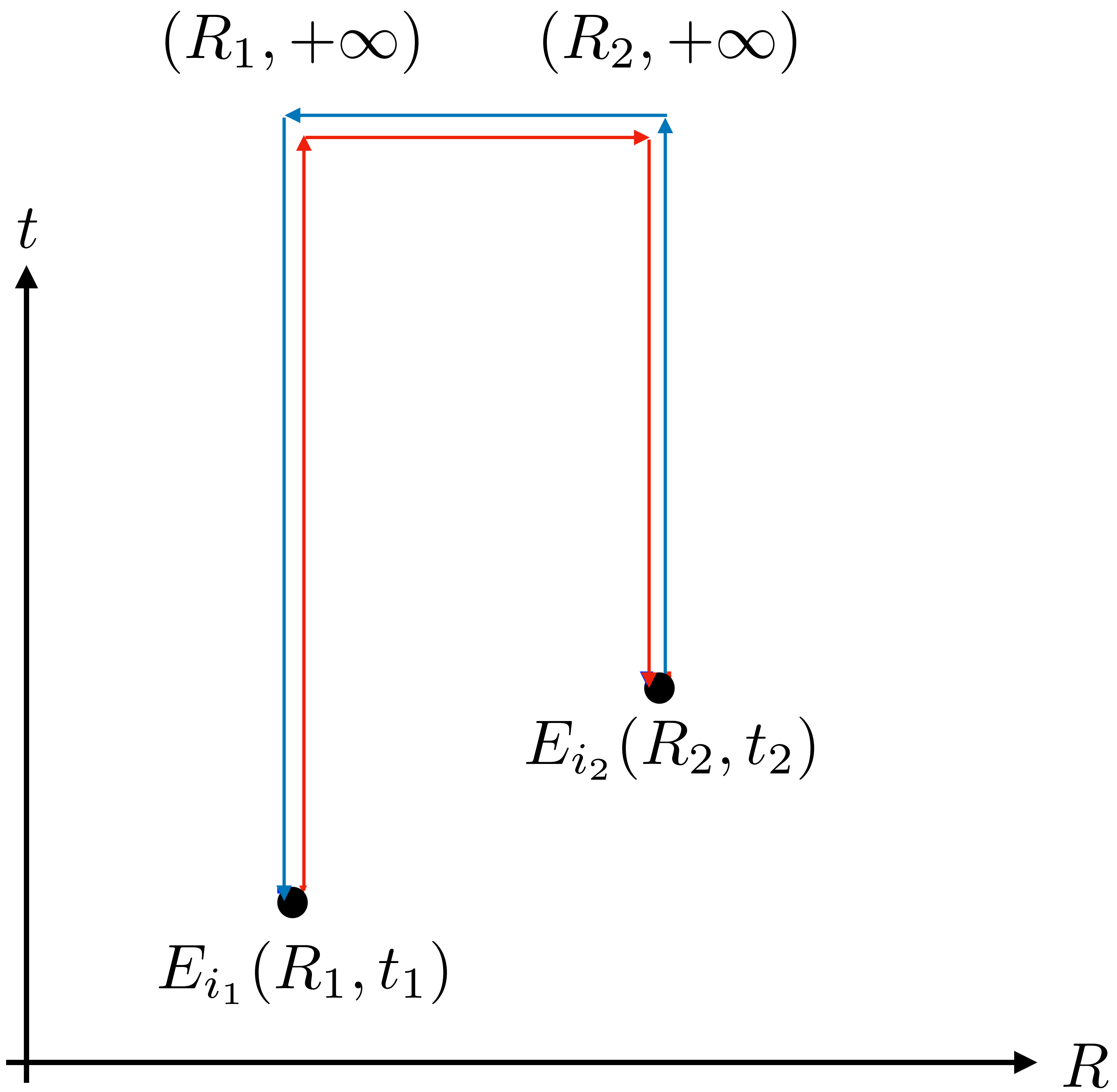

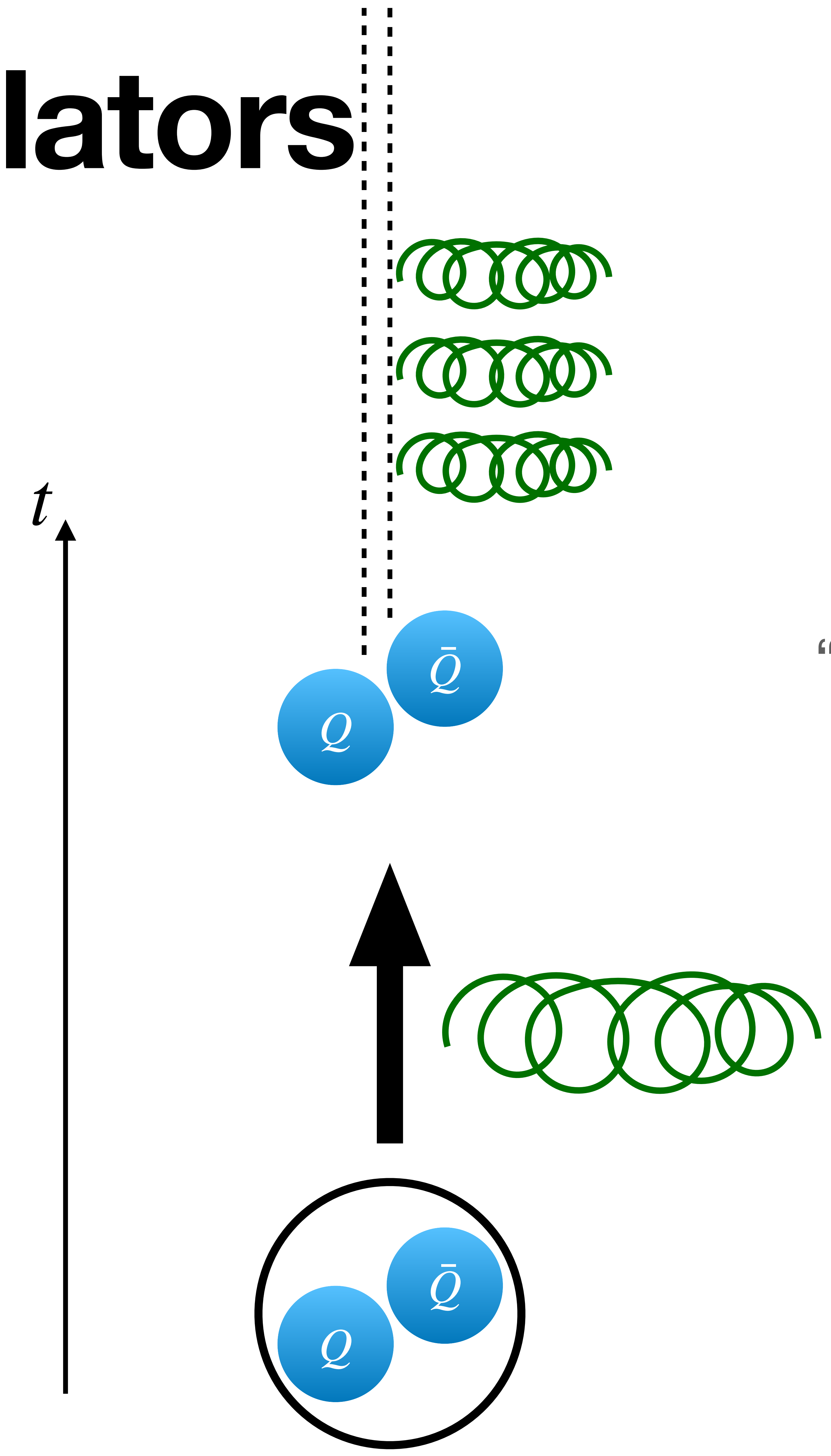

the “unbound” state carries color charge and interacts with the medium

“unbound” state: color octet

medium-induced transition

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See also: N. Brambilla et al. hep-ph/1612.07248, hep-ph/1711.04515, hep-ph/2205.10289

X. Yao and T. Mehen, hep-ph/2009.02408

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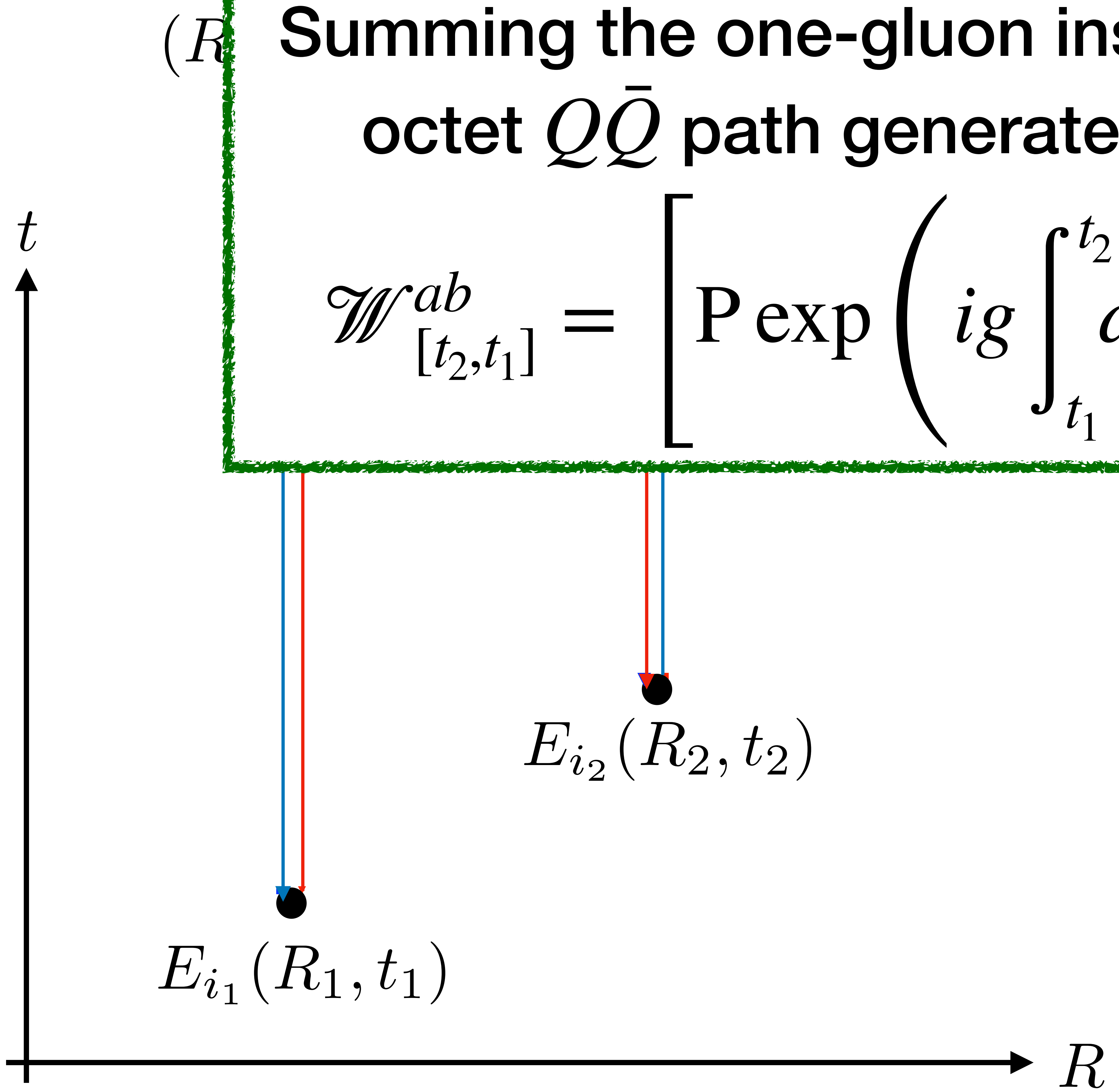
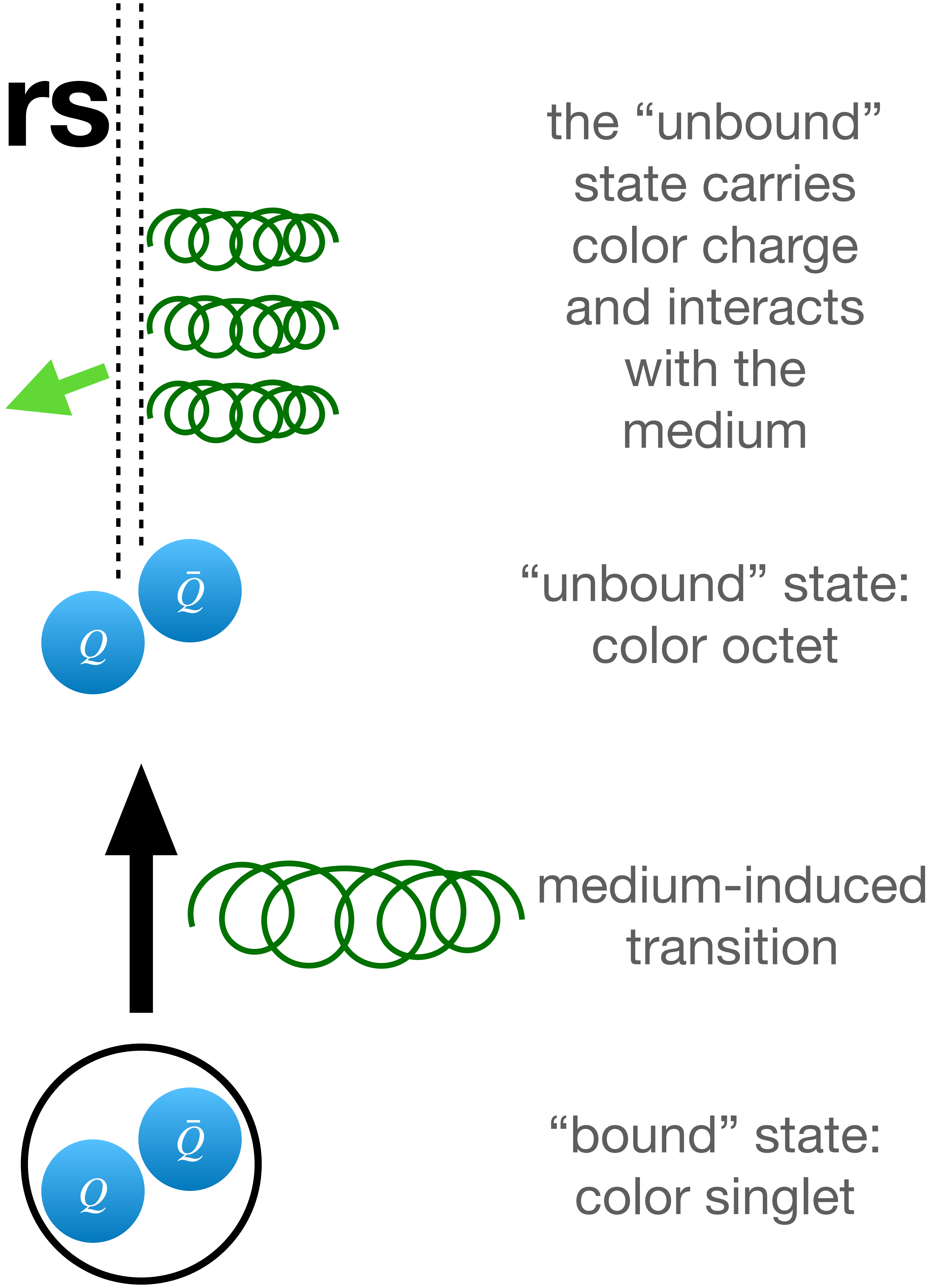
for quarkonia transport

Summing the one-gluon insertions along the octet $Q\bar{Q}$ path generates a Wilson line:

$$\mathscr{W}^{ab}_{[t_2,t_1]} = \left[\mathrm{P}\exp\left(ig\int_{t_1}^{t_2} dt\, A_0^c(t) T^c_{\mathrm{adj}} \right) \right]^{ab}$$





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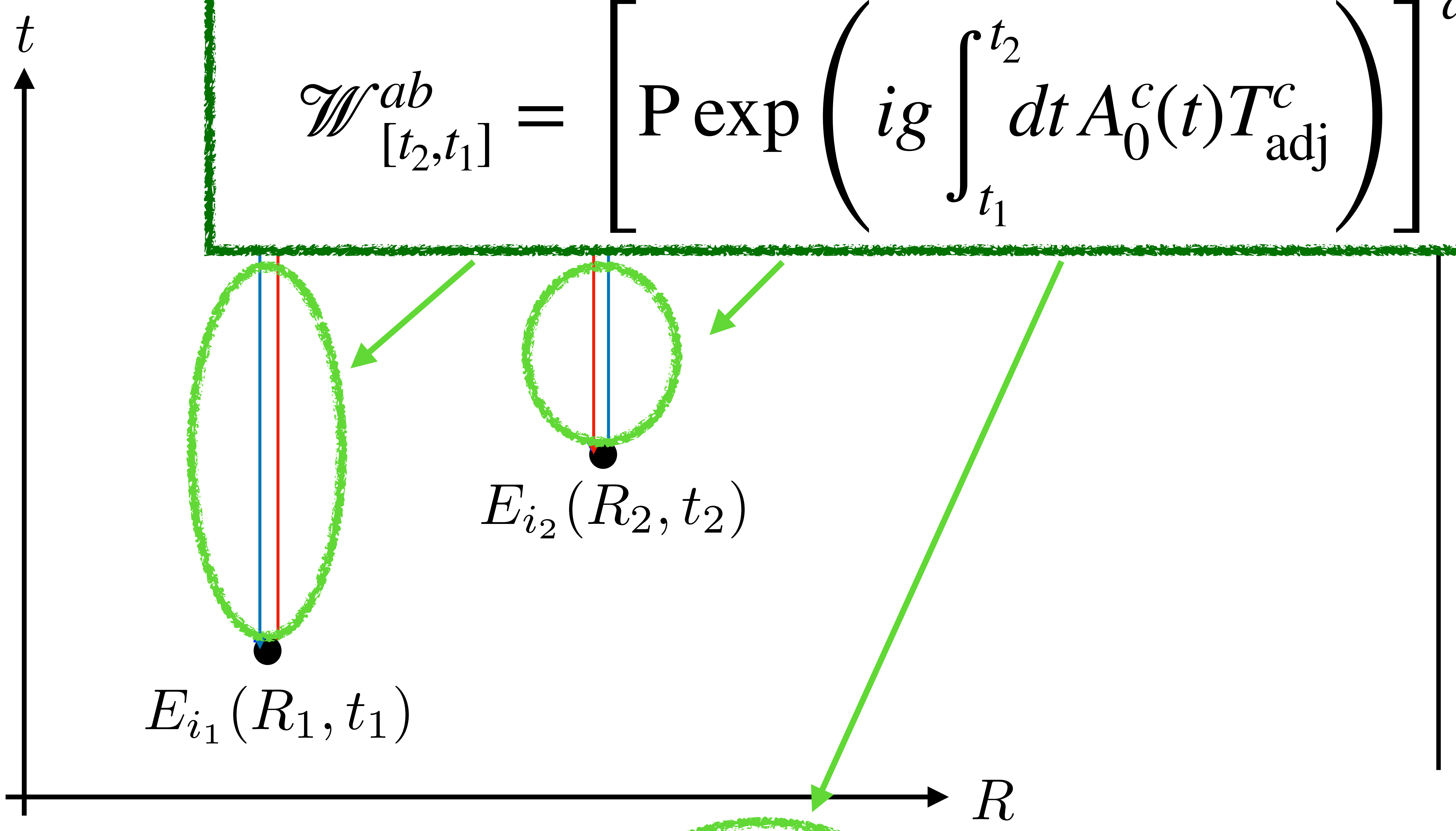

