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Comparison of Heavy quark Hadronization Models in heavy ion collisions

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In collabroation with many other heavy quark groups: Catania, Duke, LBT, Los Alamos, PHSD, TAMU, Turin

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Relativistic heavy ion collisions & Hadronization



A deconfined QCD matter — quark-gluon plasma (QGP) has been created!

Hadronization: the degree of freedom changes from quarks/gluons to hadrons

Hadronization is a non-perturbative process and can only be studied via models!

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Heavy flavor can be a nice probe:

- Mc, Mb >> Λ_{OCD} , produced by hard scattering, described by pQCD.
- Number is conserved during the evolution.
- Evolution (energy loss/gain) in the QGP is well studied.
- + Hadronization probability can be managed partly based on heavy flavor effective theory.
- Few excited states compared to light hadrons.
- The Direct and Feed-down contributions can be well separated in experiments.

Hadronization mechanisms are different in a vacuum and the hot QCD medium.

Hadronization mechanism in vacuum

✤ Fragmentation:



Fragmentation functions can be determined by the experimental data (e^+e^- , pp,...)

Hadronization in the hot QCD medium shows a huge difference compared to the vacuum case.

Hadronization mechanism in hot medium

• Enhancement of Baryon / Meson Ratio



• Quark Number Scaling of Elliptic flow



Hadronization mechanism in hot medium

Recombination:



- Enhancement of Baryon / Meson Ratio
- Quark Number Scaling of Elliptic flow

Hadronization mechanism in hot medium

Recombination + Fragmentation:



Low pT heavy flavor hadronizes via recombination, while high pT through the fragmentation! Each model with a recombination part can give a nice explanation of the experimental data!

Model comparison

Systematic studies of the parameter dependence in the various hadronization models should fix the Hadronization hypersurface and charm distribution at hadronization hypersurface.

• Given by the Fireball model for $\sqrt{s_{NN}} = 2.76 TeV Pb+Pb$ with b=7fm and T_{fo}=180MeV.

H. van Hees, V. Greco, and R. Rapp, Phys. Rev. C 73, 034913 (2006)

Nucl. Phys. A 979 (2018) 21-86.

• Uniform distribution in the coordinate space and momentum space is given by EMMI RRTF. (No Space-Momentum Correlation)



Catania, Duke, LBT, Los Alamos, Nantes, PHSD, TAMU, Turin groups/models

Model comparison

We prepared several tasks: (2021.04-2022.10)

1. Final yield H_{AA} of $D(D^+ + D^0)$, D_s and Λ_c .

$$H_{AA} = \frac{dN_D/dp_T}{dN_c/dp_T}$$

- 2. Elliptic flow v_2 of the D, D_s, Λ_c without the charm quark flow.
- 3. Elliptic flow v_2 of the D, D_s, Λ_c with the charm quark flow.
 - For pure fragmentation (assuming all c quarks proceed through fragmentation)
 - For pure recombination (assuming all c quarks proceed through recombination)
 - For mixed hadronization model (genuine process in each model)

New tasks: (2022.10-now)

Fix the parameters: $m_c = 1.5 GeV$, $m_{u/d} = 0.3 GeV$, $m_s = 0.4 GeV$, For all codes, which use the Wigner function choose $\sigma = 0.5$ fm for charmed mesons; $\sigma_{\rho} = \sigma_{\lambda} = 0.5$ fm for charmed baryons.

- 4. Final yield H_{AA} and elliptic flow v_2 of direct D^0 , D_s and Λ_c (no feed down contribution).
- 5. $dN(D^0)/dp_T$ and v_2 of direct D^0 meson produced by a c-quark with $p_T = 3GeV$ and 10GeV.