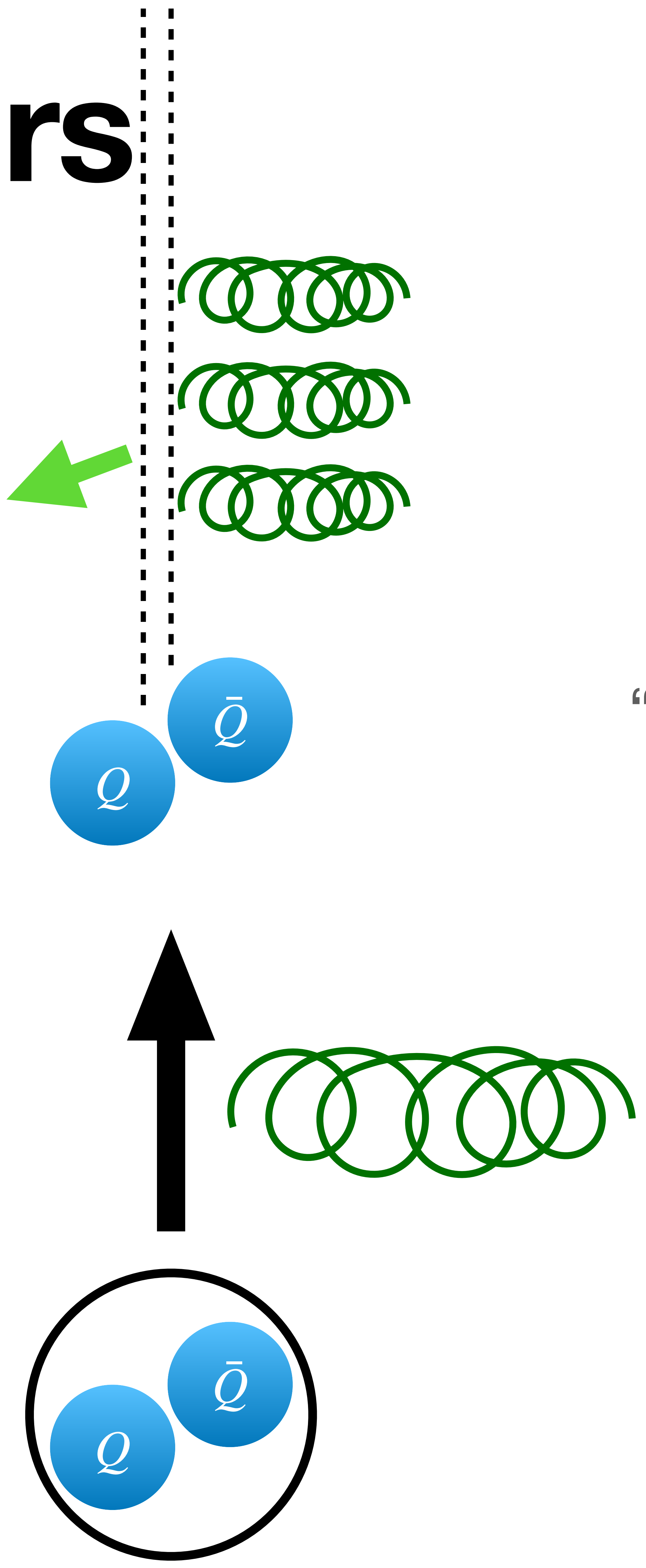

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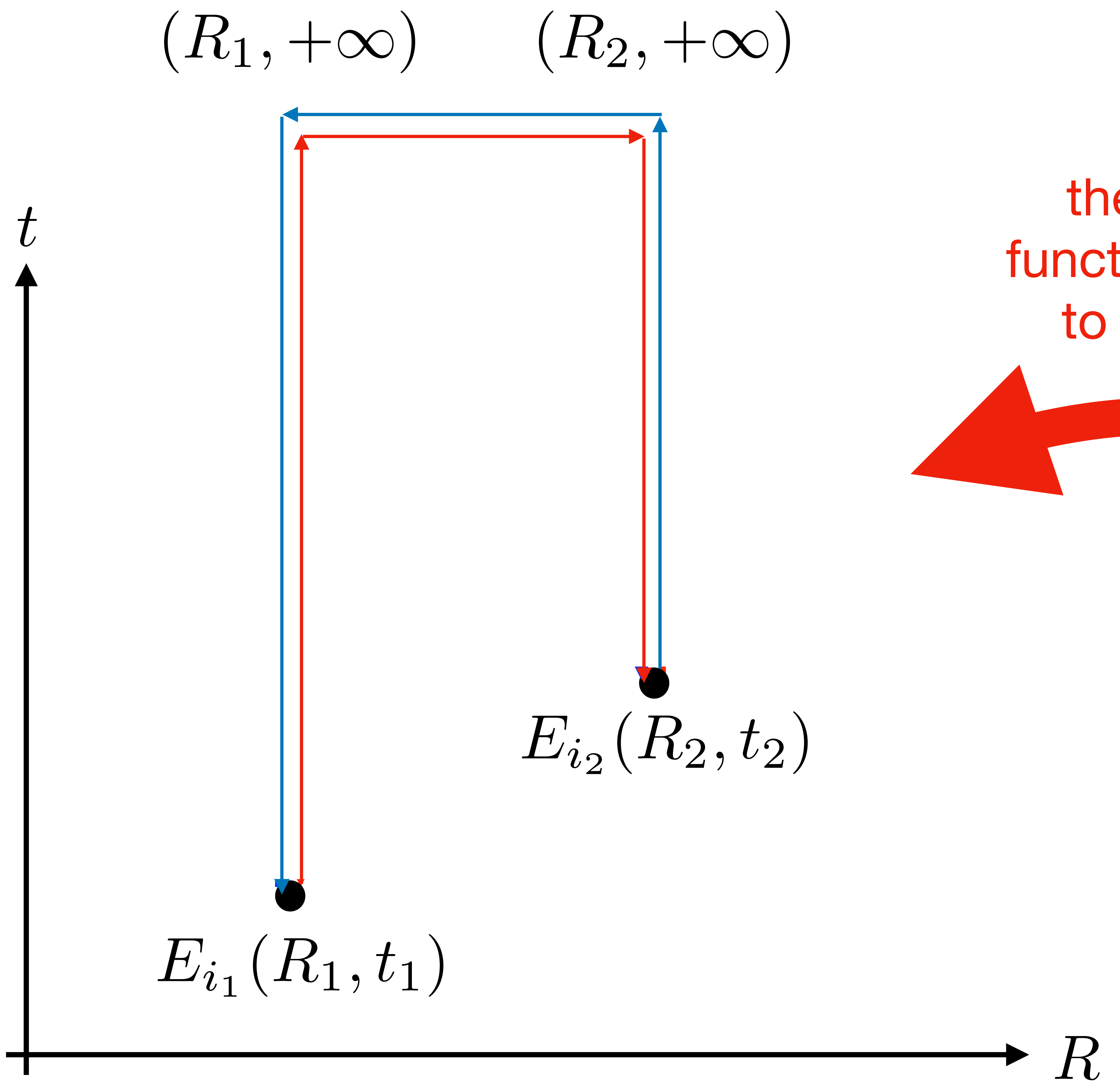

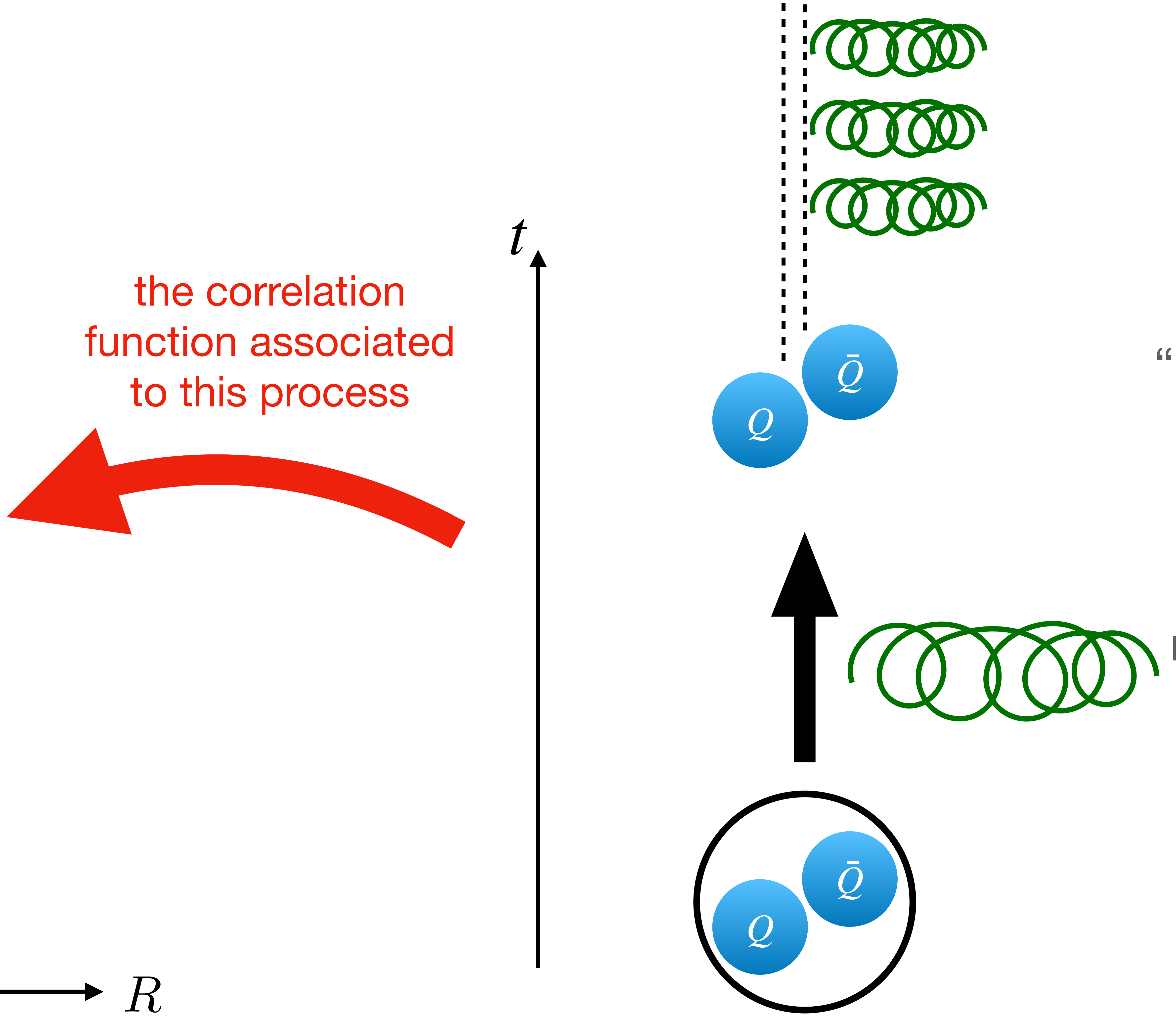

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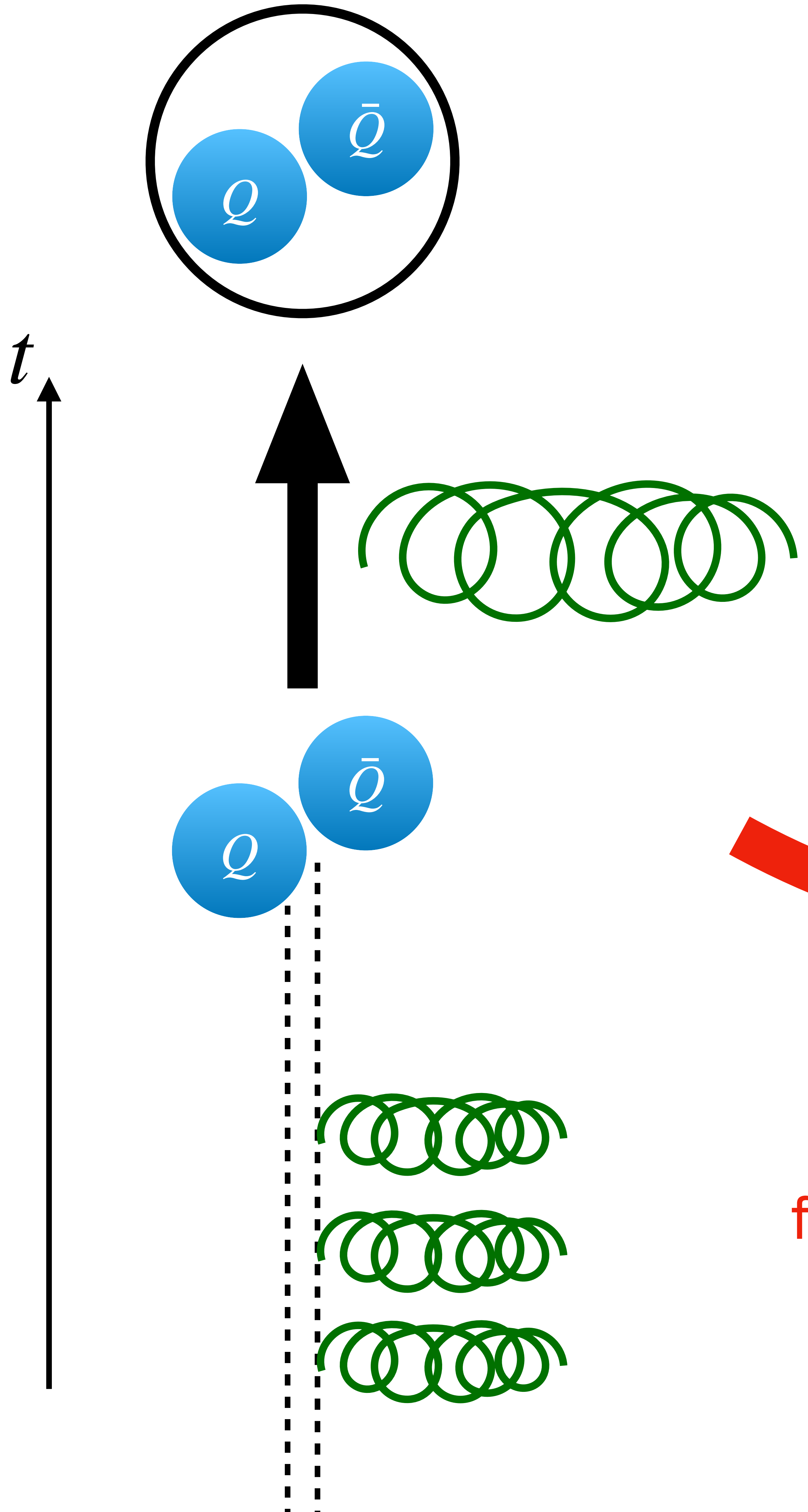
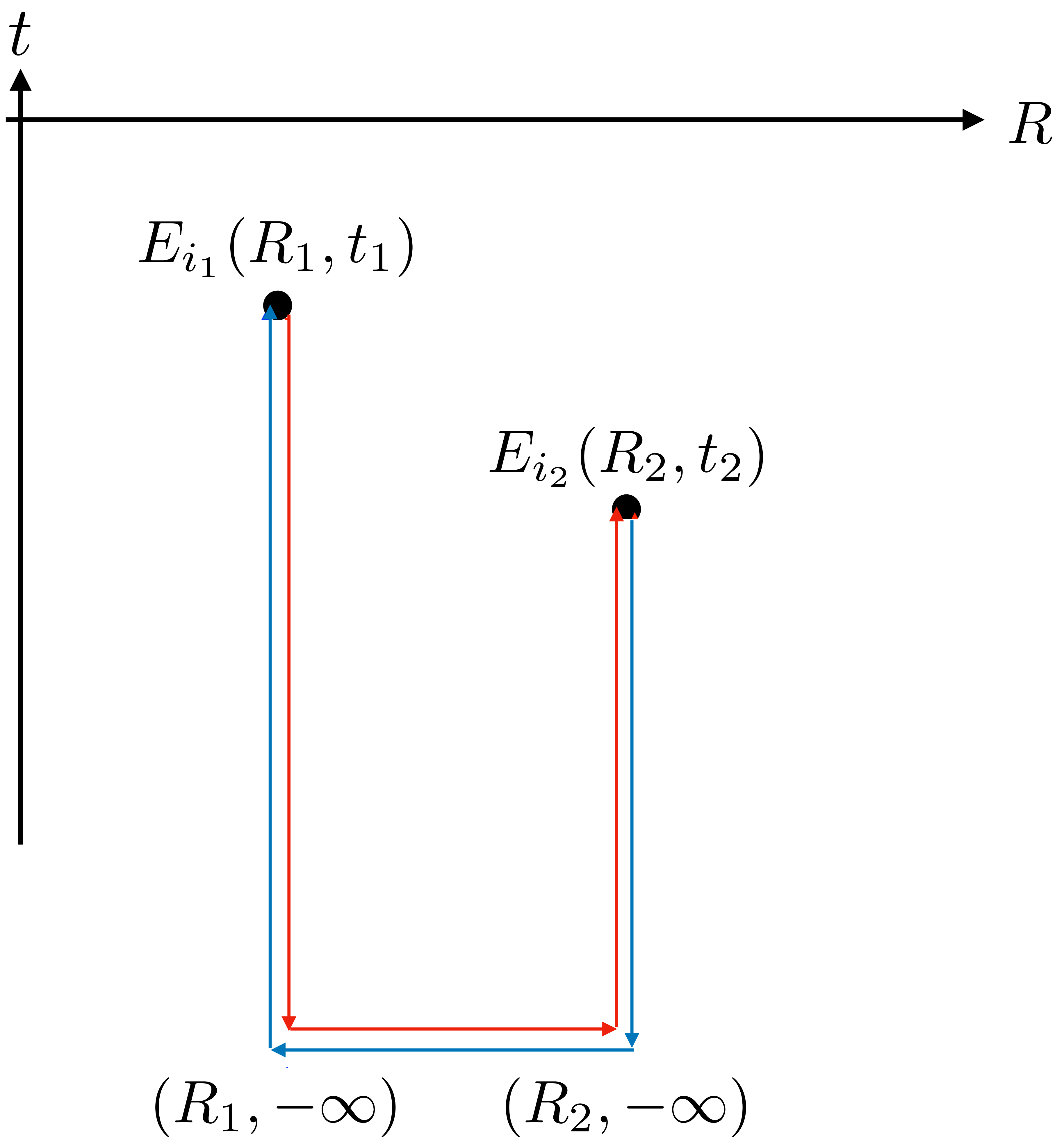


the correlation function associated to this process



Why are these correlators interesting?

Quarkonium in the quantum brownian motion limit

$Mv \gg T \gg Mv^2$ **(Brambilla et al.)**

$$\frac{d\rho_S(t)}{dt} = -i\left[H_S + \Delta H_S, \rho_S(t)\right] + \kappa_{\rm adj}\left(L_{\alpha i}\rho_S(t)L^\dagger_{\alpha i} - \frac{1}{2}\left\{L^\dagger_{\alpha i}L_{\alpha i}, \rho_S(t)\right\}\right)$$

The correlators determine the transport coefficients:

$$\gamma_{\rm adj} \equiv \frac{g^2}{6N_c}{\rm Im}\int_{-\infty}^{\infty} ds\, \langle \mathcal{T}\, E^{a,i}(s,\mathbf{0})\mathscr{W}^{ab}[(s,\mathbf{0}),(0,\mathbf{0})]E^{b,i}(0,\mathbf{0})\rangle\,,$$

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Quarkonium in the quantum optical limit

Semiclassical approximation

+ $Mv \gg Mv^2, T$ **(Yao et al.)**

$$\frac{dn_b(t,\mathbf{x})}{dt} = -\Gamma^{\rm diss}\, n_b(t,\mathbf{x}) + \Gamma^{\rm form}(t,\mathbf{x})$$

These correlators determine the dissociation and formation rates of quarkonia:

$$\Gamma^{\rm diss} \propto \int \frac{\mathrm{d}^3\mathbf{p}_{\rm rel}}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3}\,|\langle\psi_{\mathscr{B}}|\mathbf{r}|\Psi_{\mathbf{p}_{\rm rel}}\rangle|^2\,[g_E^{++}]^{>}_{ii}\left(q^0 = E_{\mathscr{B}} - \frac{\mathbf{p}^2_{\rm rel}}{M}, \mathbf{q}\right),$$

$$\Gamma^{\rm form}(t,\mathbf{x}) \propto \int \frac{\mathrm{d}^3\mathbf{p}_{\rm cm}}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{p}_{\rm rel}}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3}\,|\langle\psi_{\mathscr{B}}|\mathbf{r}|\Psi_{\mathbf{p}_{\rm rel}}\rangle|^2\,[g_E^{--}]^{>}_{ii}\left(q^0 = \frac{\mathbf{p}^2_{\rm rel}}{M} - E_{\mathscr{B}}, \mathbf{q}\right)$$
$$\times f_{\mathscr{S}}(\mathbf{x}, \mathbf{p}_{\rm cm}, \mathbf{r}=0, \mathbf{p}_{\rm rel}, t)\,.$$

A comparison with heavy quark diffusion

Different physics with the same building blocks

Heavy quark diffusion

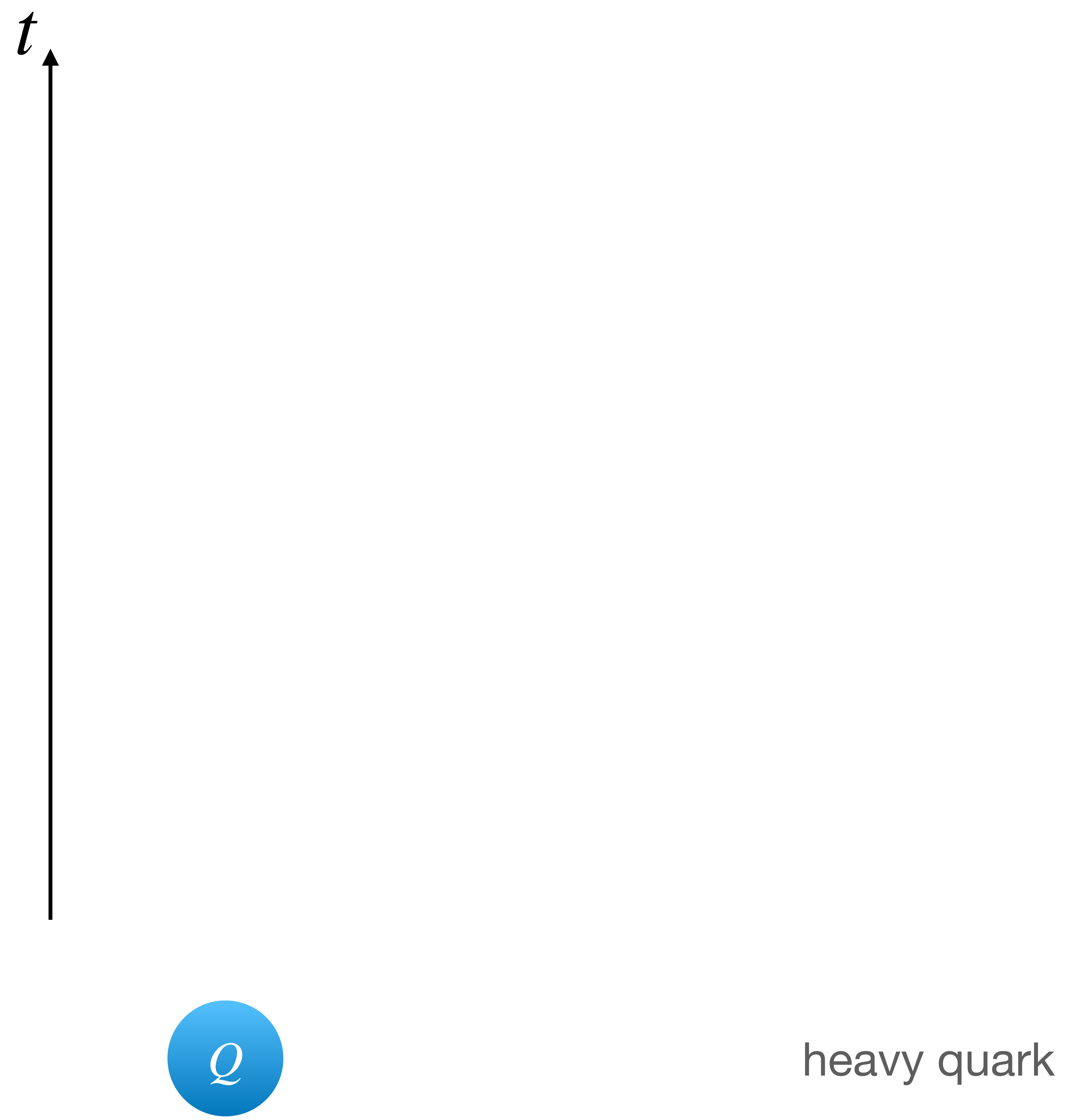
an analogous picture

J. Casalderrey-Solana and D. Teaney, hep-ph/0605199

- The heavy quark diffusion coefficient is also defined from a correlation of chromoelectric fields:

$$\langle \mathrm{Tr}\Big[(U_{[\infty,t]}E_i(t)U_{[t,-\infty]})^\dagger \times (U_{[\infty,0]}E_i(0)U_{[0,-\infty]})\Big]\rangle$$

- It reflects the typical momentum transfer $\langle p^2 \rangle$ received from “kicks” from the medium.



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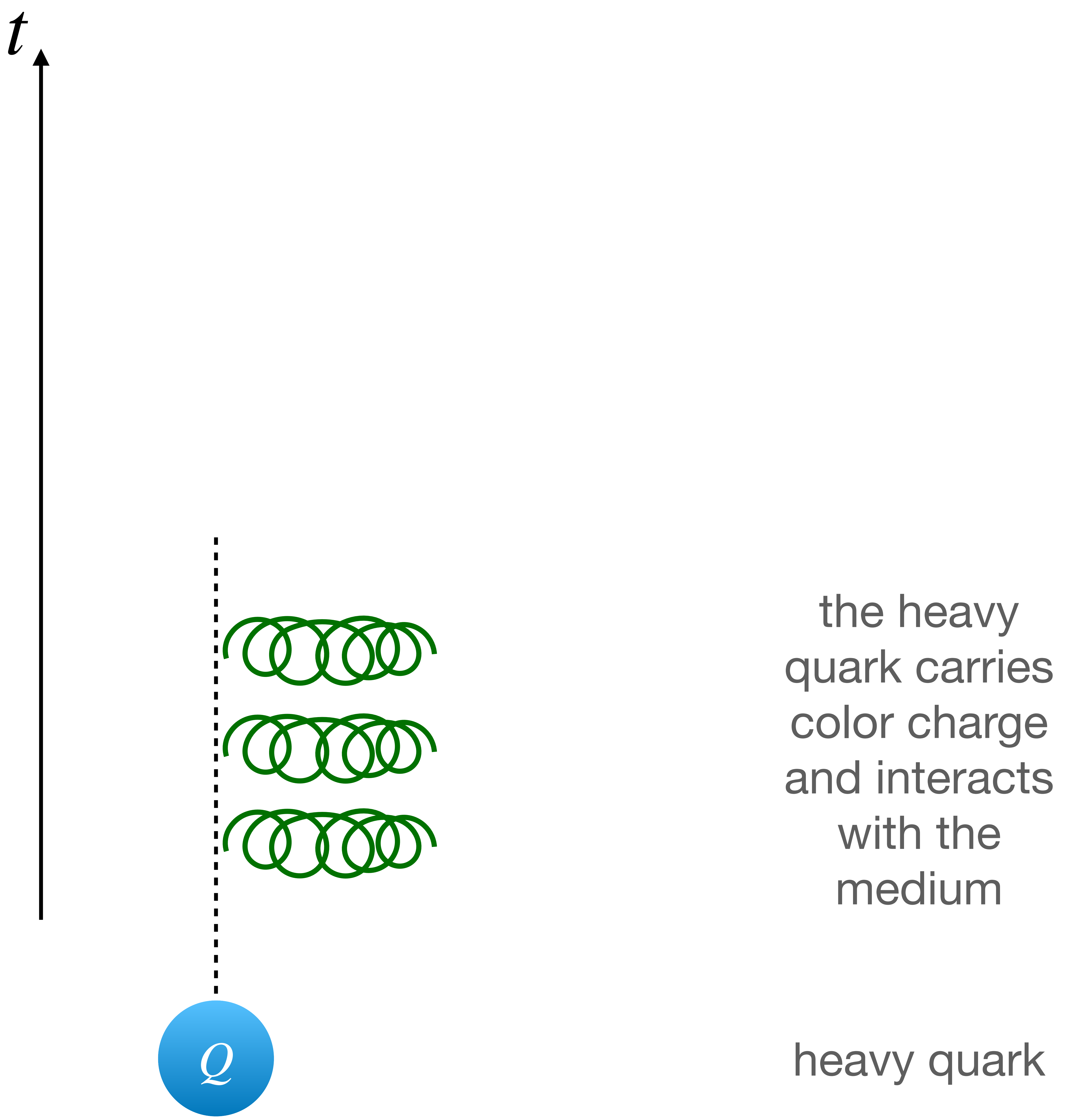

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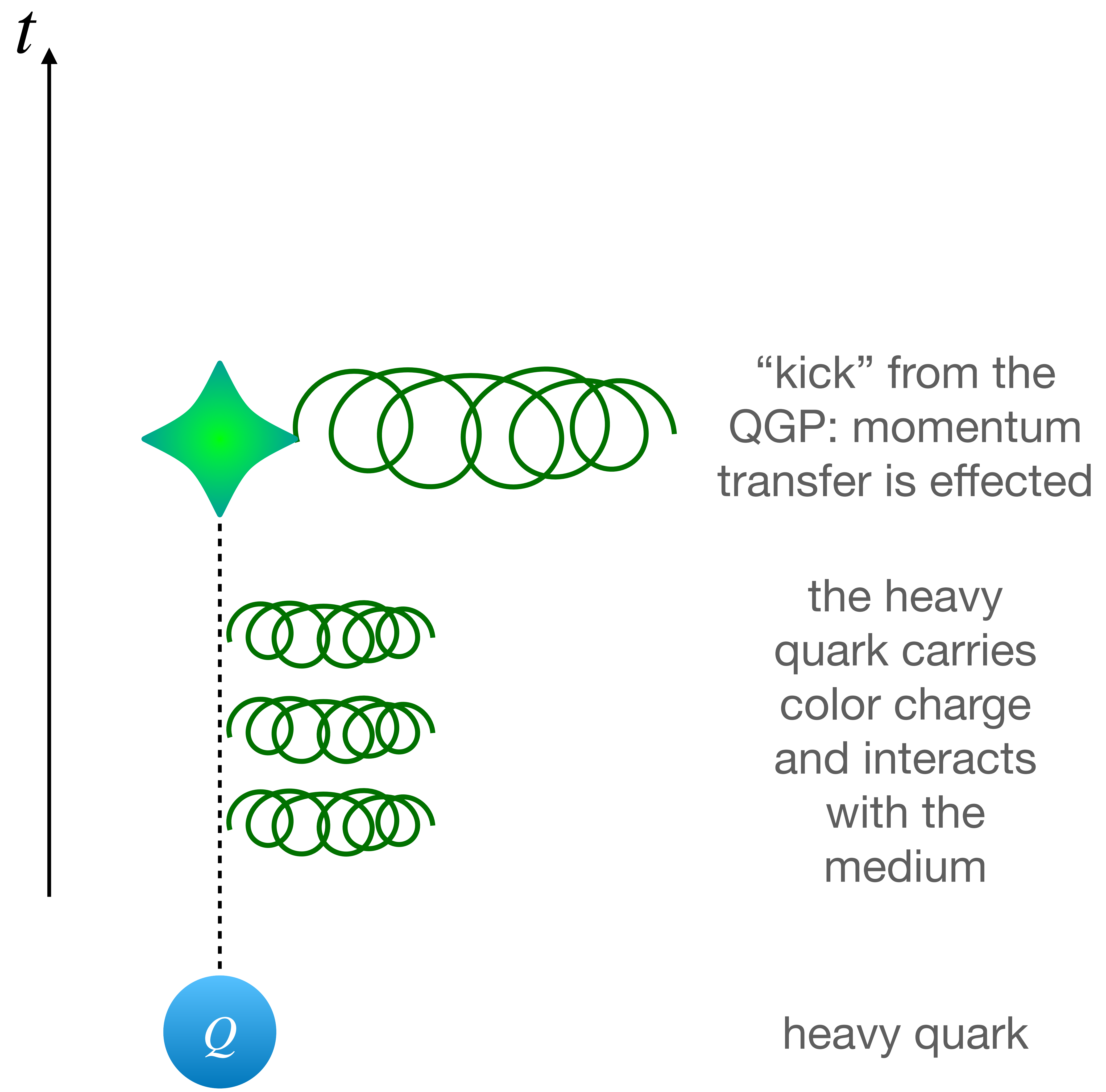

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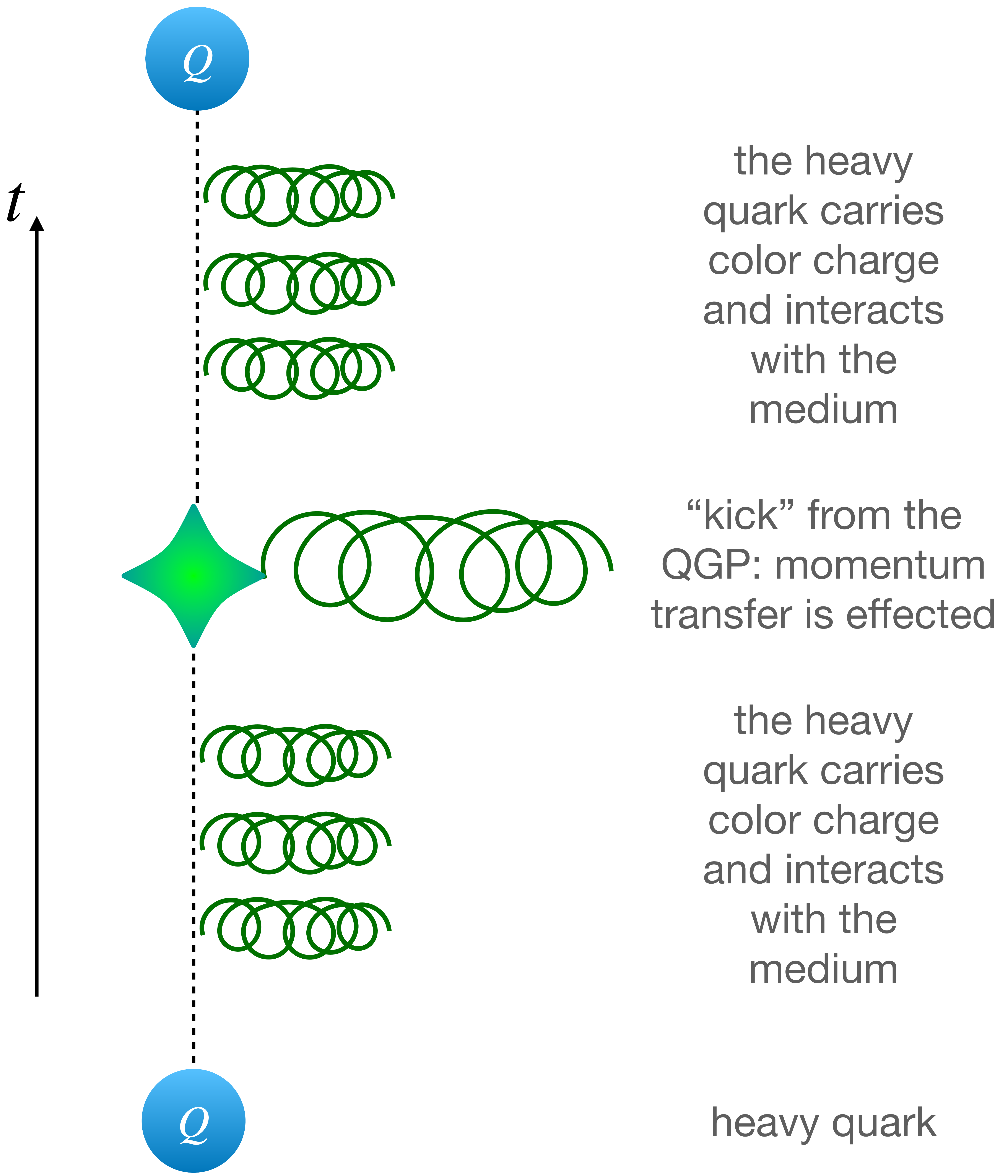


Heavy quark and quarkonia correlators

a small, yet consequential difference

The heavy quark diffusion coefficient can be defined from the real-time correlator J. Casalderrey-Solana and D. Teaney, hep-ph/0605199; see also A. M. Eller, J. Ghiglieri and G. D. Moore, hep-ph/1903.08064

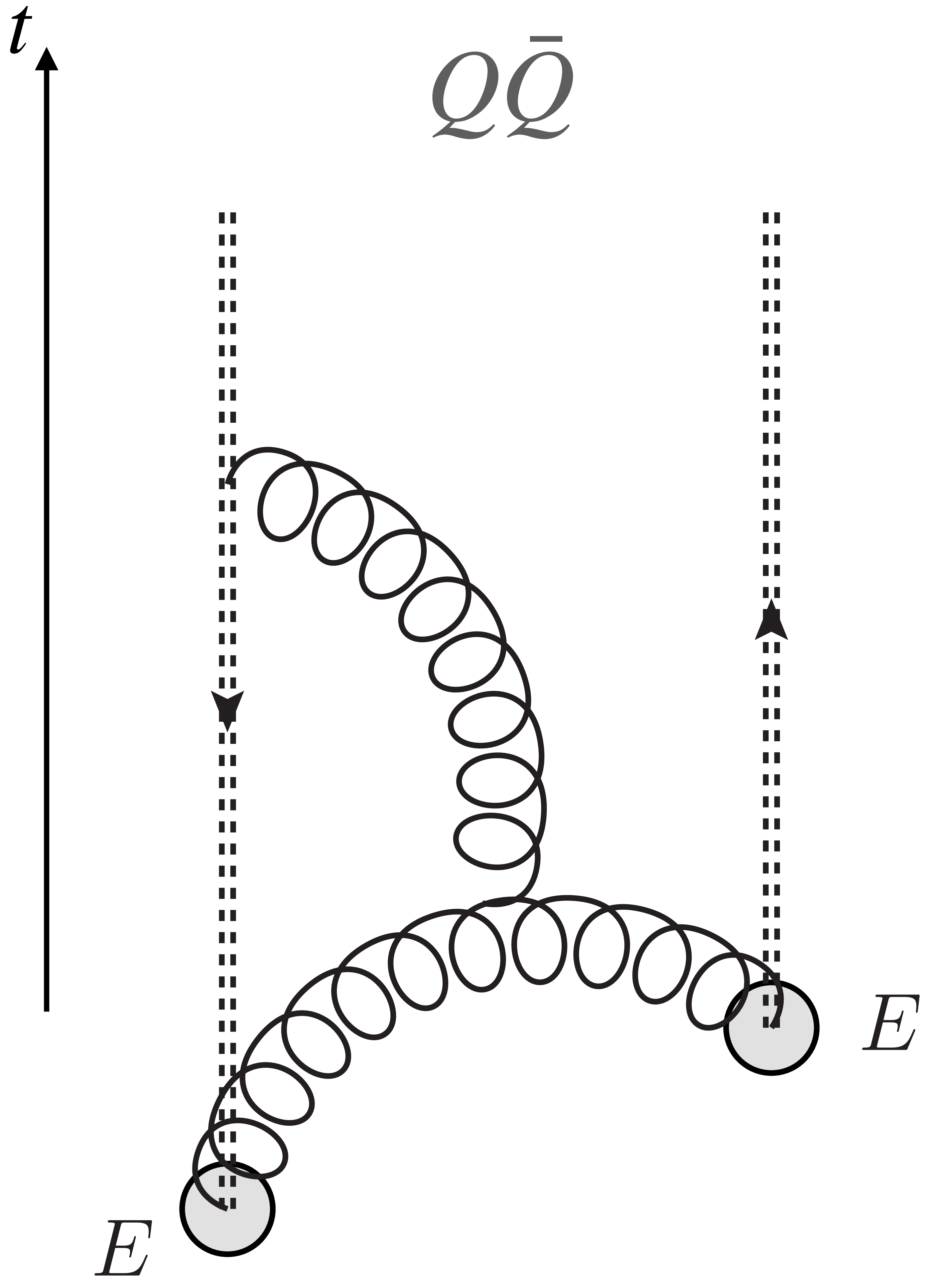
$$\left\langle \mathrm{Tr}_{\mathrm{color}} \left[U(-\infty, t) E_i(t) U(t,0) E_i(0) U(0, -\infty) \right] \right\rangle_T ,$$

whereas for quarkonia the relevant quantity is ($\mathbf{R}_1 = \mathbf{R}_2$ in the preceding discussion)