Model comparison — model description

	Frag.	Recom.	Recom. Form	Charmed hadrons involved	
Catania	Peterson	Phase space Wigner function	$W(x,p) = \prod_{i=1}^{N_q - 1} A_W \exp\left(-\frac{x_i^2}{\sigma_{ri}^2} - p_i^2 \sigma_{ri}^2\right)$	S-wave, D0,Ds, D*+,D*0,D*s,several excited states of \Lambda_c,\Sigma_c	
Duke	Pythia 6.4/ Peterson	Momentum space Wigner function	$W(p) = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-\sigma^2 p^2},$	S-wave,D,D*	
LBT	Pythia 6.4/ Peterson	Momentum space Wigner function	$W_{s}(p) = g_{h} \frac{(2\sqrt{\pi}\sigma)^{3}}{V} e^{-\sigma^{2}p^{2}},$ $W_{p}(p) = g_{h} \frac{(2\sqrt{\pi}\sigma)^{3}}{V} \frac{2}{3} \sigma^{2} p^{2} e^{-\sigma^{2}p^{2}}.$	S-wave,P-wave,D,Ds,D*, \Lambda_c,\Sigma_c,\Xi_c. \Omega_c	
Nantes	HQET	Phase space Wigner function	$W(x_Q, x_q, p_Q, p_q) = \exp\left(\frac{(x_q - x_Q)^2 - [(x_q - x_Q) \cdot u_Q]^2}{2R_c^2} - \alpha_d^2(u_Q \cdot u_q - 1)\right)$	S-wave, D0	
PHSD	Peterson	Phase space Wigner function	$W_s(r,p) = \frac{8(2S+1)}{36}e^{-\frac{r^2}{\sigma^2}-\sigma^2 p^2},$ $W_p(r,p) = \frac{2S+1}{36}\left(\frac{16}{3}\frac{r^2}{\sigma^2} + \frac{16}{3}\sigma^2 p^2 - 8\right)e^{-\frac{r^2}{\sigma^2}-\sigma^2 p^2},$	S-wave, P-wave D+,D0,Ds, D*+,D*0,D*s	
TAMU	thermal density correlated HQET	Resonance amplitude	$\frac{\gamma_M}{\Gamma} v_{rel} g_\sigma \frac{4\pi}{k^2} \frac{(\Gamma m)^2}{(s-m^2)^2 + (\Gamma m)^2}$	D+,D0,Ds and few excited states. Charm baryons+missing baryons	
Turin	Pythia 6.4/ String fragmentation	Invariant mass criterion	$M_D < M_{Cluster} < M_{max.}$	(prompt) D+,D0,Ds,\Lambda_c, \Xi_c,\Omega_c	
Los Alamos	HQET	_	_	S-wave, D+,D0,Ds, charm- baryons	

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Nantes	HQET	Phase space Wigner function	$W(x_Q, x_q, p_Q, p_q) = \exp\left(\frac{(x_q - x_Q)^2 - [(x_q - x_Q) \cdot u_Q]^2}{2R_c^2} - \alpha_d^2(u_Q \cdot u_q - 1)\right)$	S-wave, D0
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Los Alamos	HQET	_	_	S-wave, D+,D0,Ds, charm- baryons

Model comparison — Fragmentation function

There are mainly three kinds of fragmentation function used in these models



HQET fragmentation function (for the pseudoscalar and vector meson):

$$\mathcal{D}_{c \to P} \propto \frac{rz(1-z)^2}{[1-(1-r)z]^6} \Big[6 - 18(1-2r)z + (21 - 74r + 68r^2)z^2 - 2(1-r)(6 - 19r + 18r^2)z^3 + 3(1-r)^2(1-2r+2r^2)z^4 \Big]$$

$$\mathcal{D}_{c \to V} \propto \frac{rz(1-z)^2}{[1-(1-r)z]^6} \Big[2 - 2(3-2r)z + 3(3-2r+4r^2)z^2 - 2(1-r)(4-r+2r^2)z^3 + 3(1-r)^2(3-2r+2r^2)z^4 \Big]$$

r = 0.1 in Nantes and TAMU models; r = 0.2 in Los Alamos model.

Model comparison — model description

	Frag.	Recom.	Recom. Form	Charmed hadrons involved
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Nantes	HQET	Phase space Wigner function	$W(x_Q, x_q, p_Q, p_q) = \exp\left(\frac{(x_q - x_Q)^2 - [(x_q - x_Q) \cdot u_Q]^2}{2R_c^2} - \alpha_d^2(u_Q \cdot u_q - 1)\right)$	S-wave, D0
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Turin	Pythia 6.4/ String fragmentation	Invariant mass criterion	$M_D < M_{Cluster} < M_{max.}$	(prompt) D+,D0,Ds,\Lambda_c, \Xi_c,\Omega_c
Los Alamos	HQET	_		S-wave, D+,D0,Ds, charm- baryons

Model comparison – Recombination probability

There are mainly two kinds of recombination processes:

Phase space criterion:

Catania, Nantes, and PHSD model, phase-space Wigner function.

Momentum space criterion:

(Charm and light quark are at same point)

Duke and LBT model, momentum-space Wigner function.

TAMU model, Resonance amplitude, which is only related to the momentum of heavy and light quarks

Turin model, invariant mass, which is only related to the momentum of heavy and light quarks



Model comparison – Recombination probability



- Total recombination probability ~1.0 at zero p_T required by all charm quarks hadronize via recombination at $p_T \sim 0$.
- Huge difference when p_T > 3 GeV;
 Phase space criterion give a steep recombination probability ?

1. Yield
$$H_{AA} = \frac{dN_H/dp_T}{dN_c/dp_T}$$



Model comparison – H_{AA}

The large difference may come from the branching ratios between various charmed-hadrons

$$R = \frac{\int dN_c/dp_T \times H_{AA}dp_T}{\int dN_c/dp_T dp_T} \qquad H_{AA}^r \equiv \frac{dN_H/dp_T}{RdN_c/dp_T}$$

	Fragmentation			Recombination			Mixed		
R	D	D_s	Λ_c	D	D_s	Λ_c	D	D_s	Λ_c
Catania	78.3%	8.0%	13.7%	-	-	-	48.8%	6.8%	24.3%
Duke	100%	-	-	100%	-	-	100%	-	-
LBT	37.8%	5.4%	3%	50.3%	14.6%	20.8%	54.7%	12.1%	15.3%
Nantes	100%	-	-	100%	-	-	100%	-	-
Nantes(new)	100%	-	-	100%	-	-	100%	-	-
PHSD	81%	10%	-	67%	33%	-	75%	20%	-
TAMU	60.7%	11.5%	24.1%	-	-	-	50.2%	16.2%	22.8%
Turin	-	-	-	-	-	-	50.6%	17.9%	20.4%
Vitev	77.8%	10%	11.9%	-	-	-	-	-	-

Model comparison – H_{AA}^r



Model comparison – H_{AA}^r



Model comparison – H_{AA}^r





2. Yield ratio



TAMU model gives a larger Λ_c/D^0 ratio than others; may be caused by "missing" baryons

Reflects the number of charmed meson and baryons involved!

3. Elliptic flow v_2

PHSD — TAMU — LBT (New) Nantes (EMMI) — Vitev — Duke Nantes (New) — Catania C-quark

Include strong decays



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- $v_2(pure \ fragmentation) \approx v_2(charm)$
- $v_2(mixed) > v_2(pure \ recombination)$ in each model !
- $v_2(\Lambda_c, mixed) > v_2(D_s, mixed) > v_2(D, mixed)$



 $v_2(mixed \ fragmentation) \approx v_2(charm) < v_2(mixed) < v_2(mixed \ recombination)$

• For a steep recombination probability, the fragmentation domintate the hadronization at almost all pt regions.

 $v_2(mixed) \approx v_2(mixed \ fragmentation) \approx v_2(charm)$

• For a given meson (such as many resonance states, D*, Ds*,.., charmed baryons), the fragmentaion part dissapeared or is very weak. Then:

 $v_2(mixed) \approx v_2(mixed recombination)$



→ If the recombination probability is p_T – independent : $v_2(mixed recombination) \approx v_2(pure recombination)$



→ If the recombination probability is p_T -dependent: $v_2(mixed recombination) > v_2(pure recombination)$ A given p_T of D meson mainly from a recombination of softer charm and harder light quark in the mixed process; light quarks carry large v2!

• $v_2(mixed) > v_2(pure \ recombination)$

4. dN_D/dp_T of D meson

Model comparison $-dN_D/dp_T$

 dN_D/dp_T of the direct D^0 meson produced by a *c*-quark with $p_T = 3GeV$ and 10GeV.