$$T_F \left\langle E_i^a(t) \mathscr{W}^{ab}(t,0) E_i^b(0) \right\rangle_T .$$

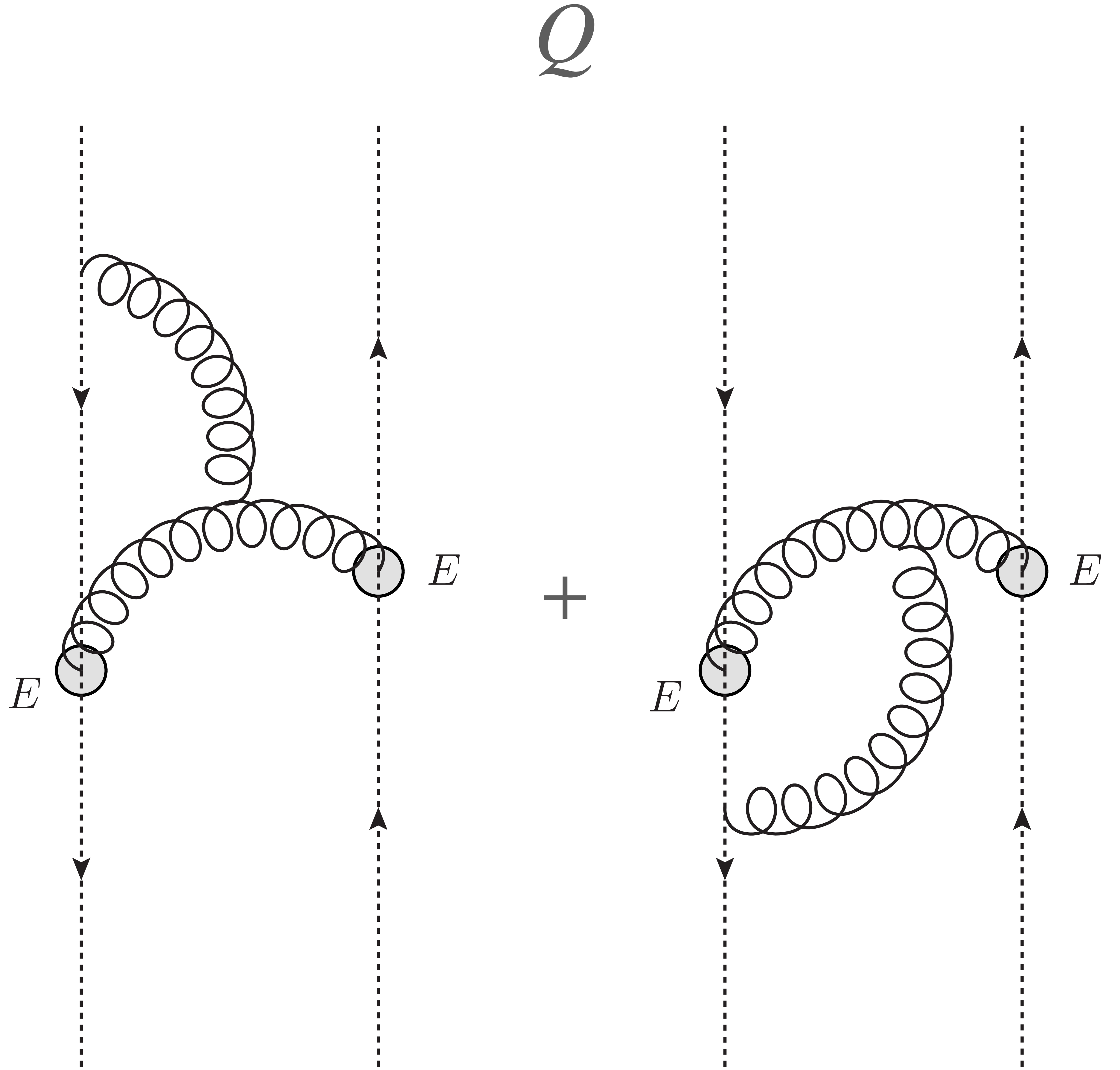
The difference in pQCD

operator ordering is crucial!



Perturbatively, one can isolate the difference between the correlators to these diagrams.

$$\Delta\rho(\omega) = \frac{g^4 N_c^2 C_F T_F}{4\pi}\omega^3$$



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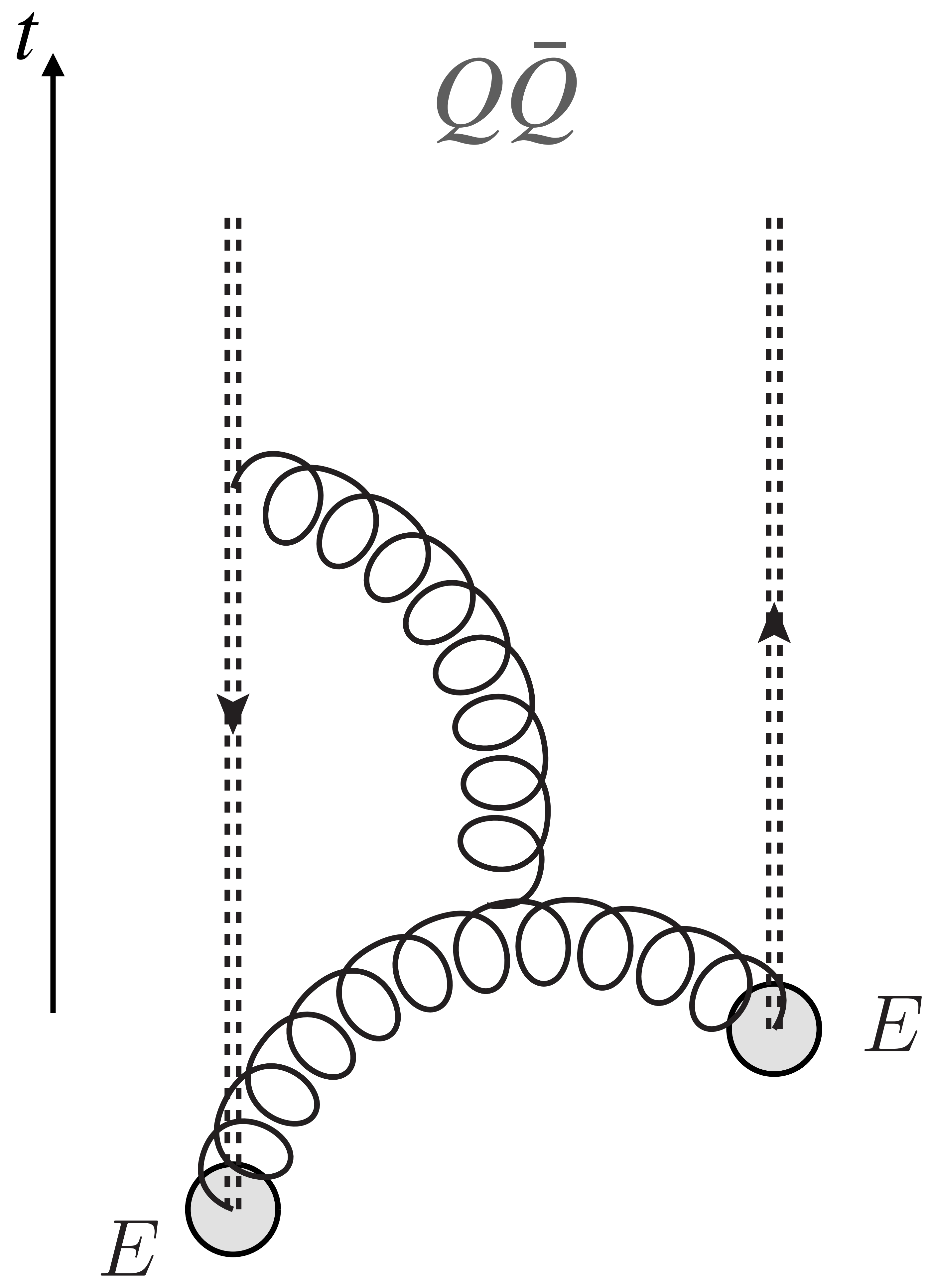

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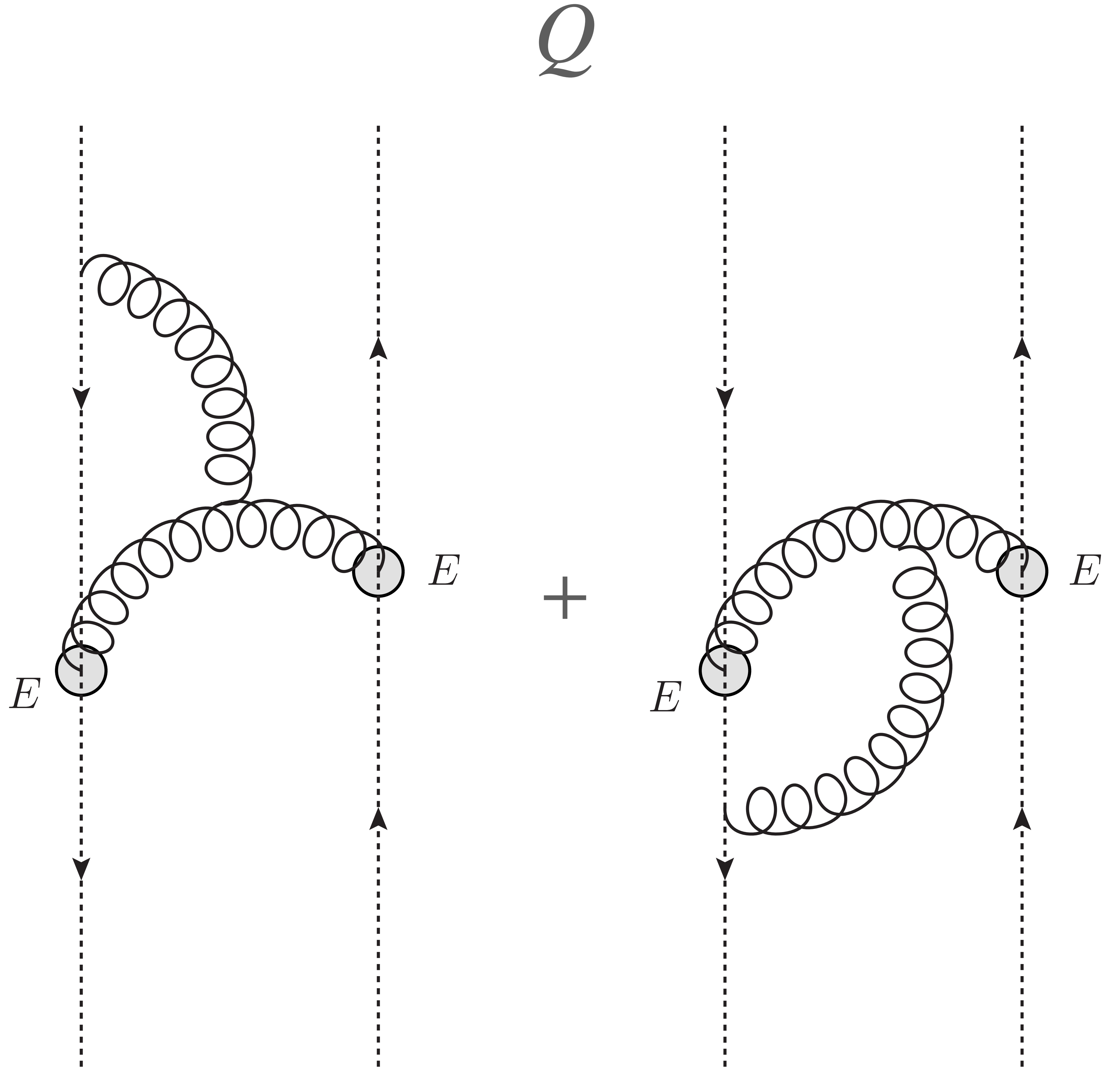
The difference is due to different operator orderings (different possible gluon insertions).



see also A. M. Eller, J. Ghiglieri and G. D. Moore, hep-ph/1903.08064

BS and X. Yao, hep-ph/2205.04477
Phys. Rev. Lett. 130, 052302

The difference in pQCD

Gauge invariant!

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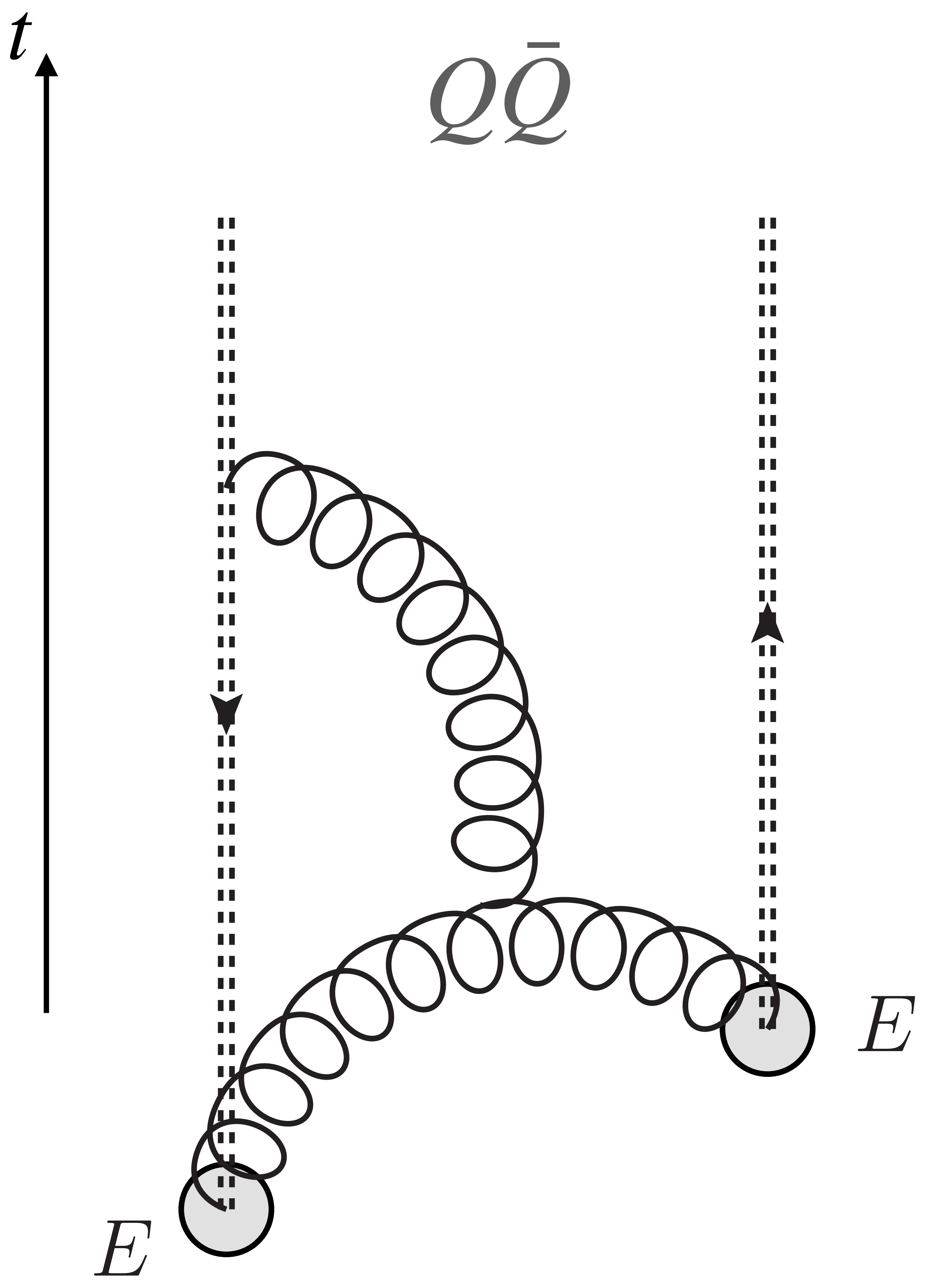

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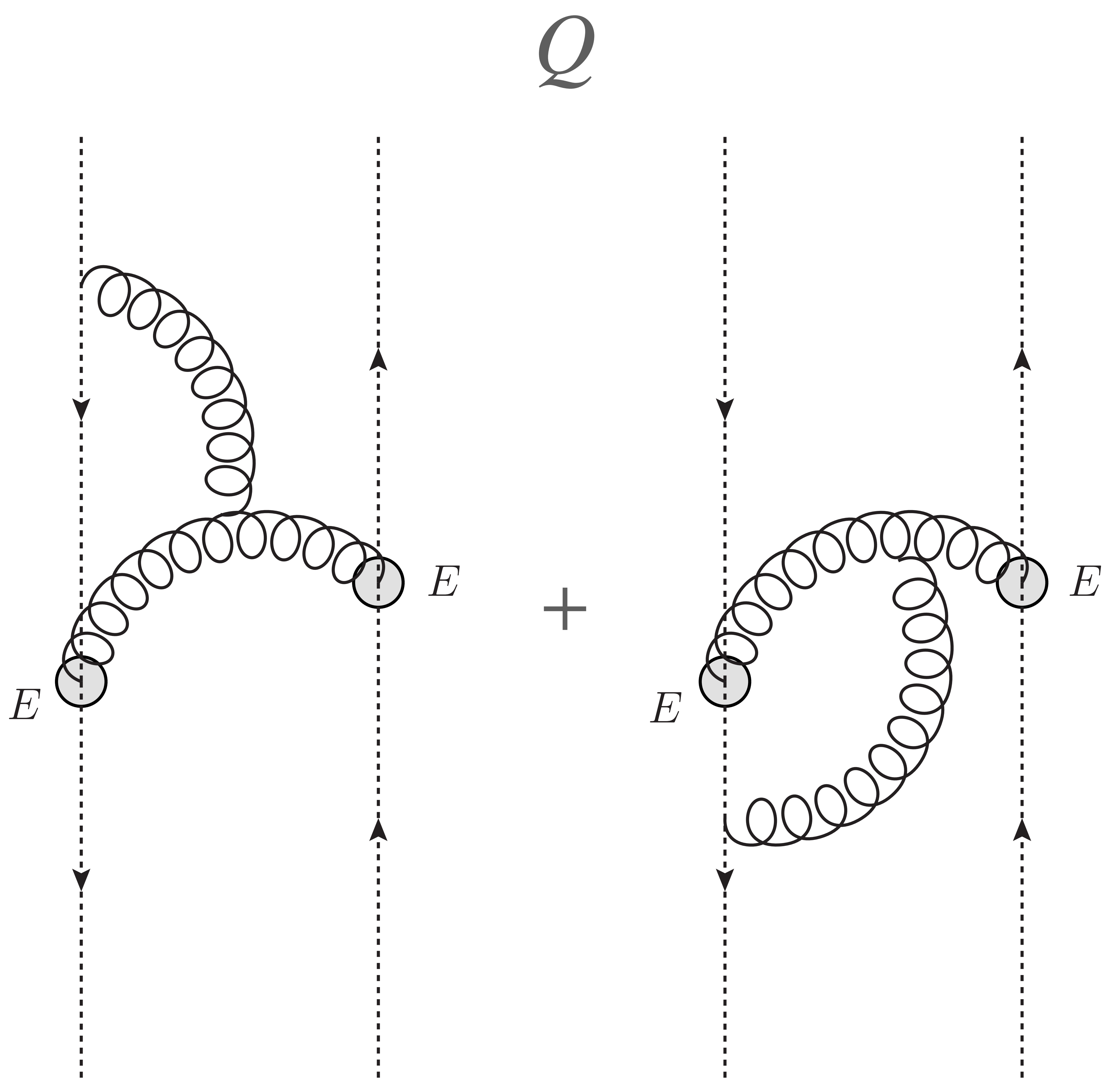

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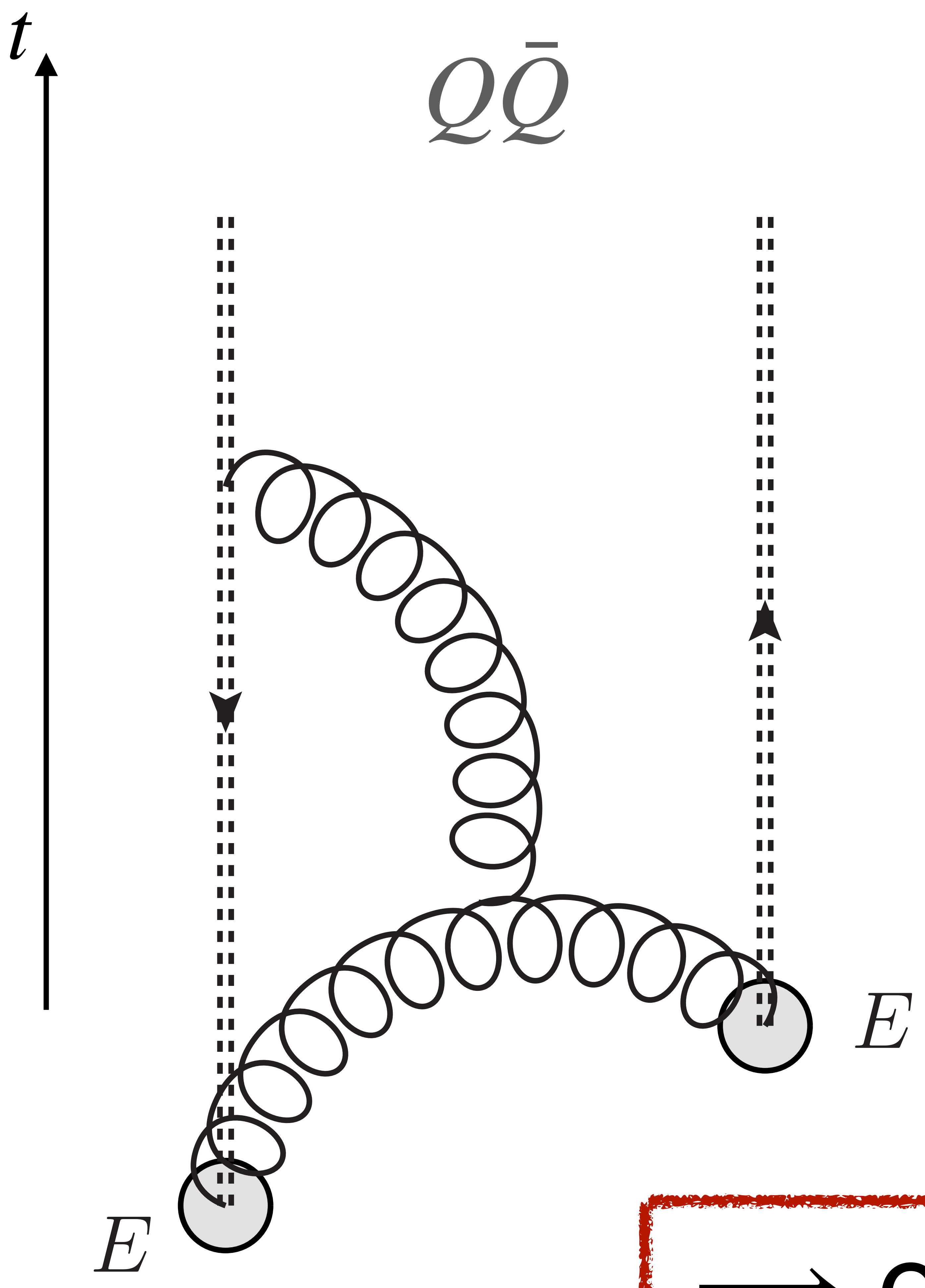

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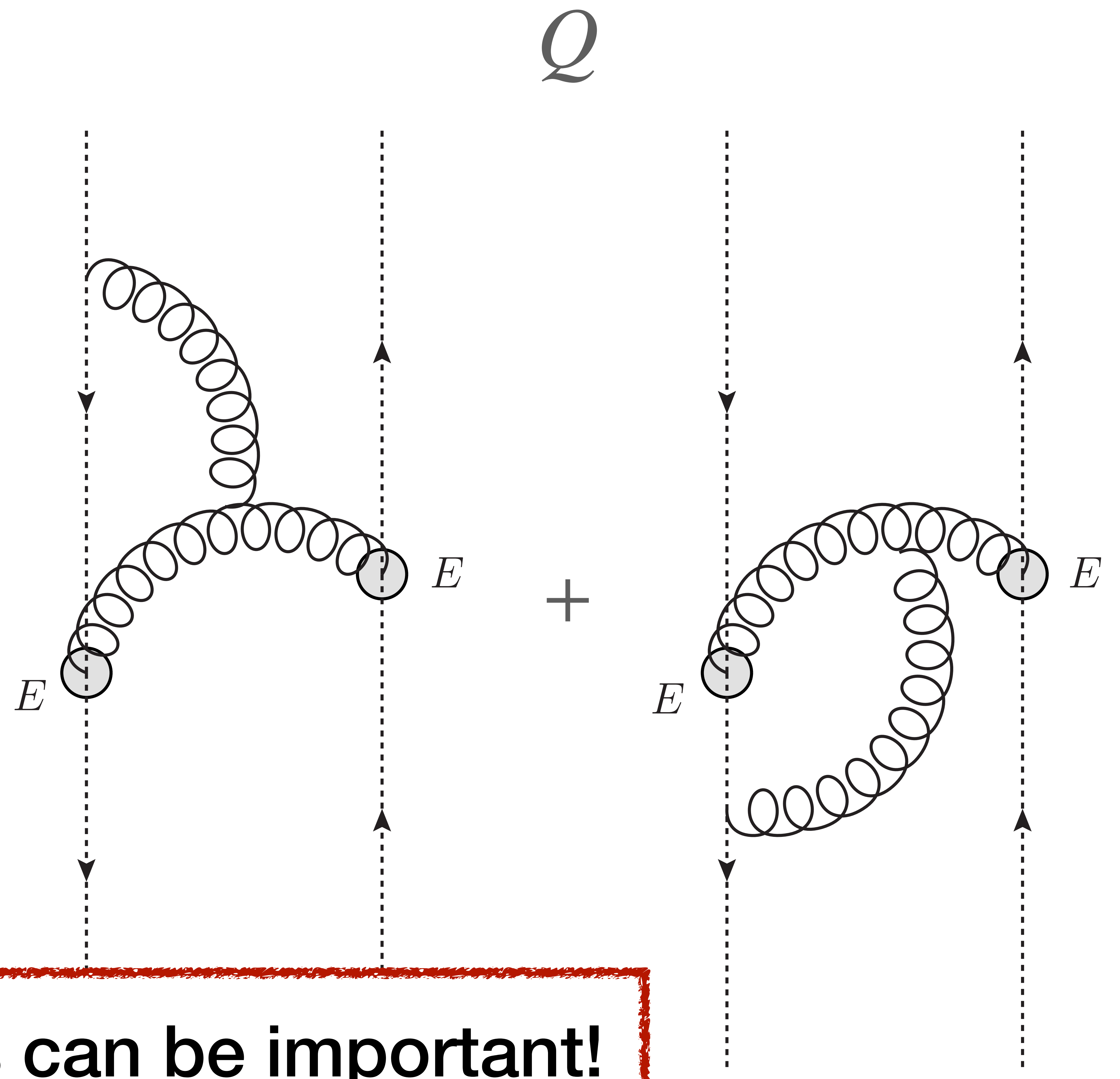


$\Longrightarrow$ Quantum color correlations can be important!

However, the QGP is not weakly coupled.

Can we make a comparison at strong coupling? In any theory?

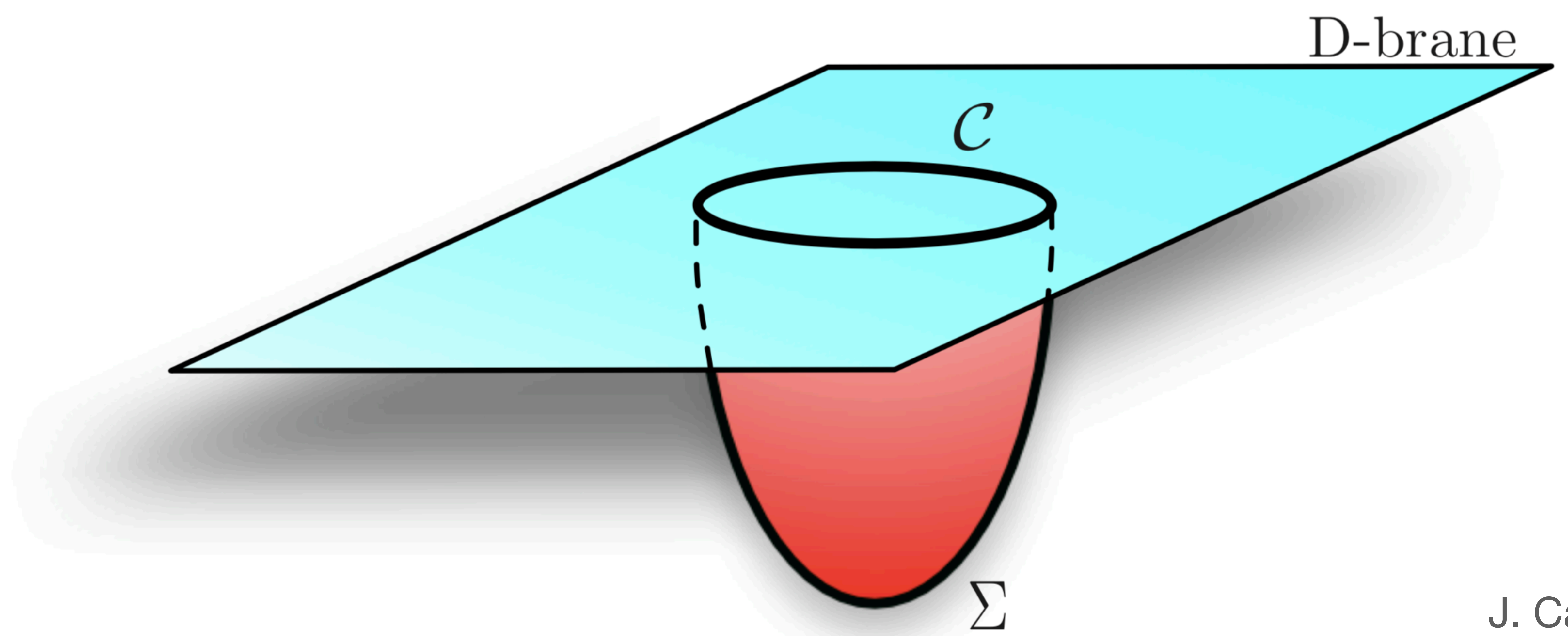
[**] J. Maldacena, hep-th/9711200

Wilson loops in AdS/CFT

setup

- The holographic duality provides a way to formulate the calculation of analogous correlators in strongly coupled theories. [**]
  - Wilson loops can be evaluated by solving classical equations of motion:

$$\langle W[\mathscr{C} = \partial\Sigma]\rangle_T = e^{iS_{\mathrm{NG}}[\Sigma]}$$



J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal and U. A. Wiedemann, hep-ph/1101.0618

How do Wilson loops help?

setup — pure gauge theory

- Field strength insertions along a Wilson loop can be generated by taking variations of the path $\mathscr{C}$:

$$\frac{\delta}{\delta f^{\mu}(s_2)}\frac{\delta}{\delta f^{\nu}(s_1)}W[\mathscr{C}_f]\bigg|_{f=0} = (ig)^2\mathrm{Tr}_{\mathrm{color}}\Big[U_{[1,s_2]}F_{\mu\rho}(\gamma(s_2))\dot{\gamma}^{\rho}(s_2)U_{[s_2,s_1]}F_{\nu\sigma}(\gamma(s_1))\dot{\gamma}^{\sigma}(s_1)U_{[s_1,0]}\Big]$$

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- Same as the lattice calculation of the heavy quark diffusion coefficient:

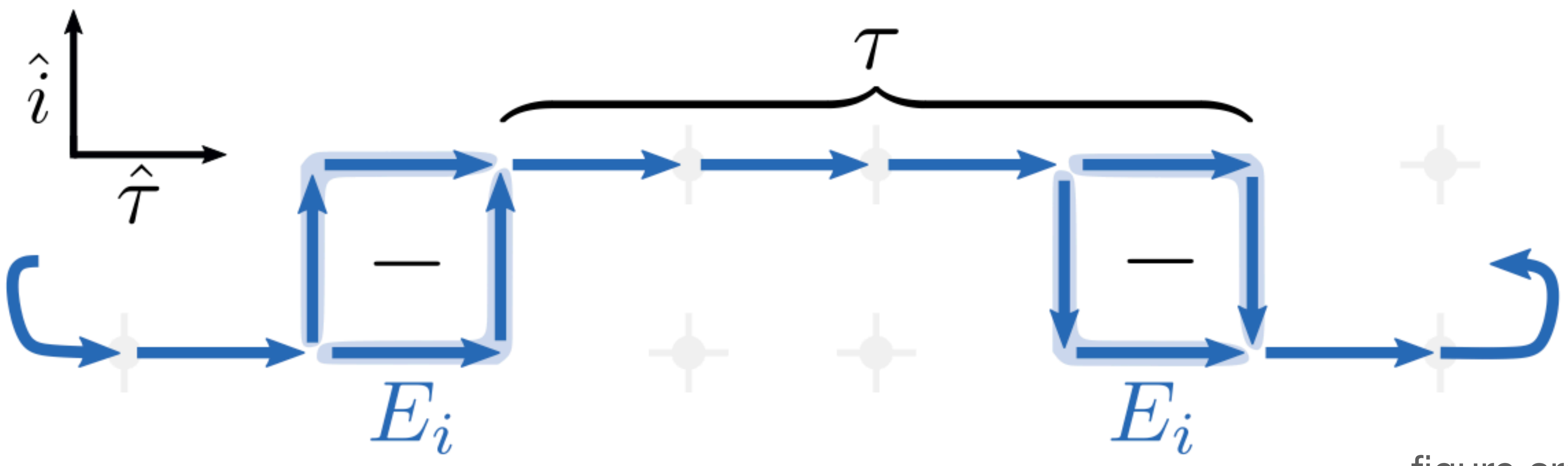

figure credit: L. Altenkort

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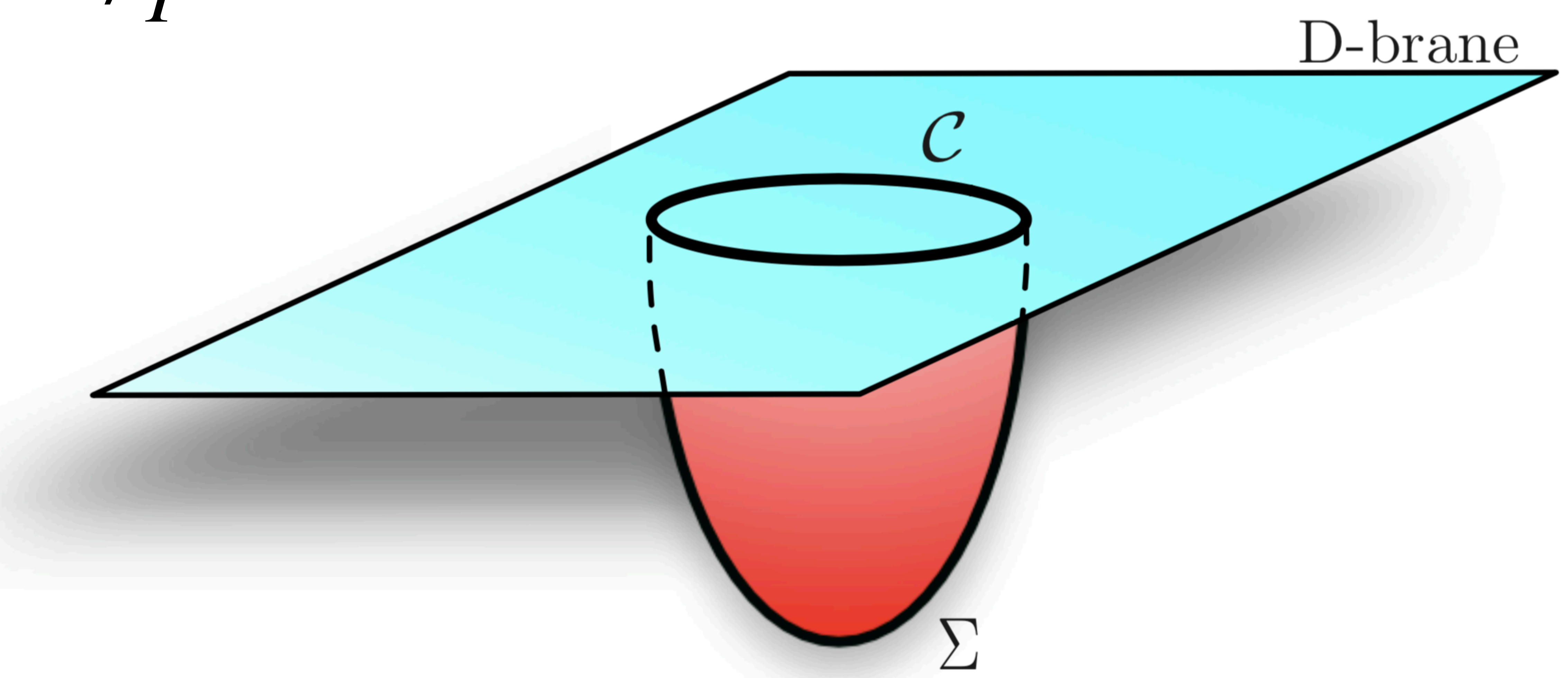
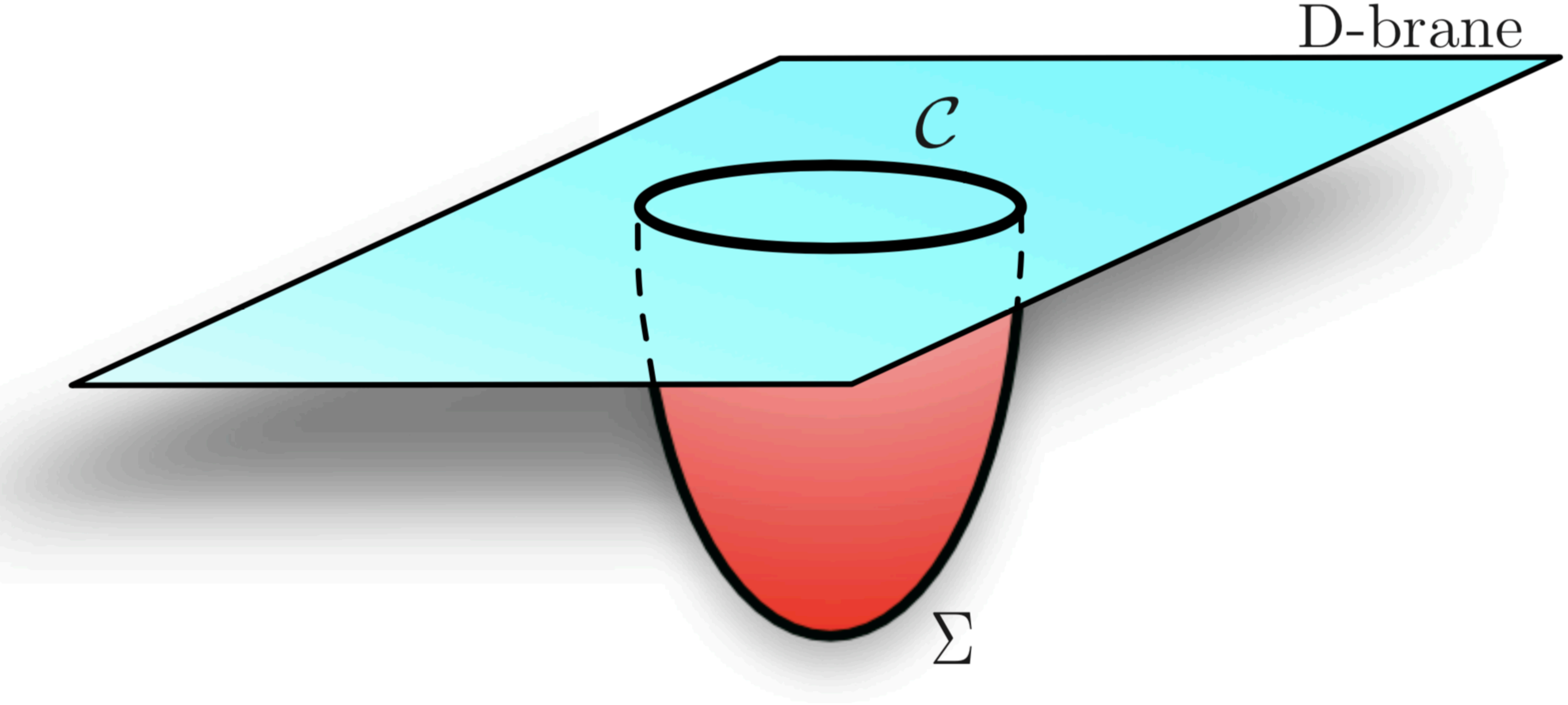