- 3GeV charm quark hadronizes via fragmentation and recombination, but 10GeV charm hadronizes almost via fragmentation.
- Peak is broadening around 10GeV than 3GeV



Summary

Heavy flavor is a nice probe to study the hadronization mechanism in HIC! Comparing different models is essential to understand the hadronization mechanism!

The hadronization model used by several groups are reviewed.

We prepared several tasks for different groups with the same hadronization hypersurface and charm distribution functions at hadronization hypersurface. After preliminary comparison, we get the following take-home messages so far:

- Hadronization changes the p_T spectra substantially, $p_T^c \neq p_T^D$.
- H_{AA} of charmed hadrons as a function of p_T strongly depends on both the fragmentation function and recombination probability.
- The prompt yield ratio is sensitive to the number of resonances involved.
- The mangnitude of hadron v_2 comes from: charm quark v_2 , light quark v_2 , recombination probability, and also fragmentaion ratio! The p_T -dependent recombination probability has an important influence on v_2 ! The existence of v_2 sequence: $v_2(\Lambda_c) > v_2(D_s) > v_2(D)$.

What next ?

- Finish the data collection and find more physics behind it.
- Considering the SMC and energy conservation effect in each model.

Hard Probes 2023, Aschaffenburg (Germany), 26-31 March.



Thanks for your attention!

Model comparison

So far, what we get:

	Pure Frag.	Pure Recom.	Mixed	$D(D^0 + D^+)$	D_s	Λ_c	New tasks
Catania		×					
Duke					×	×	×
LBT							
Nantes(new)					×	X	
Nantes					X	X	X
PHSD						X	
TAMU		X					X
Turin	X	X					X
Vitev		X	X			charm- baryons	

Related references:

S. Plumari, V. Minissale, S.K. Das, G. Coci, and V. Greco, Eur.Phys.J.C 78 (2018) 4, 348, V. Minissale, S. Plumari[,] V. Greco, Phys.Lett.B 821 (2021) 136622.

Y. Xu, S. Cao, M. Nahrgang, W. Ke, G. Qin, J. Auvinen, and S. Bass, Nucl.Part.Phys.Proc. 276-278 (2016) 225-228. S. Cao, G. Qin, and S. Bass, Phys. Rev. C92, 024907 (2015).

S. Cao, K. Sun, S. Li, S. Liu, W. Xing, G. Qin, and C. Ko, Phys.Lett.B 807 (2020) 135561. F. Liu, W. Xing, X. Wu, G. Qin, S. Cao, and X. Wang, Eur.Phys.J.C 82 (2022) 4, 350.

M. Nahrgang, J. Aichelin, P.B. Gossiaux, and K. Werner, Phys.Rev.C 93 (2016) 4, 044909, P.B. Gossiaux, R. Bierkandt and J. Aichelin, Phys.Rev.C 79 (2009) 044906.

T. Song, H. Berrehrah, D. Cabrera, J. M. Torres-Rincon, L. Tolos, W. Cassing and E. Bratkovskaya, Phys. Rev. C 92, no.1, 014910 (2015). T. Song, H. Berrehrah, D. Cabrera, W. Cassing and E. Bratkovskaya, Phys. Rev. C 93, no.3, 034906 (2016).

M. He, R. Rapp, Phys.Rev.Lett. 124 (2020) 4, 042301. M. He, R. J. Fries, and R. Rapp, Phys. Rev. C86, 014903 (2012)

A. Beraudo, A. De Pace, M. Monteno, M. Nardi and F. Prino, Eur.Phys.J.C 82 (2022) 7, 607. JHEP 05 (2021) 279.

H.T.Li, Z.L.Liu and I.Vitev, Phys. Lett. B 816,136261(2021), Z.B.Kang, R.Lashof Regas, G.Ovanesyan, P.Saad and I.Vitev, Phys. Rev. Lett. 114,no.9,092002(2015)

Model comparison – H_{AA}

Direct (no resonance decay)



PHSD

Nantes (EMMI)

Nantes (New)

LBT (New)

Duke

Turin

TAMU

Catania

Vitev





Turn off the charm flow (isotropic charm quark)





 p_T (GeV)