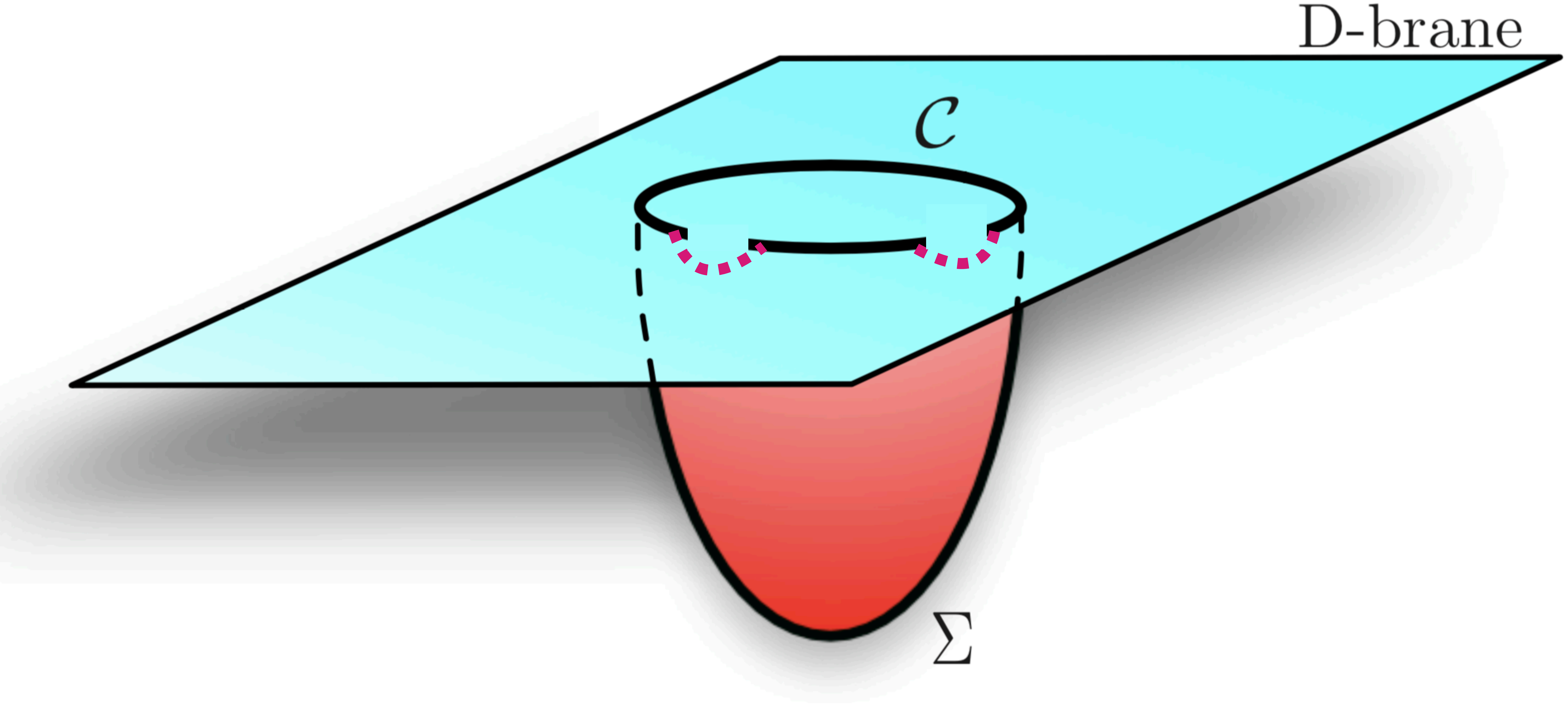
J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal and U. A. Wiedemann, hep-ph/1101.0618

Metric of interest for finite T calculations:

$$ds^2 = \frac{R^2}{z^2}\left[-f(z)\,dt^2 + d\mathbf{x}^2 + \frac{1}{f(z)}dz^2 + z^2 d\Omega_5^2\right]$$

$$f(z) = 1 - (\pi T z)^4$$


D-brane
$\mathcal{C}$
Σ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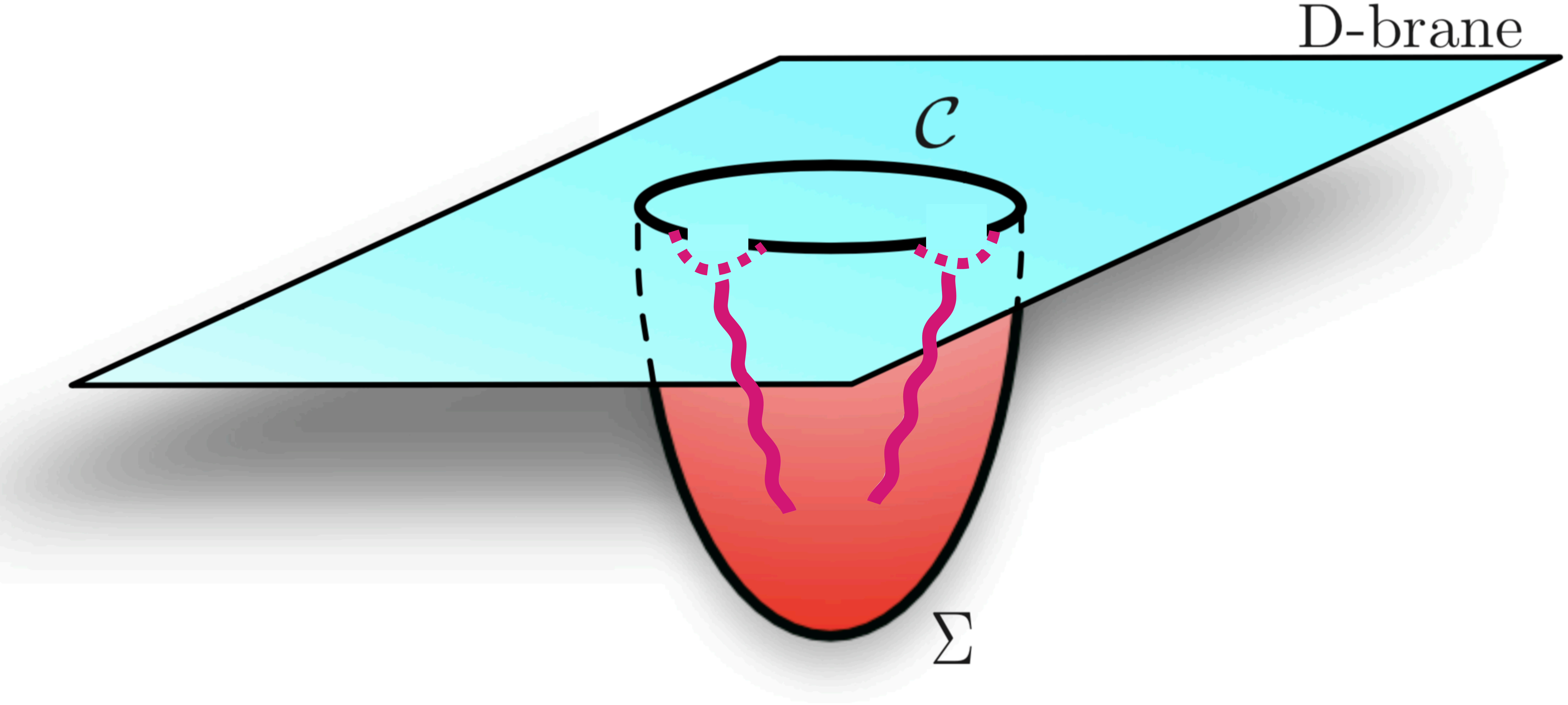
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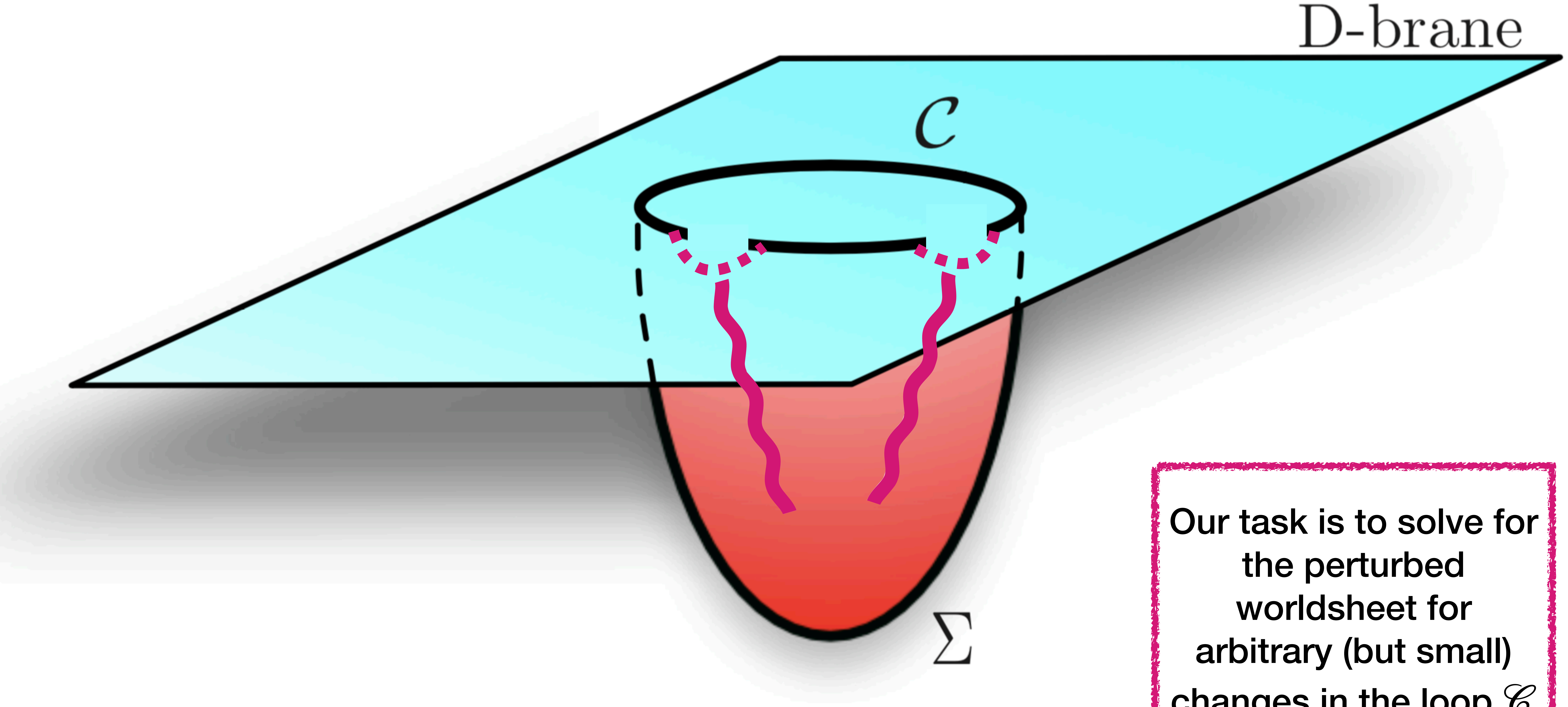
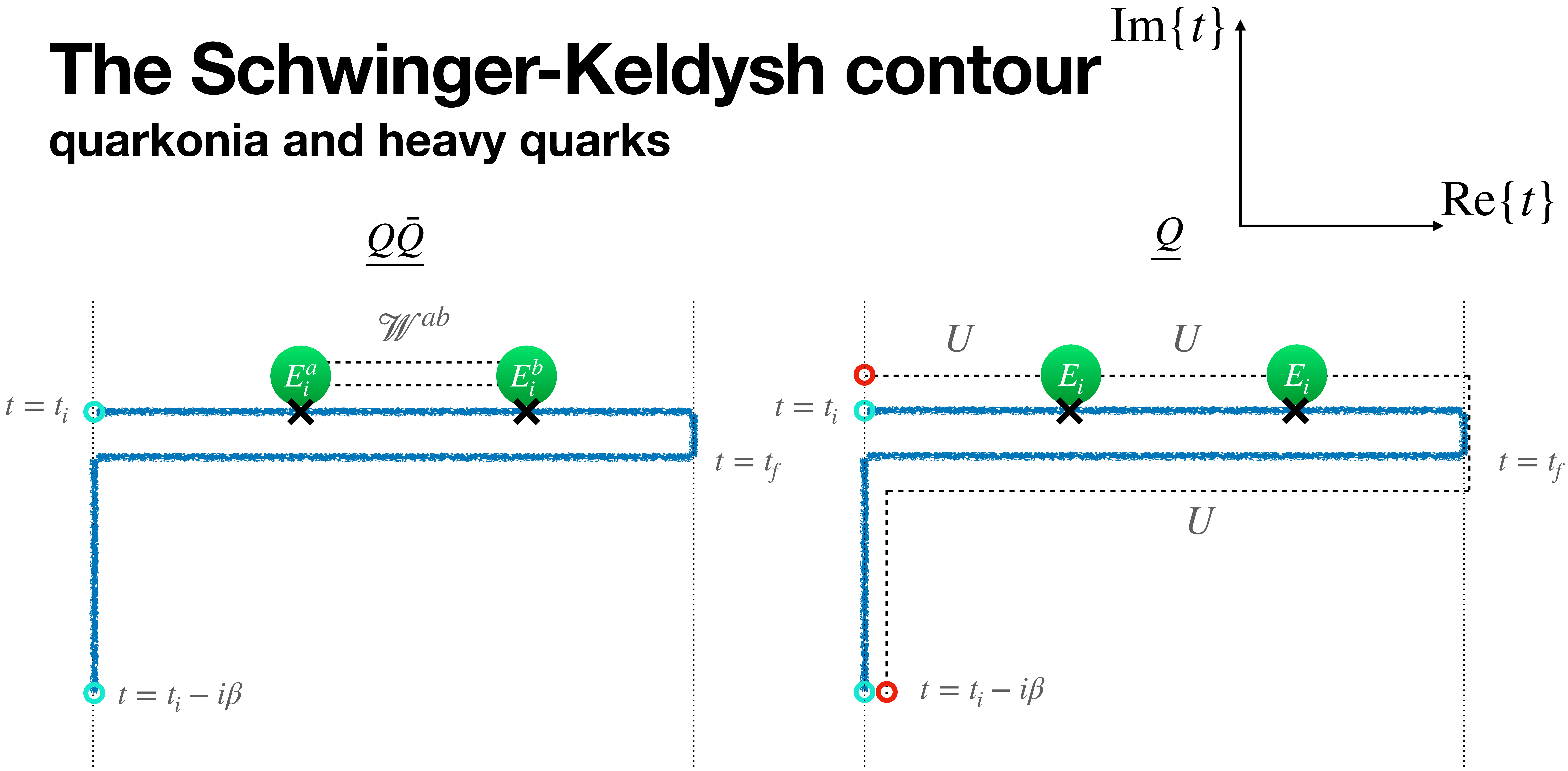


Our task is to solve for the perturbed worldsheet for arbitrary (but small) changes in the loop $\mathscr{C}$

The Schwinger-Keldysh contour

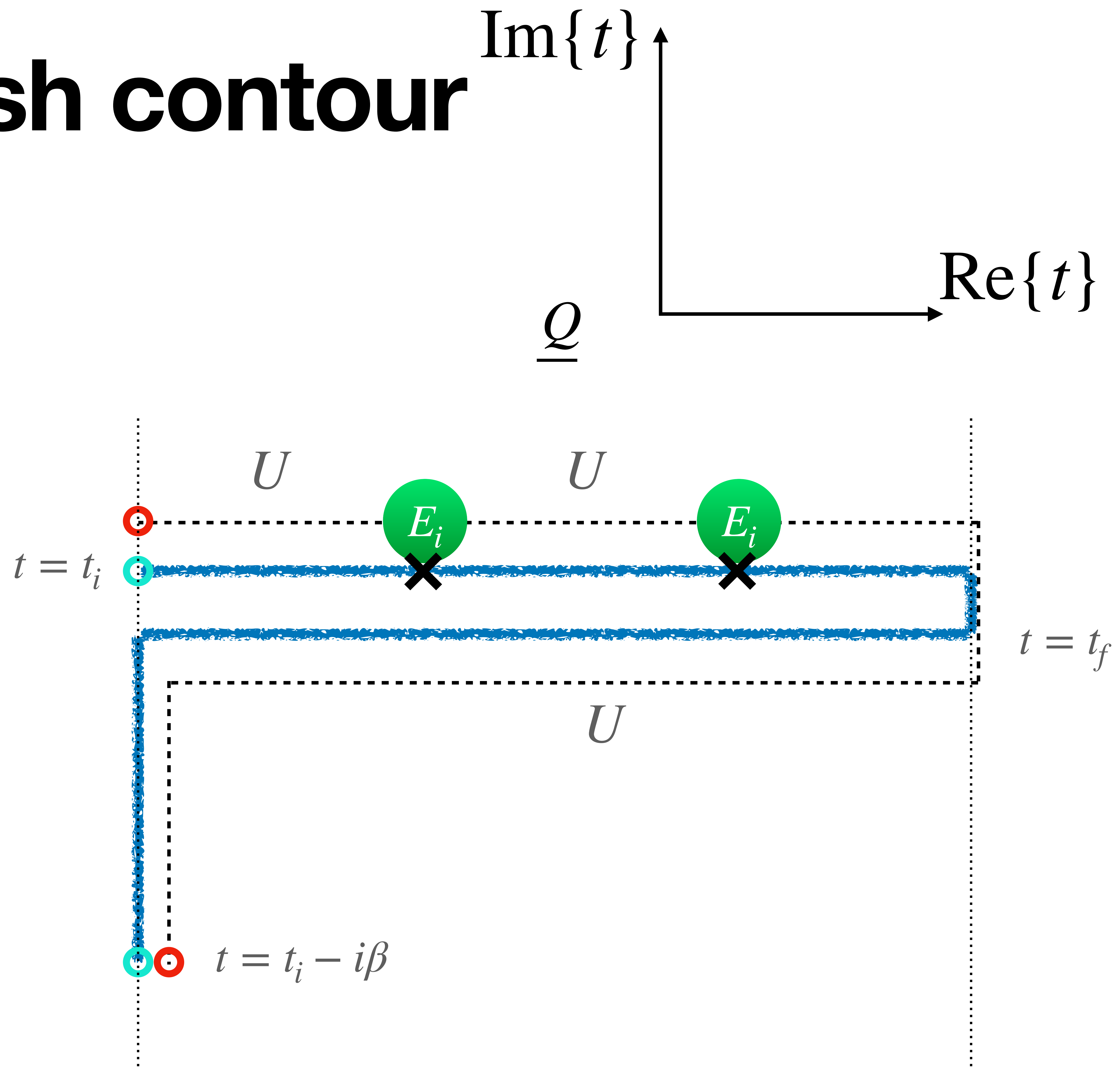
quarkonia and heavy quarks



The Schwinger-Keldysh contour

quarkonia and heavy quarks

- The heavy quark is present at all times:
  - It is part of the construction of the thermal state.
  - The Wilson line, which enforces the Gauss' law constraint due to the point charge, is also present on the Euclidean segment.



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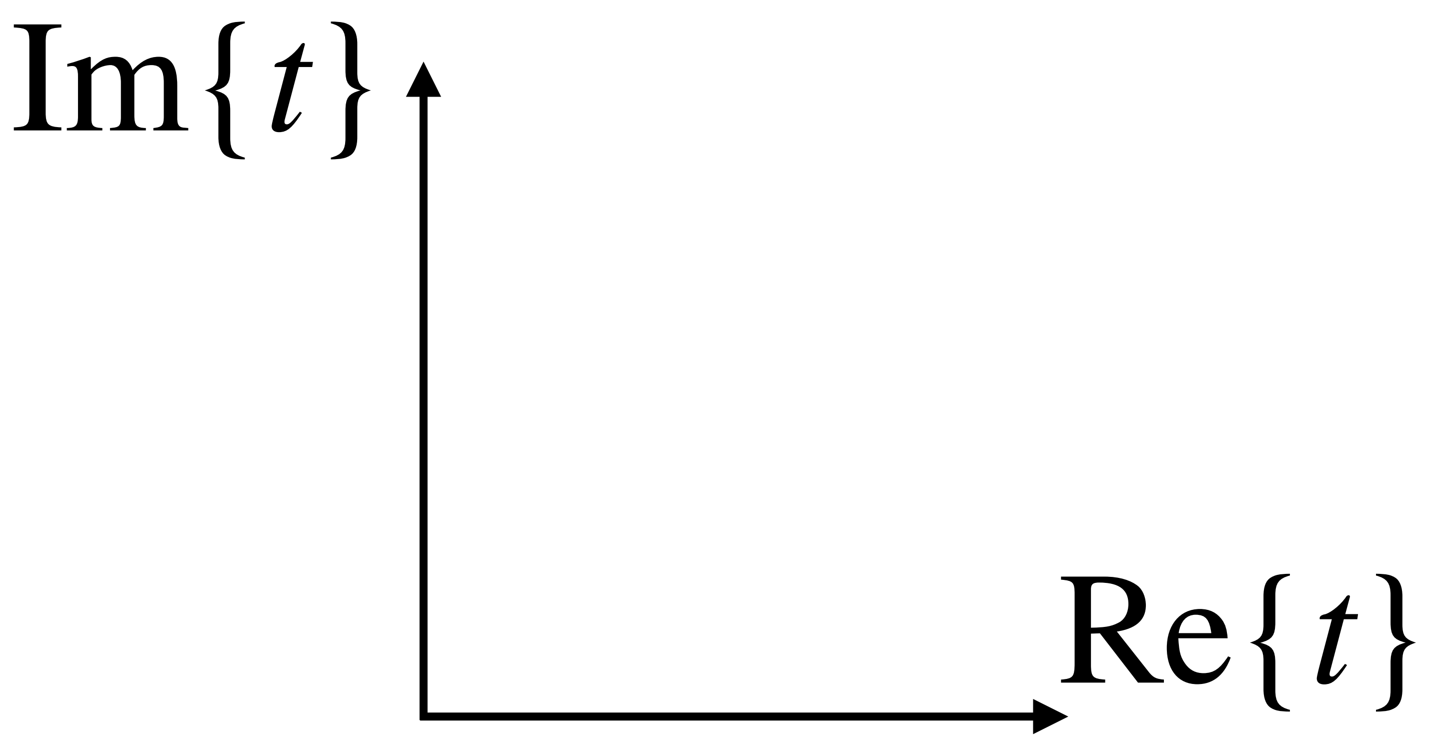

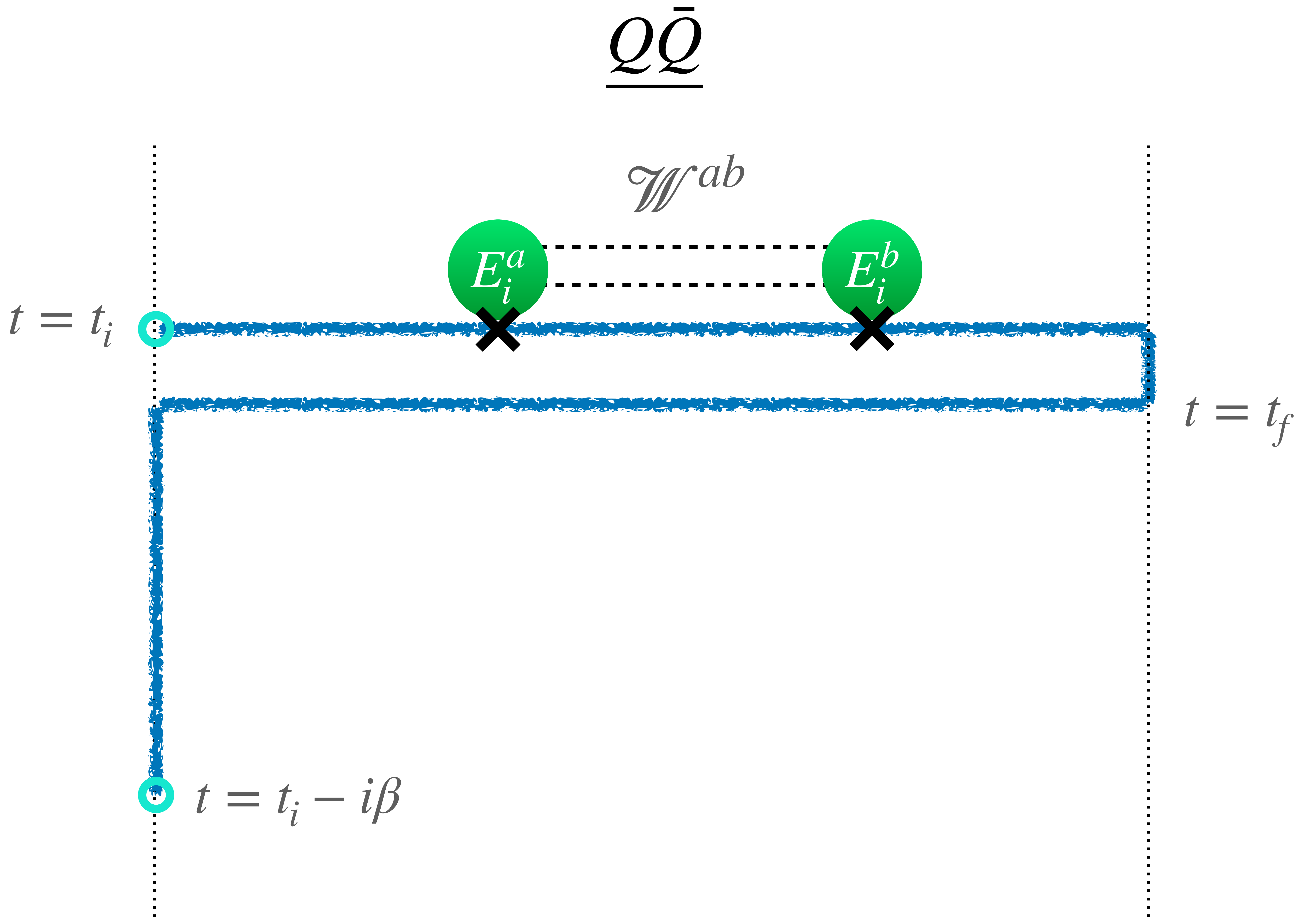

- In this correlator, the heavy quark pair is present at all times, but it is only color-charged for a finite time:
  - It is *not* part of the construction of the thermal state.
  - The adjoint Wilson line, representing the propagation of unbound quarkonium (in the adjoint representation), is only present on the real-time segment.

SK contour and Holography

Heavy quark correlator

Fluctuations are matched through the imaginary time segment solving the equations of motion $\Longrightarrow$ factors of $e^{\beta\omega}$, KMS relations

SK contour and Holography

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Fluctuations are matched through the imaginary time segment solving the equations of motion $\Longrightarrow$ factors of $e^{\beta\omega}$, KMS relations

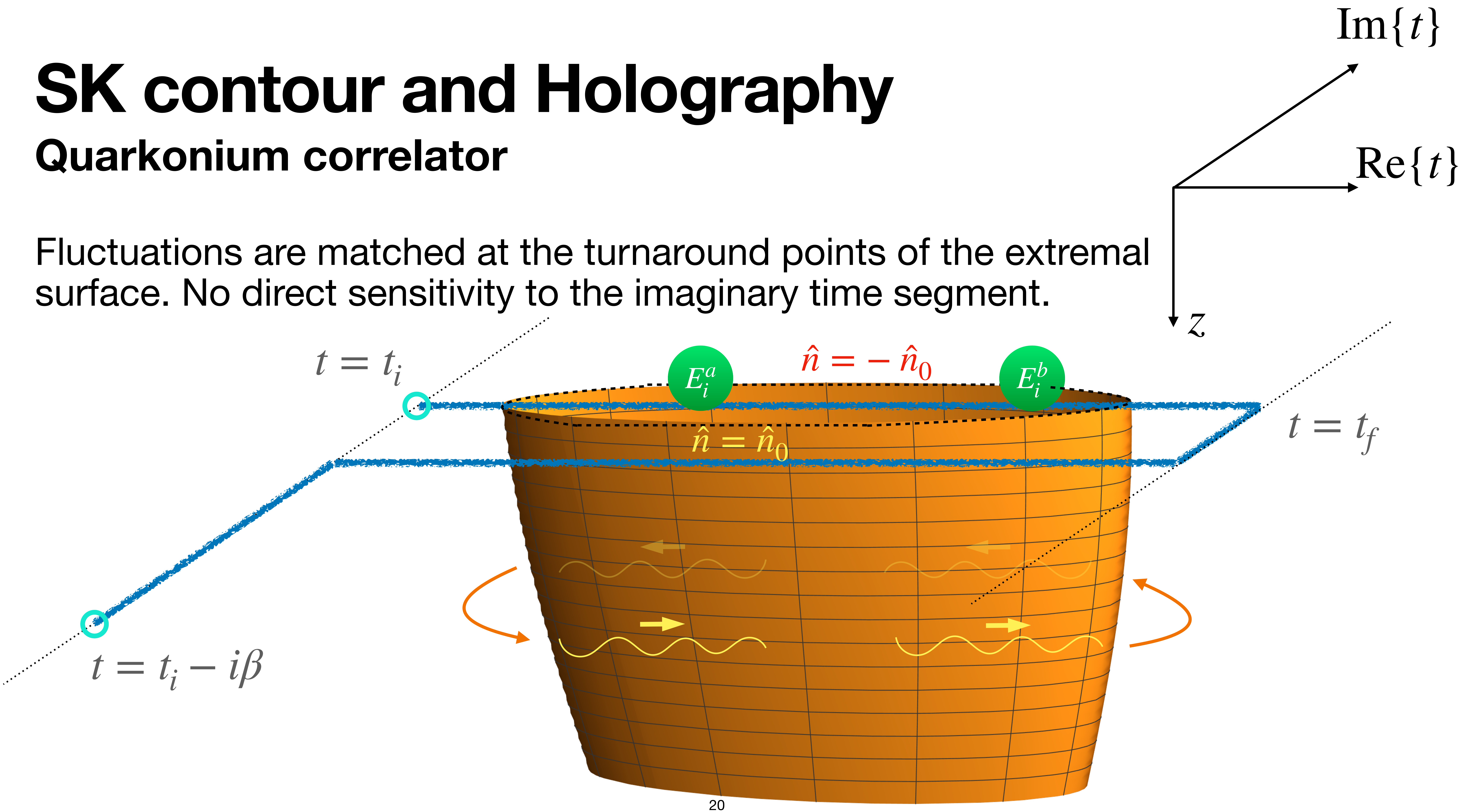
From here: $\kappa = \pi\sqrt{g^2 N_c}\, T^3$

J. Casalderrey-Solana and D. Teaney, hep-ph/0605199

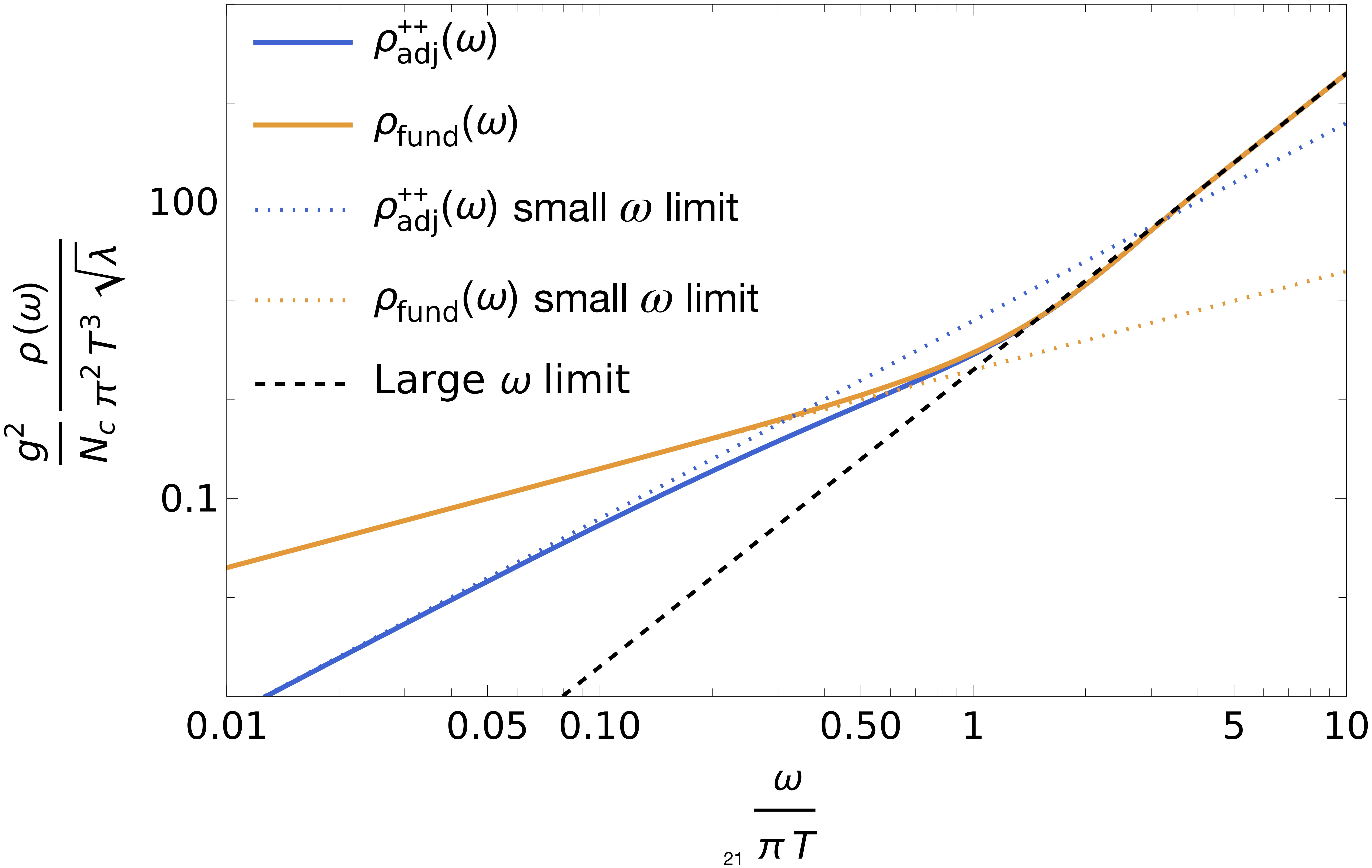
SK contour and Holography

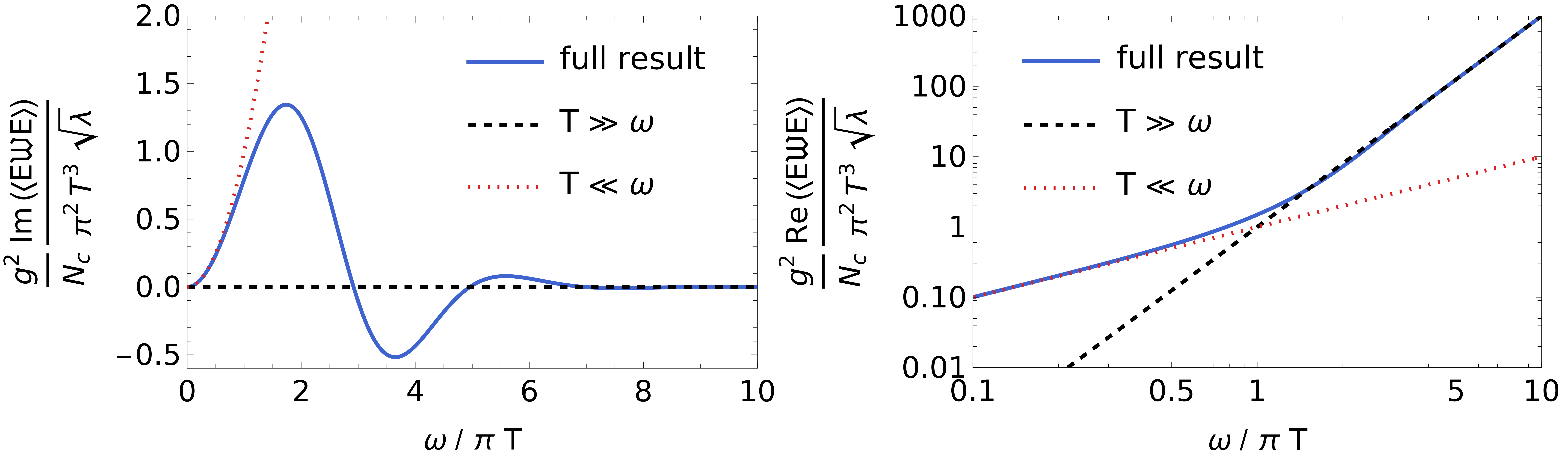
Quarkonium correlator

Fluctuations are matched at the turnaround points of the extremal surface. No direct sensitivity to the imaginary time segment.



Comparison of spectral functions





$$\gamma_{\text{adj}}^{\mathcal{N}=4} \equiv \frac{g^2}{6N_c}\text{Im}\int_{-\infty}^{\infty} ds\,\langle \mathcal{T}\, E^{a,i}(s)\mathcal{W}^{ab}_{[s,0]}E^{b,i}(0)\rangle = 0\,,$$

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2304.XXXXX G. Nijs, B. Scheihing-Hitschfeld, X. Yao

Summary and conclusions

- We have discussed how to calculate the chromoelectric correlators of the QGP that govern quarkonium transport at strong coupling in $\mathcal{N} = 4$ SYM.
  - Interesting prospects for interpolating between weak & strong coupling.
- Next steps:
  - Generalize the calculations to include a boosted medium.
  - Calculate the chromo-magnetic correlators $\langle B^a(t)\mathscr{W}^{ab}_{[t,0]}B^b(0)\rangle_T$.
  - Use them as input for quarkonia transport codes.
- Stay tuned!

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Thank you!

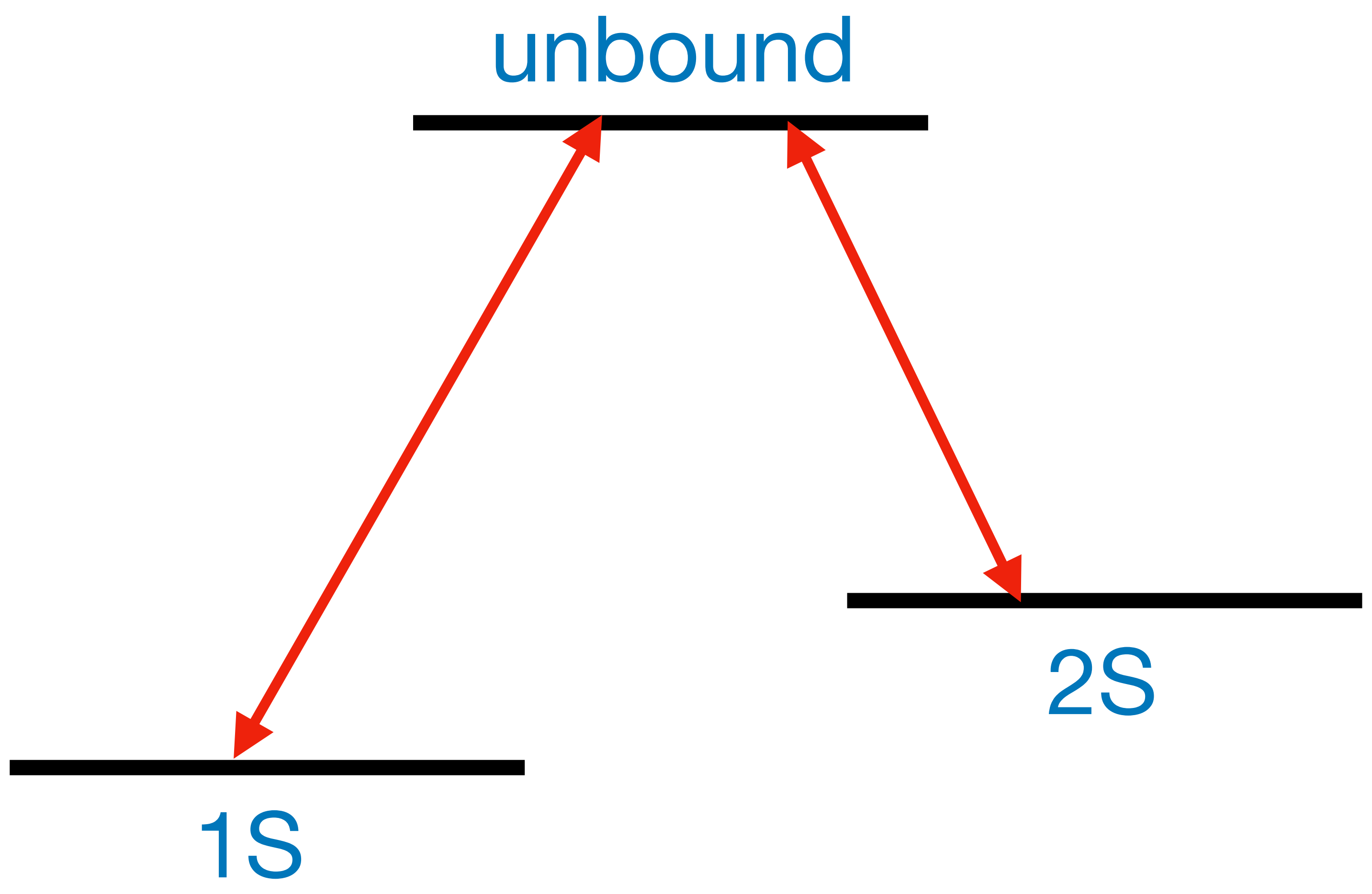
Extra slides

[*] N. Brambilla, A. Pineda, J. Soto, A. Vairo hep-ph/9707481, hep-ph/9907240, hep-ph/0410047

X. Yao, hep-ph/2102.01736

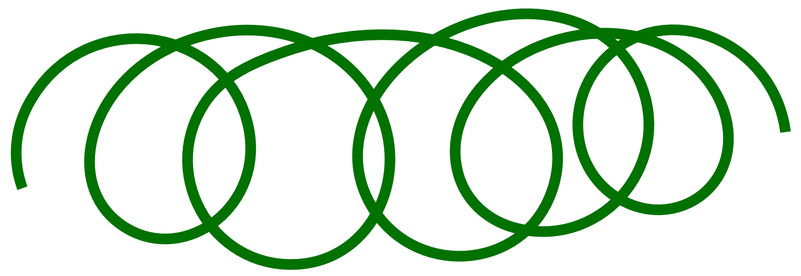
Time scales of quarkonia

Transitions between quarkonium energy levels (the system)



$$\frac{1}{\tau_S} \sim \Delta E_n \sim Mv^2$$

Interaction with the environment



$$\frac{1}{\tau_I} \sim \frac{H_{\text{int}}^2}{T} \sim T\frac{T^2}{(Mv)^2}$$

QGP (the environment)

$$\frac{1}{\tau_E} \sim T$$

$$\mathscr{L}_{\text{pNRQCD}} = \mathscr{L}_{\text{light quarks}} + \mathscr{L}_{\text{gluon}} + \int d^3r \mathrm{Tr}_{\text{color}}\Big[S^\dagger(i\partial_0 - H_s)S + O^\dagger(iD_0 - H_o)O$$

$$+V_A\big(O^\dagger \mathbf{r}\cdot g\mathbf{E}S + \mathrm{h.c.}\big) + \frac{V_B}{2}O^\dagger\{\mathbf{r}\cdot g\mathbf{E}, O\} + \cdots\Big]$$

Open quantum systems

"tracing/integrating out" the QGP

- Given an initial density matrix $\rho_{\rm tot}(t=0)$, quarkonium coupled with the QGP evolves as

$$\rho_{\rm tot}(t) = U(t)\rho_{\rm tot}(t=0)U^\dagger(t).$$

- We will only be interested in describing the evolution of quarkonium and its final state abundances

$$\implies \rho_S(t) = {\rm Tr}_{\rm QGP}\left[U(t)\rho_{\rm tot}(t=0)U^\dagger(t)\right].$$

- Then, one derives an evolution equation for $\rho_S(t)$, assuming that at the initial time we have $\rho_{\rm tot}(t=0) = \rho_S(t=0) \otimes e^{-H_{\rm QGP}/T}/\mathcal{Z}_{\rm QGP}$.

Open quantum systems

"tracing/integrating out" the QGP: semi-classic description

Unitary evolution of environment + subsystem

⇩ Trace out the environment degrees of freedom

OQS: ρ_S has non-unitary, time-irreversible evolution

⇩ Markovian approximation $\Longleftrightarrow$ weak coupling in H_I

OQS: Lindblad equation

⇩ Wigner transform: $f(\mathbf{x}, \mathbf{k}, t) \equiv \int_{k'} e^{i\mathbf{k}'\cdot\mathbf{x}} \left\langle \mathbf{k} + \frac{\mathbf{k}'}{2} \middle| \rho_S(t) \middle| \mathbf{k} - \frac{\mathbf{k}'}{2} \right\rangle$

Semi-classic subsystem: Boltzmann/Fokker-Planck equation

Lindblad equations for quarkonia at low $T \ll Mv$

quantum Brownian motion limit & quantum optical limit in pNRQCD

- After tracing out the QGP degrees of freedom, one gets a Lindblad-type equation:

$$\frac{\partial \rho}{\partial t} = -i[H_{\text{eff}}, \rho] + \sum_j \gamma_j \left(L_j \rho L_j^\dagger - \frac{1}{2}\left\{ L_j^\dagger L_j, \rho \right\} \right)$$

- This can be done in two different limits within pNRQCD:

Quantum Brownian Motion:

see works by Brambilla et al.

$$\tau_I \gg \tau_E$$
$$\tau_S \gg \tau_E$$

relevant for $Mv \gg T \gg Mv^2$

Quantum Optical:

$$\tau_I \gg \tau_E$$
$$\tau_I \gg \tau_S$$

see works by Yao et al.

relevant for $Mv \gg Mv^2, T$

Quantum Brownian Motion limit details

$$\frac{d\rho_S(t)}{dt} = -i\left[H_S + \Delta H_S, \rho_S(t)\right] + \kappa_{\text{adj}}\left(L_{\alpha i}\rho_S(t)L_{\alpha i}^\dagger - \frac{1}{2}\left\{L_{\alpha i}^\dagger L_{\alpha i}, \rho_S(t)\right\}\right)$$

$$H_S = \frac{\mathbf{p}_{\text{rel}}^2}{M} + \begin{pmatrix} -\frac{C_F\alpha_s}{r} & 0 \\ 0 & \frac{\alpha_s}{2N_c r} \end{pmatrix}, \qquad \Delta H_S = \frac{\gamma_{\text{adj}}}{2} r^2 \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix}$$

$$L_{1i} = \left(r_i + \frac{1}{2MT}\nabla_i - \frac{N_c}{8T}\frac{\alpha_s r_i}{r}\right)\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$L_{2i} = \sqrt{\frac{1}{N_c^2-1}}\left(r_i + \frac{1}{2MT}\nabla_i + \frac{N_c}{8T}\frac{\alpha_s r_i}{r}\right)\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$L_{3i} = \sqrt{\frac{N_c^2-4}{2(N_c^2-1)}}\left(r_i + \frac{1}{2MT}\nabla_i\right)\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Heavy quark and quarkonia correlators

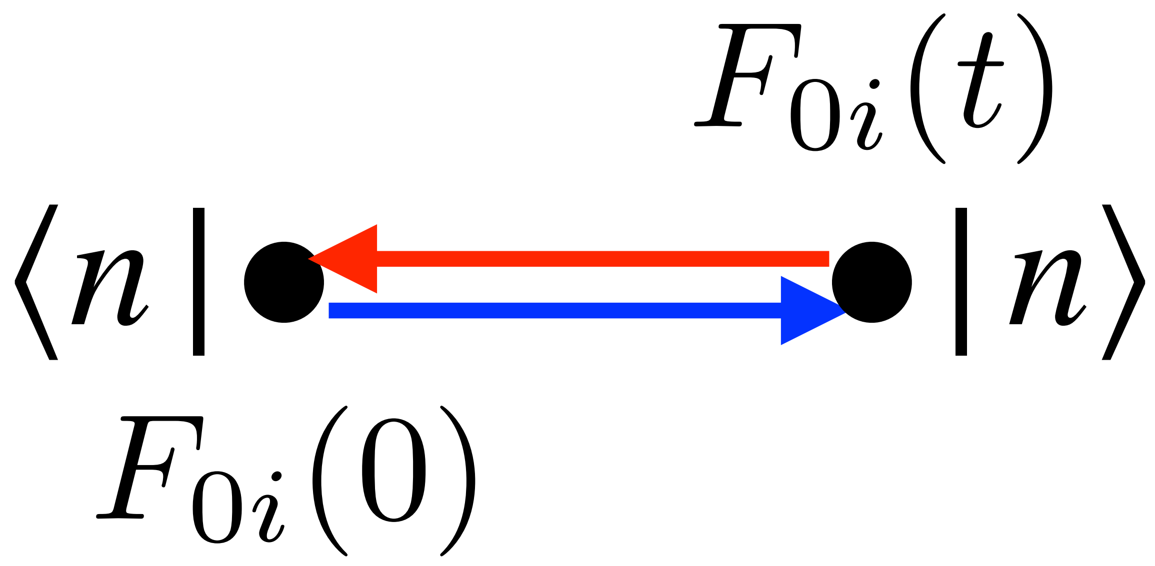
a small, yet consequential difference

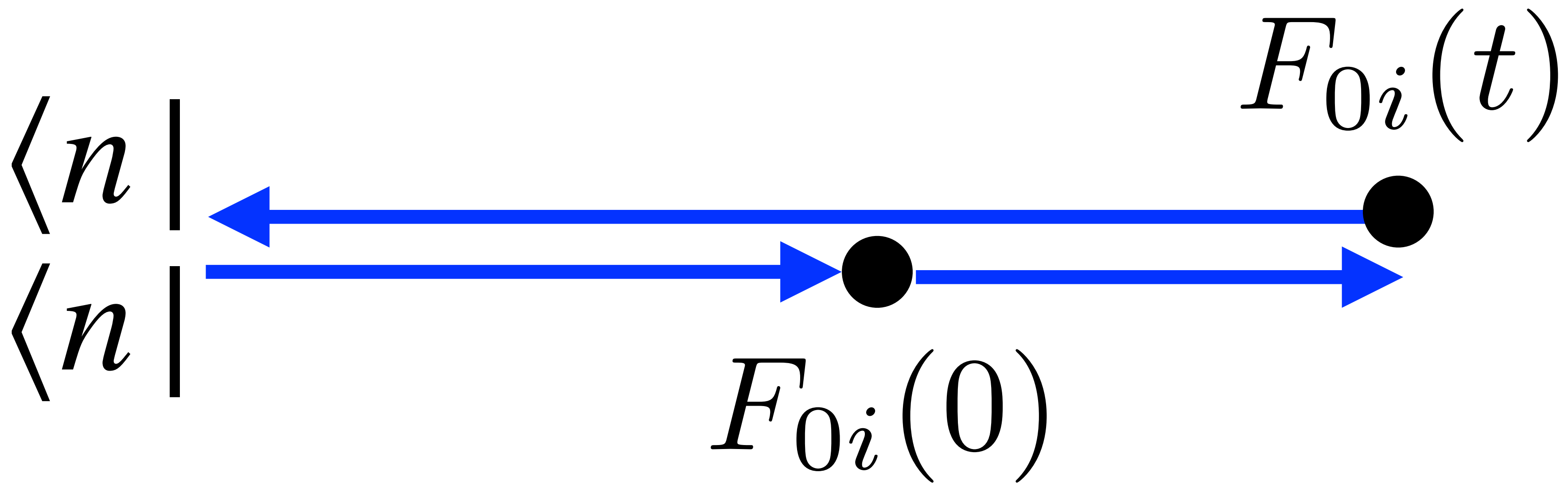
A. M. Eller, J. Ghiglieri and G. D. Moore, hep-ph/1903.08064

What we just found, and had been noticed even earlier by Eller, Ghiglieri and Moore, is simply stating that:

They compared M. Eidemuller and M. Jamin, hep-ph/9709419 with
Y. Burnier, M. Laine, J. Langelage and L. Mether, hep-ph/1006.0867

$$T_F \left\langle E_i^a(t)\mathscr{W}^{ab}(t,0)E_i^b(0)\right\rangle_T \neq \left\langle \mathrm{Tr}_{\mathrm{color}}\left[U(-\infty,t)E_i(t)U(t,0)E_i(0)U(0,-\infty)\right]\right\rangle_T$$





An axial gauge puzzle

an apparent (but not actual) inconsistency

- This finding presents a puzzle:
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BS and X. Yao, hep-ph/2205.04477

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We verified that this difference between the correlators is gauge invariant using an interpolating gauge condition:

$$G_M^a[A] = \frac{1}{\lambda}A_0^a(x) + \partial^\mu A_\mu^a(x)$$

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Wilson loops in $\mathscr{N} = 4$ SYM

a slightly different observable

- A holographic dual in terms of an extremal surface exists for

$$W_{\text{BPS}}[\mathscr{C};\hat{n}] = \frac{1}{N_c}\text{Tr}_{\text{color}}\left[\mathscr{P}\exp\left(ig\oint_{\mathscr{C}} ds\, T^a\left[A^a_\mu\,\dot{x}^\mu + \hat{n}(s)\cdot\vec{\phi}^a\sqrt{\dot{x}^2}\,\right]\right)\right],$$

which is *not* the standard Wilson loop.

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- What to do with this extra parameter? For a single heavy quark, just set $\hat{n} = \hat{n}_0$.

Choosing $\hat{n}$

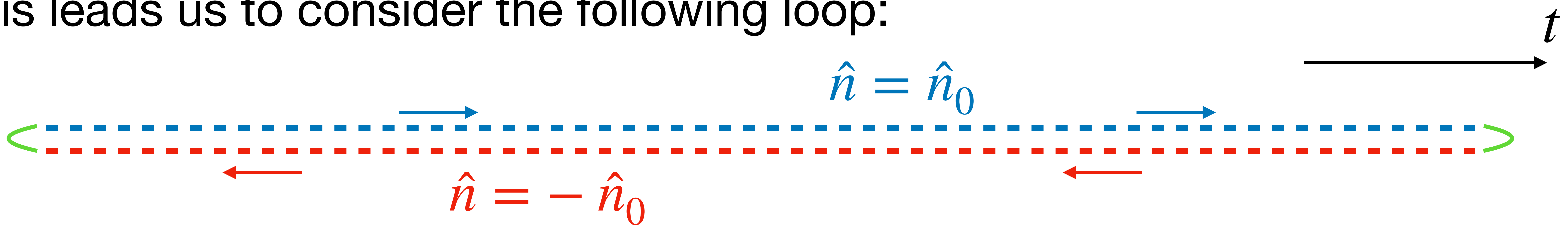
what is the best proxy for an adjoint Wilson line?

- A key property of the adjoint Wilson line is

$$\mathscr{W}^{ab}_{[t_2,t_1]} = \frac{1}{T_F}\mathrm{Tr}\left[\mathscr{T}\left\{T^a U_{[t_2,t_1]} T^b U^\dagger_{[t_2,t_1]}\right\}\right],$$

which means that we can obtain the correlator we want by studying deformations of a Wilson loop of the form $W = \dfrac{1}{N_c}\mathrm{Tr}\left[UU^\dagger\right] = 1.$

- This leads us to consider the following loop:



How the calculation proceeds

what equations do we need to solve?

- The classical, unperturbed equations of motion from the Nambu-Goto action to determine Σ:

$$S_{\mathrm{NG}} = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma \sqrt{-\det\left(g_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu\right)}\ .$$

- The classical, linearized equation of motion with perturbations in order to be able to calculate derivatives of $\langle W[\mathscr{C}_f]\rangle_T = e^{iS_{\mathrm{NG}}[\Sigma_f]}$:

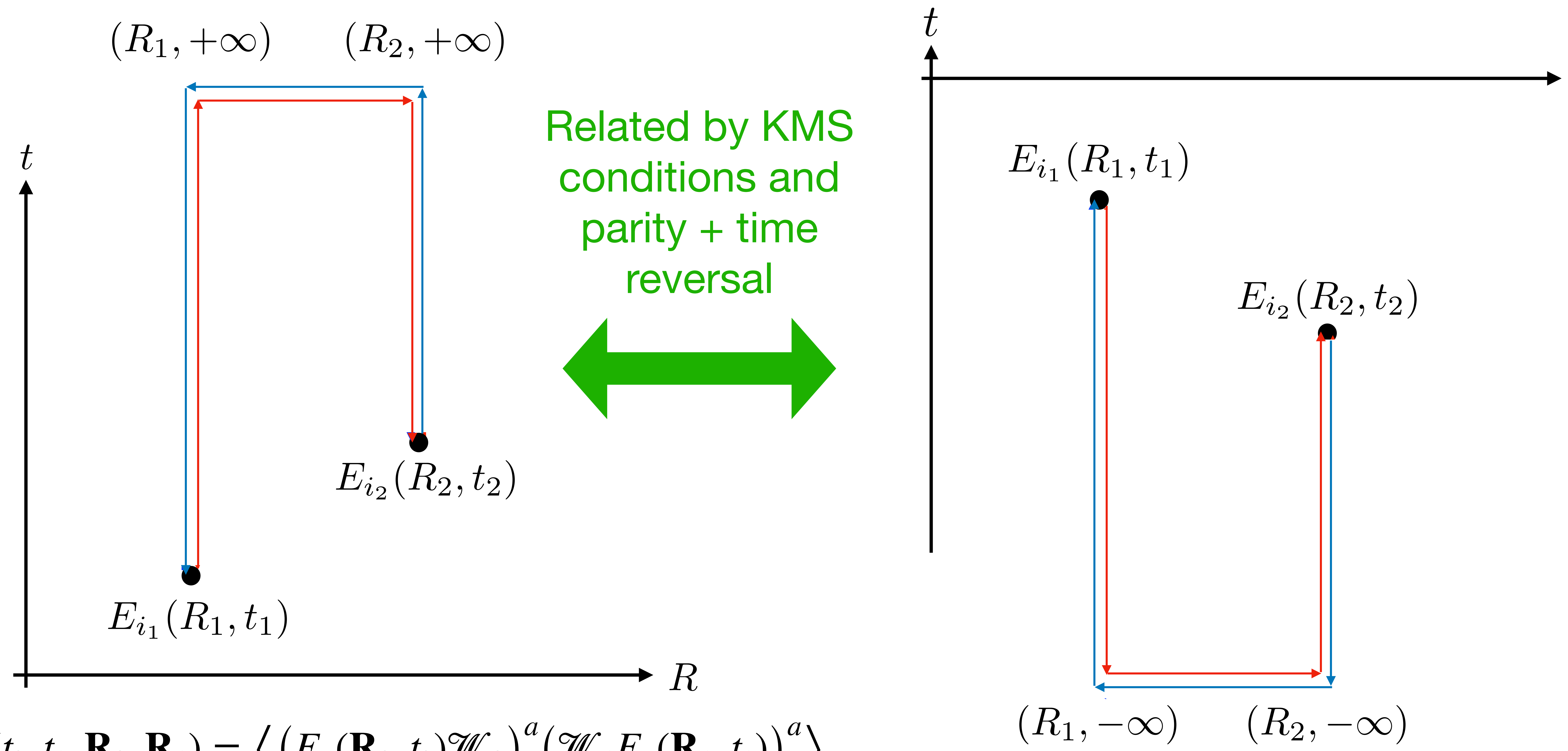
$$S_{\mathrm{NG}}[\Sigma_f] = S_{\mathrm{NG}}[\Sigma] + \int dt_1 dt_2 \left.\frac{\delta^2 S_{\mathrm{NG}}[\Sigma_f]}{\delta f(t_1)\delta f(t_2)}\right|_{f=0} f(t_1)f(t_2) + O(f^3)\ .$$

- In practice, the equations are only numerically stable in Euclidean signature, so we have to solve them and analytically continue back.

QGP chromoelectric correlators

for quarkonia transport

$$[g_E^{--}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(\mathscr{W}_{2'} E_{i_2}(\mathbf{R}_2, t_2)\right)^a \left(E_{i_1}(\mathbf{R}_1, t_1)\mathscr{W}_{1'}\right)^a \right\rangle_T$$



$$[g_E^{++}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(E_{i_2}(\mathbf{R}_2, t_2)\mathscr{W}_2\right)^a \left(\mathscr{W}_1 E_{i_1}(\mathbf{R}_1, t_1)\right)^a \right\rangle_T$$

The spectral function of quarkonia

symmetries and KMS relations

The KMS conjugates of the previous correlators are such that

$$[g_E^{++}]_{ji}^{>}(q) = e^{q^0/T}[g_E^{++}]_{ji}^{<}(q)\,, \quad [g_E^{--}]_{ji}^{>}(q) = e^{q^0/T}[g_E^{--}]_{ji}^{<}(q)\,,$$

and one can show that they are related by

$$[g_E^{++}]_{ji}^{>}(q) = [g_E^{--}]_{ji}^{<}(-q)\,, \quad [g_E^{--}]_{ji}^{>}(q) = [g_E^{++}]_{ji}^{<}(-q)\,.$$

The spectral functions $[\rho_E^{++/--}]_{ji}(q) = [g_E^{++/--}]_{ji}^{>}(q) - [g_E^{++/--}]_{ji}^{<}(q)$ are not necessarily odd under $q \leftrightarrow -q$. However, they do satisfy:

$$[\rho_E^{++}]_{ji}(q) = -[\rho_E^{--}]_{ji}(-q)\,.$$